# Section 8: Electronic Transport

Drude model

The simplest treatment of the electrical conductivity was given by Drude. There are four major assumptions within the Drude model.

1. Electrons are treated as classical particles within a free-electron approximation. Thus, in the absence of external electromagnetic fields each electron is taken to move uniformly in a straight line, neglecting the interactions with other electrons and ions. In the presence of external fields each electron is taken to move according to Newton's laws of motion.

2. Electrons move free only between collisions with scattering centers. Collisions, as in kinetic theory, are instantaneous events that abruptly alter the velocity of an electron. Drude attributed them to the electrons scattering by ion cores. However, as we will see later, this is not a correct picture of electron scattering on ordered periodic structures. A particular type of scattering centers does not matter in the Drude model. An understanding of metallic conduction can be achieved by simply assuming that there is *some* scattering mechanism, without inquiring too closely into just what that mechanism might be.
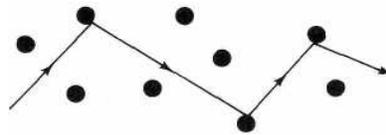

Fig.1 Trajectory of a conduction electron scattering off the ions, according to the picture of Drude.

3. An electron experiences a collision, resulting in an abrupt change in its velocity, with a probability per unit time $1/\tau$. This implies that the probability of an electron undergoing a collision in any infinitesimal time interval of length dt is just dt/τ. The time τ is therefore an average time between the two consecutive scattering events. It known as, the *collision time* (relaxation time), it plays a fundamental role in the theory of metallic conduction. It follows from this assumption that an electron picked at random at a given moment will, on the average, travel for a time τ before its next collision. The relaxation time τ is taken to be independent of an electron's position and velocity.

4. Electrons are assumed to achieve thermal equilibrium with their surroundings only through collisions. These collisions are assumed to maintain local thermo-dynamic equilibrium in a particularly simple way: immediately after each collision an electron is taken to emerge with a velocity that is not related to its velocity just before the collision, but randomly directed and with a speed appropriate to the temperature prevailing at the place where the collision occurred.

Now we consider the application of the Drude model for electrical conductivity in a metal.

According to *Ohm's law,* the current I flowing in a wire is proportional to the potential drop $V=V_2-V_1$ along the wire: $V = IR,$ where $R,$ the resistance of the wire, depends on its dimensions. It is much more convenient to express the *Ohm's law* in a form which is independent of the dimensions of the wire because these factors are irrelevant to the basic physics of the conduction.

Fig.2
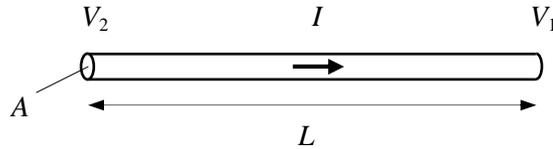
We define the conductivity which is the proportionality constant between the current density **j** and the electric field **E** at a point in the metal:

$$\mathbf{j} = \sigma\mathbf{E} \tag{1}$$

The current density **j** is a vector, parallel to the flow of charge, whose magnitude is the amount of charge per unit time crossing a unit area perpendicular to the flow. Thus if a uniform current I flows through a wire of length L and cross-sectional area A, the current density will be $j = I/A$. Since the potential drop along the wire will be $V = EL$, Eq. (1) gives $I/A = \sigma V/L$, and hence $R = L/\sigma A = \rho L/A$, where we introduced resistivity $\rho = 1/\sigma$. Unlike R, σ and ρ, is a property of the material, since it does not depend on the shape and size.

Now we want to express σ is terms of the microscopic properties using the Drude model. If n electrons per unit volume all move with velocity **v**, then the current density they give rise to will be parallel to **v**. Furthermore, in a time dt the electrons will advance by a distance vdt in the direction of v, so that $n(\text{v}dt)A$ electrons will cross an area A perpendicular to the direction of flow. Since each electron carries a charge $-e$, the charge crossing A in the time dt will be $-ne\text{v}Adt$, and hence the current density is

$$\mathbf{j} = -ne\mathbf{v}. \tag{2}$$

At any point in a metal, electrons are always moving in a variety of directions with a variety of thermal energies. The net current density is thus given by Eq.(2), where **v** is the average electronic velocity or *drift velocity*. In the absence of an electric field, electrons are as likely to be moving in any one direction as in any other, **v** averages to zero, and, as expected, there is no net electric current density. In the presence of a field **E**, however, there will be a drift velocity directed opposite to the field (the electronic charge being negative), which we can compute as follows.

Consider a typical electron at time zero. Let t be the time elapsed since its last collision. Its velocity at time zero will be its velocity $\mathbf{v}_0$ immediately after that collision plus the additional velocity $-eEt/m$ it has subsequently acquired. Since we assume that an electron emerges from a collision in a random direction, there will be no contribution from $\mathbf{v}_0$ to the average electronic velocity, which must therefore be given entirely by the average of $-eEt/m$. However, the average of t is the relaxation time τ. Therefore

$$\mathbf{v} = -\frac{e\mathbf{E}\tau}{m}, \tag{3}$$

$$\mathbf{j} = -\frac{ne^2\tau}{m}\mathbf{E}. \tag{4}$$

The conductivity is, therefore, given by

$$\sigma = \frac{ne^2\tau}{m} \tag{5}$$

We see that the conductivity is proportional to the density of electrons, which is not surprising since the higher the number of carriers, the more the current density. The conductivity is inversely proportional to the mass because the mass determine the acceleration of an electron in electric field. The proportionality to τ follows because τ is the time between two consecutive collisions. Therefore, the larger τ is, the more time for electron to be accelerated between the collisions and consequently the larger the drift velocity.

The values of relaxation time can be obtained from the measured values of electrical conductivity. For example at room temperature the resistivity of many metals lies in the range of 1-10 $\mu\Omega$cm. The corresponding relaxation time is of the order of 10^{-14}s.

In this discussion of electrical conductivity we treated electrons on a classical basis. How are the results modified when the quantum mechanics is taken into account? Let us refer to Fig.3. In the absence of an electric field, the Fermi sphere is cantered at the origin (Fig. 3a). The various electrons are all moving - some at very high speeds - and they carry individual currents. But the total current of the system is zero, because, for every electron at velocity **v** there exists another electron with velocity -**v**, and the sum of their two currents is zero. Thus the total current vanishes due to pair wise cancellation of the electron currents.
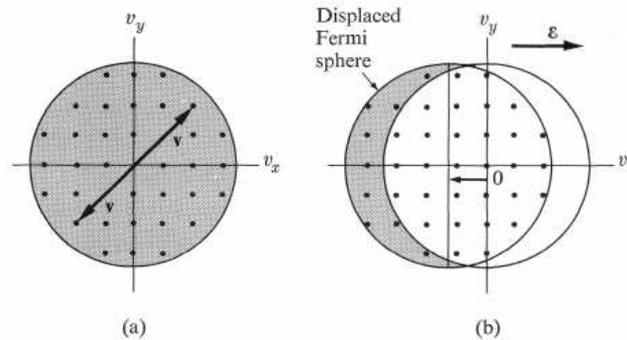
Fig. 3 (a) The Fermi sphere at equilibrium, (b) Displacement of the Fermi sphere due to an electric field.

The situation changes when a field is applied. If the field is in the positive x-direction, each electron acquires a drift velocity, as given by Eq.(3). Thus the whole Fermi sphere is displaced to the left, as shown in Fig.3(b). Although the displacement is very small, and although the great majority of the electrons still cancel each other pairwise, some electrons - in the shaded crescent in the figure - remain uncompensated. It is these electrons which produce the observed current.

The very small displacement is due to a relatively small drift velocity. If we assume that the electric field is 0.1V/cm, we obtain the drift velocity of 1cm/s, which is by 8 orders in magnitude smaller the Fermi velocity of electrons.

Let us estimate the current density: The fraction of electrons which remain uncompensated is approximately v/v_F. The concentration of these electrons is therefore $n(v/v_F)$, and since each electron has a velocity of approximately v_F, the current density is given by

$$j = -en(v/v_F)v_F = -nev.$$ (6)

This is the same expression we obtained before. Therefore, formally the conductivity is expressed by the same formula (5). However, the actual picture of electrical conduction is thus quite different from the classical one. In the classical picture, we assumed that the current is carried equally by all electrons, each moving with a very small drift velocity v. In the quantum-mechanical picture the current is carried only by very small fraction of electrons, all moving with the Fermi velocity. The relaxation time is determined only by electrons at the Fermi surface, because only these electrons can contribute to the transport properties. Both approaches lead to the same result, but the latter is conceptually the more accurate.

Since only electrons at the Fermi surface contribute to the conductance, we can define the mean free path of electrons as $l = \tau v_F$. We can make an estimate of the mean free path for metal at room temperature. This estimate gives a value of 100Å. So it is of the order of a few tens interatomic distances. At low temperatures for very pure metals the mean free path can be made as high as a few cm.

The origin of collision time

We see that between two collisions, the electron travels a distance of more than 20 times the interatomic distance. This is much larger than one would expect if the electron really *did* collide with the ions whenever it passed them. This paradox can be explained only using quantum concepts according to which an electron has a wave character. It is well known from the theory of wave propagation in periodic structures that, when a wave passes through a periodic lattice, it continues propagating indefinitely without scattering. The effect of the atoms in the lattice is to absorb energy from the wave and radiate it back, so that the net result is that the wave continues without modification in either direction or intensity.

Therefore we see that, if the ions form a perfect lattice, there is no collision at all - that is, $l = \infty$ - and hence $\tau = \infty$, which in turn leads to infinite conductivity. It has been shown, however, that the observed l is about 10^2 A. The finiteness of σ must thus be due to the deviation of the lattice from perfect periodicity; this happens either because of (1) thermal vibration of the ions, or because of (2) the presence of imperfections or foreign impurities.
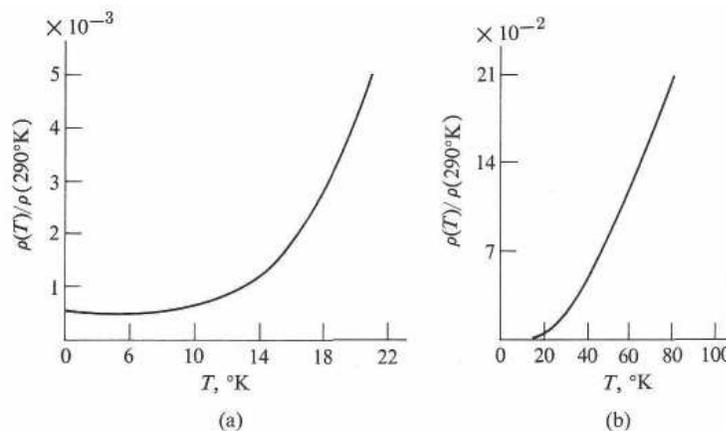
Fig. 4 The normalized resistivity $\rho(T)/\rho(290°K)$ versus T for Na in the low-temperature region (a), and at higher temperatures (b). $\rho (290°K) \sim 2\mu\Omega$cm.

In order to consider their contribution we examine the temperature dependence of the electrical conductivity. The electrical conductivity of a metal varies with temperature in a characteristic manner. This variation is usually discussed in terms of the behavior of the resistivity ρ versus T. Figure 4 shows the observed curve for Na. At $T \sim 0°K$, ρ has a small *constant* value; above that, ρ increases with T, slowly at first, but afterward ρ increases linearly with T. The linear behavior continues essentially until the melting point is reached. This pattern is followed by most metals, and usually room temperature falls into the linear range.

We want to explain this behavior of in terms of the Drude formula. Recalling that $\rho = \sigma^{-1}$, we have

$$\rho = \frac{m}{ne^2\tau}. \tag{7}$$

As we have discussed earlier $1/\tau$ which enters equation (7), is the probability of the electron scattering per unit time. Thus, if $\tau = 10^{-14}$s, then the electron undergoes 10^{14} collisions in one second. We found that the electron undergoes collisions only because the lattice is not perfectly regular. We group the deviations from a perfect lattice into two classes.

a) Lattice vibrations (phonons) of the ions around their equilibrium position due to thermal excitation of the ions.

b) All static imperfections, such as impurities or crystal defects. Of this latter group we shall take impurities as an example.

The total probability for an electron to be scattered in a unit time is the sum of the probabilities of scattering by phonons and by impurities. This is because these two mechanisms are assumed to act independently. Therefore we may write

$$\frac{1}{\tau} = \frac{1}{\tau_i} + \frac{1}{\tau_{ph}} \tag{8}$$

where the first term on the right is due to impurities and the second is due to phonons. The scattering by impurities is essentially independent of temperature, whereas the scattering by phonons is temperature dependent because the number of phonons increases with temperature. When (8) is substituted into (7), we readily find

$$\rho = \rho_i + \rho_{ph} = \frac{m}{ne^2\tau_i} + \frac{m}{ne^2\tau_{ph}} \tag{9}$$

We see that ρ has split into two terms. A term ρ_i due to scattering by impurities, which is independent of T, is called the *residual resistivity*. Another term, $\rho_{ph}(T)$, is due to scattering by phonons; hence it is temperature dependent. Sometimes it is called the *lattice resistivity*.

At very low T, scattering by phonons is negligible because the amplitudes of oscillation are very small; in that region $\tau_{ph} \to \infty$, $\rho_{ph} \to 0$, and hence $\rho = \rho_i$ is a constant. This is in agreement with Fig.4. As T increases, scattering by phonons becomes more effective, and $\rho_{ph}(T)$ increases; this is why ρ increases. When T becomes sufficiently large, scattering by phonons dominates and $\rho \sim \rho_{ph}(T)$. The statement that ρ can be split into two parts, is known as the *Matthiessen rule*. This rule is embodied in (9).

In general, the *Matthiessen rule* predicts that if there are two distinguishable sources of scattering (like in the case above – phonons and impurities) the resistivity is the sum of the resistivities due to the first and the second mechanism of scattering. The Mathissen rule is sort of empirical observation which can be used for a qualitative understanding of the contribution from different scattering mechanisms. However, one must always bear in mind the possibility a failure of this rule. In particular, in the case when the relaxation time depends on the wavevector **k**, the Matthissen rule becomes invalid.

Now let us derive approximate expressions for τ_i and τ_{ph} using arguments from the kinetic theory of gases. Consider first the collision of electrons with impurities. We write

$$\tau_i = \frac{l_i}{v_F}, \tag{10}$$

where l_i is the mean free path for collision with impurities. In order to find the mean free path we shall assume, for simplicity, that the collision is of the hard-spheres (billiard-ball) type and introduce the *scattering cross section* of an impurity Σ_i, which is the area an impurity atom presents to the incident electron. Then, we can argue that the product of the mean free path and the cross section of impurity, $l_i\Sigma_i$, is equal to the average volume per impurity, $1/n_i$, where n_i is the impurity concentration, i.e.

$$l_i\Sigma_i = \frac{1}{n_i}, \tag{11}$$

and therefore

$$l_i = \frac{1}{n_i\Sigma_i}. \tag{12}$$

The scattering cross section Σ_i is of the same magnitude as the actual geometrical area of the impurity atom. That is, that $\Sigma_i \sim 1\text{Å}^2$. Calculations of the exact value of Σ_i require quantum scattering theory. By substituting Eqs. (12) and (10) into (9), we find

$$\rho_i = \frac{mv_F}{ne^2} n_i\Sigma_i. \tag{13}$$

As expected, ρ_i is proportional to n_i the concentration of impurities.

Calculating ρ_{ph} is much more difficult, but equations similar to (10) and (12) still hold. In particular, one may write

$$l_{ph} = \frac{1}{n_a\Sigma_a}, \tag{14}$$

where n_a is the concentration of the host atoms in the lattice, and Σ_a is the scattering cross section per atom. We should note here that Σ_a has no relation to the geometrical cross section of the atom. Rather it is the area presented by the thermally fluctuating atom to the passing electron. Suppose that the distance of deviation from equilibrium is x, then the average scattering cross section is

$$\Sigma_a \propto \langle x^2 \rangle, \tag{15}$$

where $\langle x^2 \rangle$ is the average of x^2. We can easily estimate this value at high temperatures, when the classical approach is valid. Since the ion is a harmonic oscillator, the value $\langle x^2 \rangle$ is proportional to the average of its potential energy is equal to half the total energy. Thus,

$$\Sigma_a \propto \langle x^2 \rangle \propto \frac{k_B}{2C} T, \tag{16}$$

where C is interatomic force constant introduced earlier and we used the formula for the energy of a classical oscillator. We see therefore that at high temperatures the resistivity is linear in T,

$$\rho_{ph} \propto \frac{m v_F n_a}{ne^2} \frac{k_B}{2C} T. \tag{17}$$

which is in agreement with experiment.

In the low-temperature range the lattice resistivity varies with temperature in a different way. Using the Debye model at low temperature range one can find that $\rho_{ph} \sim T^5$.
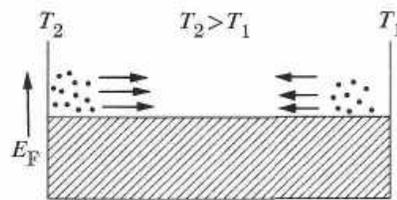
Thermal conductivity

When the ends of a metallic wire are at different temperatures, heat flows from the hot to the cold end. The basic experimental fact is that the heat current density, j_Q, i.e. the amount of thermal energy crossing a unit area per unit time is proportional to the temperature gradient,

$$j_Q = -K \frac{dT}{dx} \tag{18}$$

where K is the thermal conductivity. In insulators, heat is carried entirely by phonons, but in metals heat may be transported by both electrons and phonons. The thermal conductivity K is therefore equal to the sum of the two contributions,

$$K = K_e + K_{ph} \tag{19}$$

where K_e and K_{ph} refer to electrons and phonons, respectively. In most metals, the contribution of the electrons greatly exceeds that of the phonons, because of the great concentration of electrons. Typically $K_e \sim 10^2 K_{ph}$.

Fig. 5 The physical basis for thermal conductivity. Energetic electrons on the left carry net energy to the right.

The physical process by which heat conduction takes place via electrons is illustrated in Fig.5. Electrons at the hot end (to the left) travel in all directions, but a certain fraction travel to the right and carry energy to the cold end. Similarly, a certain fraction of the electrons at the cold end (on the

right) travel to the left, and carry energy to the hot end. Since on the average electrons at the hot end are more energetic than those on the right, a net energy is transported to the right, resulting in a current of heat. Note that heat is transported entirely by electrons having the Fermi energy, because those well below this energy cancel each other's contributions.

To evaluate the thermal conductivity K quantitatively, we use the formula $K = 1/3C_{el}v_Fl$, where C_{el} is the electronic specific heat per unit volume, v is the Fermi velocity of electrons, l is the mean free path of electrons at the Fermi energy. Using expression for the heat capacity derived earlier, we find

$$K = \frac{1}{3}\left(\frac{\pi^2}{2}n\frac{k_B^2T}{E_F}\right)v_Fl \ . \tag{20}$$

Noting that $E_F = \frac{1}{2}mv_F^2$ and that $l/v_F = \tau$, we can simplify this expression for K to

$$K = \frac{\pi^2nk_B^2\tau T}{3m}, \tag{21}$$

which expresses thermal conductivity in terms of the electronic properties of the metal.

Many of the parameters appearing in the expression for K were also included in the expression for electrical conductivity σ. Recalling that $\sigma = ne^2\tau/m$, we find

$$\frac{K}{\sigma} = \frac{1}{3}\left(\frac{\pi k_B}{e}\right)^2 T = LT \tag{22}$$

We see from here that the ratio of the thermal conductivity to the electrical conductivity is directly proportional to the temperature. This is called the *Wiedemann-Franz law*. The constant of proportionality L, which is called the Lorentz number, is independent of the particular metal. It depends only on the universal constants k_B and e, should be the same for *all* metals. Its numerical value is $2.45 \cdot 10^{-8}$ WΩ/K^2. This conclusion suggests that the electrical and thermal conductivities are intimately related, which is to be expected, since both electrical and thermal current are carried by the same agent: electrons.

Motion in a magnetic field: cyclotron resonance and Hall effect

The application of a magnetic field to a metal gives rise to several interesting phenomena due to conduction electrons. The *cyclotron resonance* and the *Hall effect* are two which we consider.

Cyclotron resonance

If a magnetic field is applied to a metal the Lorentz force $\mathbf{F} = -e[\mathbf{E}+(\mathbf{v}\times\mathbf{B})]$ acts on each electron. For a perfect metal in the absence of electric field the equation of motion takes the form

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{v}\times\mathbf{B} \ . \tag{23}$$

If the magnetic field lies along the *z*-direction this results in

$$\frac{dv_x}{dt} = -\omega_c v_y$$
$$\frac{dv_y}{dt} = \omega_c v_x$$

$$(24)$$

where

$$\omega_c = \frac{eB}{m}$$

$$(25)$$

is the *cyclotron frequency* in SI system of units (in CGS $\omega_c = \frac{eB}{mc}$). For magnetic fields of the order of a few kG the cyclotron frequencies lie in the range of a few GHz. For example for $B=1$kG, the cyclotron frequency is $v_c = \omega_c / 2\pi = 2.8 GHz$. Therefore, the magnetic field causes electrons to move in a counterclockwise circular fashion with the cyclotron frequency in a plane normal to the field.
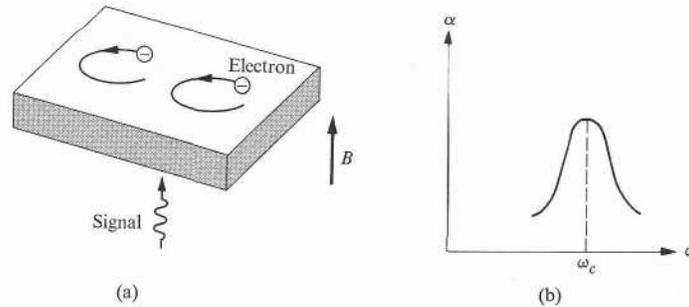
Fig. 6 (a) Cyclotron motion, (b) The absorption coefficient versus ω

Suppose now that an electromagnetic signal is passed through the slab in a direction parallel to **B**, as shown in figure 6. The electric field of the signal acts on the electrons, and some of the energy in the signal is absorbed. The rate of absorption is greatest when the frequency of the signal is exactly equal to the frequency of the cyclotron (see Fig.6b), i.e.

$$\omega = \omega_c .$$

$$(26)$$

This is so because, when this condition holds true, each electron moves synchronously with the wave throughout the cycle, and therefore the absorption continues all through the cycle. Thus, Eq.(26) is the condition for *cyclotron resonance*. On the other hand, when Eq.(26) is not satisfied, the electron is in phase with the wave through only a part of the cycle, during which time it absorbs energy from the wave. In the remainder of the cycle, the electron is out of phase and returns energy to the wave.

Cyclotron resonance is commonly used to measure the electron mass in metals and semiconductors. The cyclotron frequency is determined from the absorption curve, and this value is then substituted in (25) to evaluate the effective mass.

The Hall effect

First we derive an equation of motion of an electron in applied magnetic and electric field in the presence of scattering.

Assume that that the momentum of an electron is $\mathbf{p}(t)$ at time t, let us calculate the momentum per electron $\mathbf{p}(t + dt)$ an infinitesimal time dt later. An electron taken at random at time t will have a collision before time $t + dt$, with probability dt/τ, and will therefore survive to time $t + dt$ without suffering a collision with probability $1 - dt/\tau$. If it experiences no collision, however, it simply evolves under the influence of the force $\mathbf{F}$ (due to the spatially uniform electric and/or magnetic fields) and will therefore acquire an additional momentum $\mathbf{F}\,dt$. The contribution of all those electrons that do not collide between t and $t + dt$ to the momentum per electron at time $t + dt$ is the fraction $(1 - dt/\tau)$ they constitute of all electrons, times their average momentum per electron, $\mathbf{p}(t) + \mathbf{F}dt$.

Thus neglecting for the moment the contribution to $\mathbf{p}(t + dt)$ from those electrons that *do* undergo a collision in the time between t and $t + dt$, we have

$$\mathbf{p}(t+dt) = \left(1 - \frac{dt}{\tau}\right)\left(\mathbf{p}(t) + \mathbf{F}dt\right), \tag{27}$$

Note that if the force is not the same for every electron it should be averaged.

The correction to (27) due to those electrons that have had a collision in the interval t to $t + dt$ is only of the order of $(dt)^2$. To see this, first note that such electrons constitute a fraction dt/τ of the total number of electrons. Furthermore, since the electronic velocity (and momentum) is randomly directed immediately after a collision, each such electron will contribute to the average momentum $\mathbf{p}(t + dt)$ only to the extent that it has acquired momentum from the force $\mathbf{F}$ since its last collision. Such momentum is acquired over a time no longer than dt, and is therefore of order $\mathbf{F}dt$. Thus the correction to (27) is of order $(dt/\tau)\mathbf{F}dt$, and does not affect the terms of linear order in dt. We may therefore write:

$$\frac{\mathbf{p}(t+dt) - \mathbf{p}(t)}{dt} = \frac{d\mathbf{p}}{dt} = \mathbf{F} - \frac{\mathbf{p}(t)}{\tau}. \tag{28}$$

This simply states that the effect of individual electron collisions is to introduce a frictional damping term into the equation of motion for the momentum per electron. We apply this equation to discuss the Hall effect in metals using a free electron model.

The physical process underlying the Hall effect is illustrated in Fig.7. Suppose that an electric current J_x is flowing in a wire in the x-direction, and a magnetic field B_z is applied normal to the wire in the z-direction. We shall show that this leads to an additional electric field, normal to both J_x and B_z, that is, in the y-direction.

Before the magnetic field is applied, there is an electric current flowing in the positive x-direction, which means that the conduction electrons are drifting with a velocity $\mathbf{v}$ in the negative x-direction. When the magnetic field is applied, the Lorentz force $\mathbf{F} = -e(\mathbf{v} \times \mathbf{B})$ causes the electrons to bend downward, as shown in the figure. As a result, electrons accumulate on the lower surface, producing a net negative charge there. Simultaneously a net positive charge appears on the upper surface, because of the deficiency of electrons there. This combination of positive and negative surface charges creates a downward electric field E_H, which is called the *Hall field*.
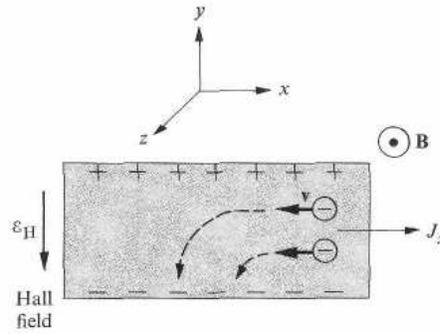
Fig. 7 Origin of the Hall field and Hall effect.

Let us evaluate this Hall field. We start from the Lorentz force acting on each electron $\mathbf{F} = -e[\mathbf{E}+(\mathbf{v} \times \mathbf{B})]$. According to (28) we find

$$m\frac{d\mathbf{v}}{dt} = -e[\mathbf{E}+\mathbf{v}\times\mathbf{B}] - m\frac{\mathbf{v}}{\tau}, \qquad (29)$$

where τ is the relaxation time. Note that the Lorentz force is not the same for all electrons because they move with different velocities, therefore it is averaged over ensemble. We are looking for the solution of this equation in the steady state when the current is independent of time and therefore $d\mathbf{v}/dt=0$.

$$0 = -eE_x - eB\mathbf{v}_y - m\frac{\mathbf{v}_x}{\tau}$$
$$0 = -eE_y + eB\mathbf{v}_x - m\frac{\mathbf{v}_y}{\tau} \qquad (30)$$

We multiply these equations by $-ne\tau/m$ to introduce current densities components $j_x = -en\mathbf{v}_x$ and $j_y = -en\mathbf{v}_y$, so that

$$\sigma E_x = \omega_c\tau j_y + j_x$$
$$\sigma E_y = -\omega_c\tau j_x + j_y \qquad (31)$$

where σ is the Drude conductivity in the absence of a magnetic field. In the steady state there is no electric current flowing perpendicular to the wire. Therefore the Hall field $E_H=E_y$ can be determined by the requirement that there be no transverse current j_y. Setting j_y to zero in the second equation of (31) we find that

$$E_y = -\left(\frac{\omega_c\tau}{\sigma}\right)j_x = -\frac{1}{ne}j_xB, \qquad (32)$$

The proportionality constant $-1/ne$, is known as the *Hall constant*, and is usually denoted by R_H.. Therefore,

$$R_H = -\frac{1}{ne}. \qquad (33)$$

This is a very striking result, which predicts that the Hall coefficient depends on no parameters of the metal except the density of carriers. Since R_H is inversely proportional to the electron concentration n, it follows that we can determine n by measuring the Hall field. Since we have already calculated n assuming that the atomic valence electrons become the metallic conduction electrons, a measurement of the Hall constant provides a direct test of the validity of this assumption.

Some Hall coefficients are listed in a table. Note the occurrence of cases in which R_H is actually positive, apparently corresponding to carriers with a positive charge.

METAL	VALENCE	$- 1/R_H ne$
Li	1	0.8
Na	1	1.2
K	1	1.1
Rb	1	1.0
Cs	1	0.9
Cu	1	1.5
Ag	1	1.3
Au	1	1.5
Be	2	-0.2
Mg	2	-0.4
In	3	-0.3
Al	3	-0.3

Table. Hall coefficients of selected elements. Evidently the alkali metals obey the Drude result reasonably well, the noble metals (Cu, Ag, Au) less well, and the remaining entries, not at all.

It is found experimentally that the experimental values for sodium and potassium are in excellent agreement with values calculated for one conduction electron per atom. However, for some metals the experimental values strongly disagree with the predictions of the free electron model. For example, for aluminum experiment shows that there is one positive carrier per atom, rather then three negative, as we would expect assuming the free electron model and taking into account three valence electrons. This disagreement can be explained using a band theory of solids.