Midterm Exam, 3/19/2012, 3:00-5:30PM
**(closed book, 1 sheet of notes double sided allowed)**

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1. (10 pt) Answer following questions briefly: (a) In X-ray CT, where is the x-ray source located (inside or outside the body)? what is measured at the detector? What properties of the tissue does the reconstructed image reveal? (b) answer the same set of questions for nuclear imaging (SPECT or PET).

2. (10 pt) Consider a CT imaging system. Suppose the imaged slice has a circular area of diameter D (cm), and the maximum spatial frequency along any direction is f_{max} (cycles/cm). (a) What is the maximum spacing (τ) between projection lines (i.e. spacing between adjacent lines with the same angle) that will enable proper image reconstruction? (b) What should be the reconstructed image size (in terms of number of rows M and columns N)? (c) What should be the number of DFT points K to use along each projection angle if we were to use the filtered backprojection method for image reconstruction? Note that you should express τ, M, N, K in terms of D and f_{max}.

3. (10 pt) Compare SPECT and PET: (a) For SPECT, what type of radioactive decay does the radio tracer undergo? Do you need a collimator at the detector? Why? (b) Answer the same questions for PET. (c) How does the half-life of the radio tracer affects the imaging process?

4. (10 pt) Suppose you have an imaging system that can do CT imaging followed by PET imaging for the same patient location. Describe the major steps that can be used to reconstruct the CT image and the PET image. Assume in both CT and PET imaging, the photon detectors surround the imaged slice and obtained data (photon intensity) at different detector locations can be written as $I(\theta_m, l_n), m = 1,2,...,M; n = 1,2,...,N$, where θ_m, l_n denote the projection angle and distance in the projection line. You can use either filtered backprojection algorithm or the convolution backprojection algorithm for the inverse radon transform.

5. (10 pt) Assuming the probability density functions of a blood test result for patients with and without a disease are described by Gaussian distributions, where we assume $\mu_n < \mu_d$:

$$Normal: p_N(x) = \frac{1}{\sqrt{2\pi}\sigma_d} \exp\left\{-(x-\mu_n)^2/\sigma_n^2\right\} \quad Diseased: p_D(x) = \frac{1}{\sqrt{2\pi}\sigma_d} \exp\left\{-(x-\mu_d)^2/\sigma_d^2\right\}$$

Assume the diagnosis is determined based on a threshold t. For a patient with test value below t, we call it normal. Otherwise, we call it diseased. Determine the sensitivity (true positive fraction) and the specificity (true negative fraction) in terms of the threshold t. When you increase t, how is the sensitivity and specificity affected? Propose a method to choose t to achieve a good tradeoff between the sensitivity and specificity.

Note: you could just write down all the equations clearly without doing the actual integration.

6. (15 pt) Consider the x-ray imaging of a slab that consists of two different materials with different linear attenuation coefficients μ_1 and μ_2, respectively, as illustrated below. Determine the intensity of detected photons along the y axis on the detector plane. Express your solution in terms of the y-coordinate. You should consider the inverse square law and the oblique effect. Assume the x-ray source is an ideal point source with intensity I_0. For simplicity, assume the slab is infinitely long in the y direction.
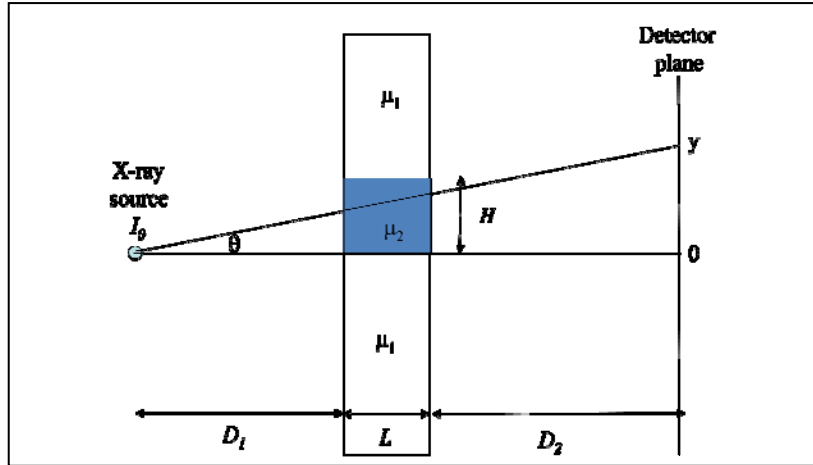
7. (25 pt) Suppose the tissue slice (with dimension 6x6cm) being imaged by a parallel beam x-ray CT scanner contains distribution of the linear attenuation coefficients as shown below. (a) Assume the detector is a point detector. Sketch the projection $g(l, \theta)$ as a function of l, for $\theta=0$, 45, 90 degrees, respectively. You should indicate the magnitudes of the projected values where necessary on your sketch. Also clearly specify any transition points in the l-axis. (b) Sketch the image obtained by backprojections from both 0 and 90 degree projections. You should assume that you know the dimension of the tissue being imaged and normalize your backprojection using the known dimensions. (c) Determine the Fourier transform of the original image along a line with orientation $\theta=0$ degree in the frequency domain. (d) What will be the projected function for $\theta=0$ if the detector is an area detector with width 0.2 cm. Sketch the projection function. (e) How is the image recovered from the readings obtained with such a detector related to the image recovered from using an ideal point detector? (assume that in both cases you use the convolution projection method with the same filter). (Note: for your solution, if you need to know the Hankel transform of a function, you can just state that it is the Hankel transform of that function, without giving the actual function form).
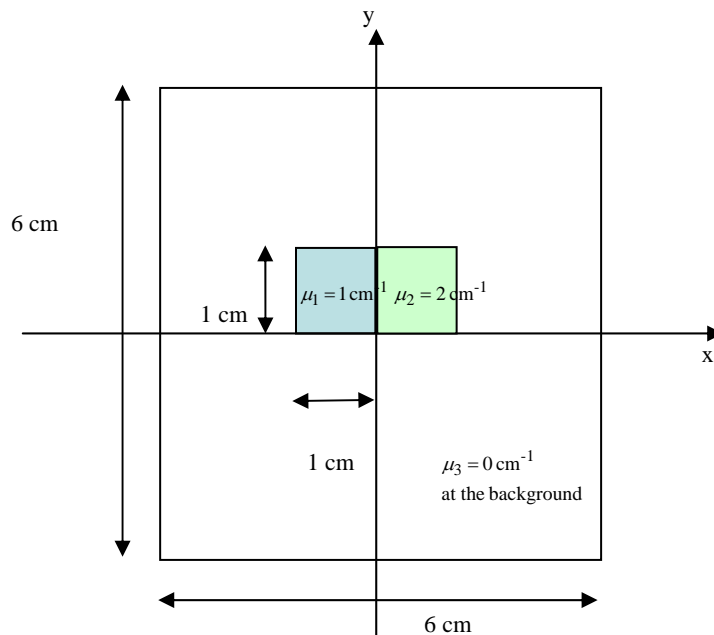
8. (10 pt) A 2-D slice to be imaged is shown in Fig. P7, which consists of two regions with linear attenuation coefficients μ_1 and μ_2. The background has a linear attenuation coefficient of μ_3

   a. Suppose a solution containing a gamma ray emitting radio tracer with an initial radioactivity of A_1 and half life time of T_1 is injected into both regions. We image the radioactivity distribution in this slice using a rotating SPECT camera. Compute the measured signal by the camera at positions A and B, respectively at time t after the injection of the radionuclide solution.
   b. Now suppose the radio tracer in (a) is replaced by a positron emitting radio tracer with the initial radioactivity A_2 and half-life T_2. This time the slice is imaged using a PET scanner. Compute the measured signal by the pair of cameras positioned at A and B, at time t after the injection of the radionuclide solution.