Atomic and nuclear physics

- force, energy, and light
- atomic structure and transition
- nuclear structure and transition
- interaction between particles and matter
- photon attenuation in medium
- detection of photons
- production of radioactive nuclides

Atomic and nuclear physics

Textbook:

Chapters 1 – 7

Cherry SR, Sorenson JA, Phelps ME, "Physics in Nuclear Medicine" 3rd ed (2003)

Force

push or pull between two objects

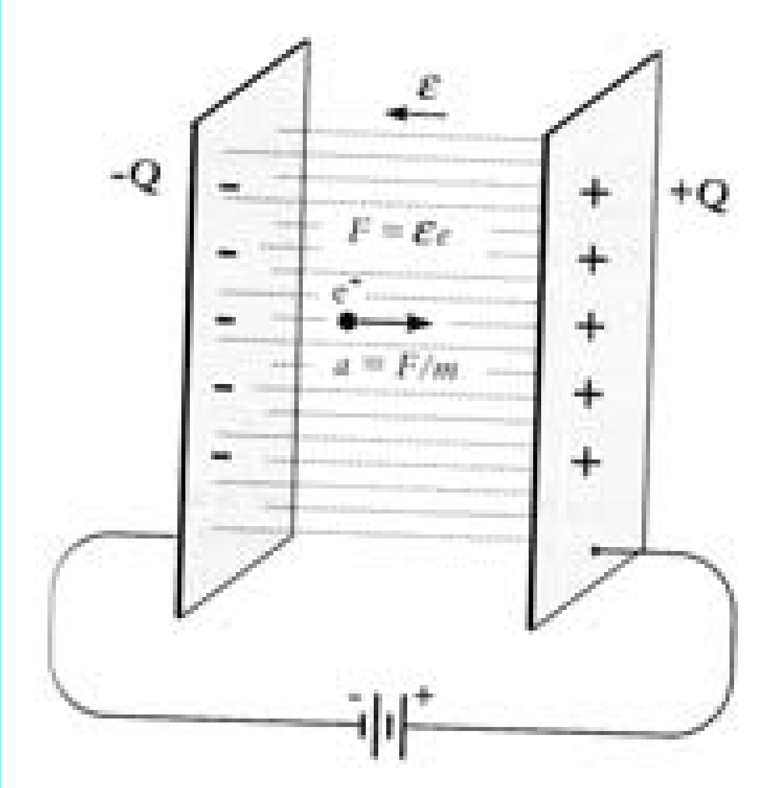
- **contacted: mechanical force**
- **over a distance:**
  - **gravity force**
  - **electric force**
  - **magnetic force**
  - **nuclear force**

Energy and mass

- **Energy closely relates to force.**
  higher energy → stronger force
- **Energy can be converted to mass or vice versa:** $E = m \cdot C^2$
- **energy and mass unit:**
  - **electron volt (eV, keV, MeV)**
  - **1 atomic unit (u) = 1.66 x 10^{-27} kg = 931.5 MeV ≈ 1 GeV**
- **Energy is conserved but form can be changed in a process.**

Electric force

- **attractive or repulsive**
- **stronger than gravity**
- **medium range:**
  inverse square law
- **electric energy = e·V**
- **binding the orbital electrons to the nucleus in an atom**

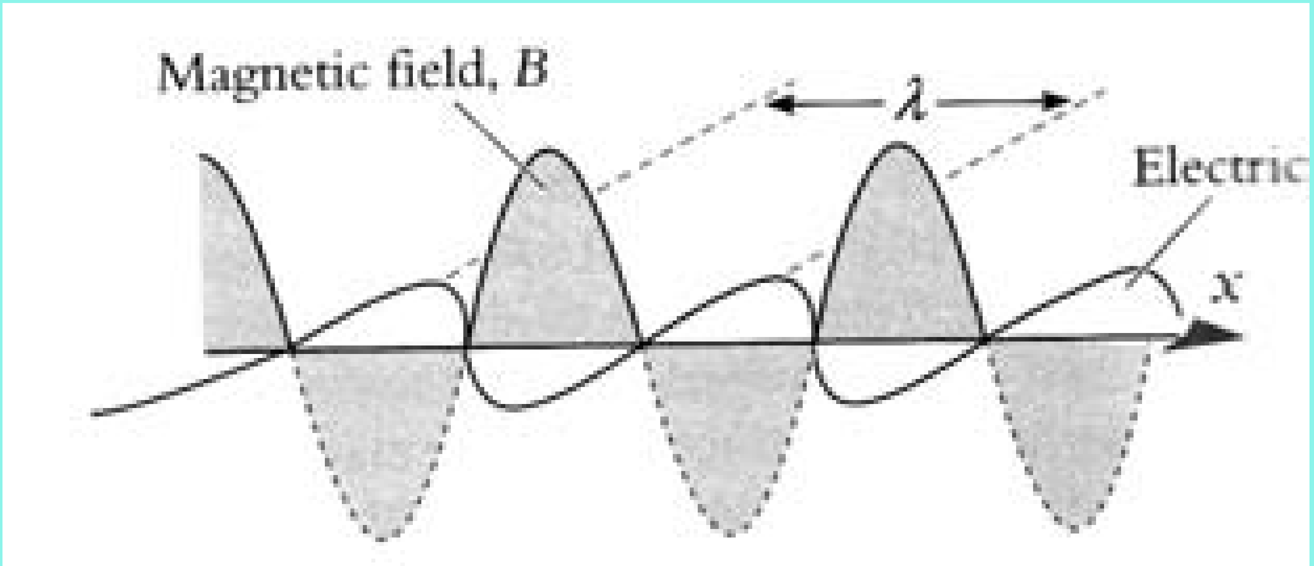


Light: electromagnetic wave

oscillating electric field and perpendicularly oscillating magnetic field propagating perpendicularly to the electric and magnetic fields at a speed c = 3 x 10^8 m/sec

- **amplitude**
- **phase**
- **wave length λ**
- **frequency ν**

$$\lambda \cdot \nu = c$$



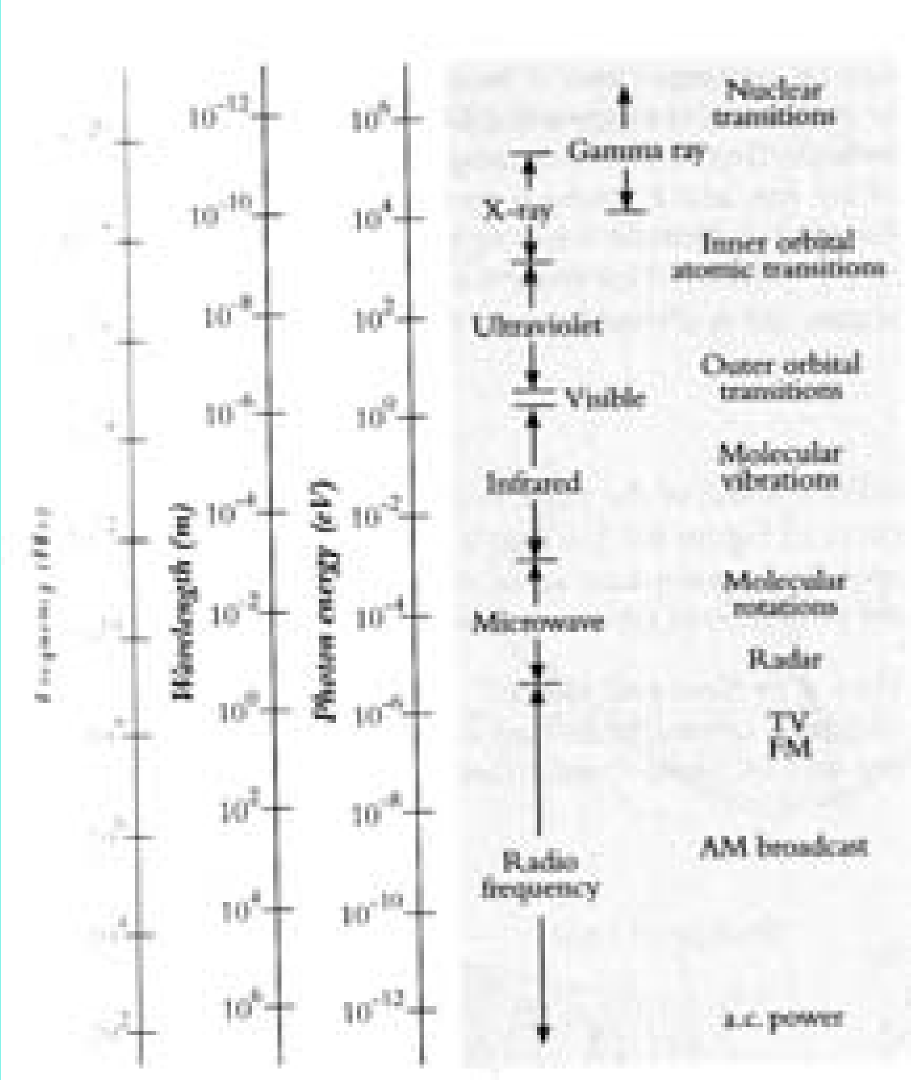
Light: photons

- e-m wave quantized → photons
- Photons are particles like electrons and protons.
- Photons carry energy and momentum, so always move, but no mass and no charge.
- photon energy: $E = h\nu = hc/\lambda$

  photon momentum: $p = E/c$

  Planck constant: $h = 6.62 \times 10^{-34}$ J-sec

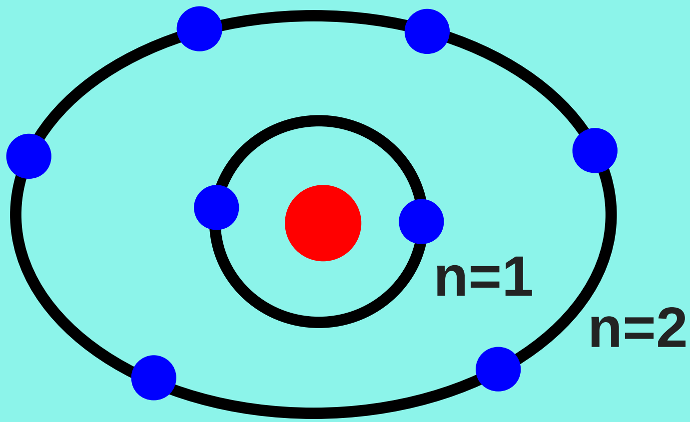
Light spectrum



Atomic structure

atoms: the smallest part of element

- **size ~ 10^{-8} cm**
- **mass ~ 1 - 260 u**
- **charge = 0**
- **orbital electrons, nucleus**



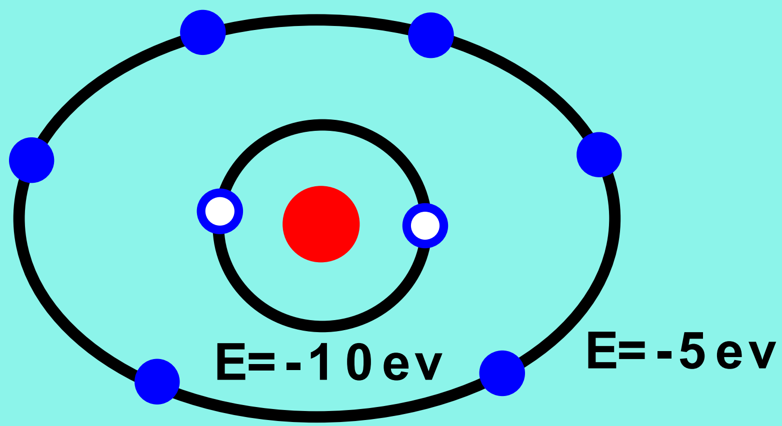
charge -1 electron charge, mass: 511 keV, spin 1/2

1 electron charge = 1.6×10^{-19} Coulomb

- **Pauli exclusion principle:**
  - **electrons allowed for an inner shell $\leqslant 2n^2$ (n = 1, 2, 3, …)**
  - **electrons allowed for an outer shell $\leqslant 8$**

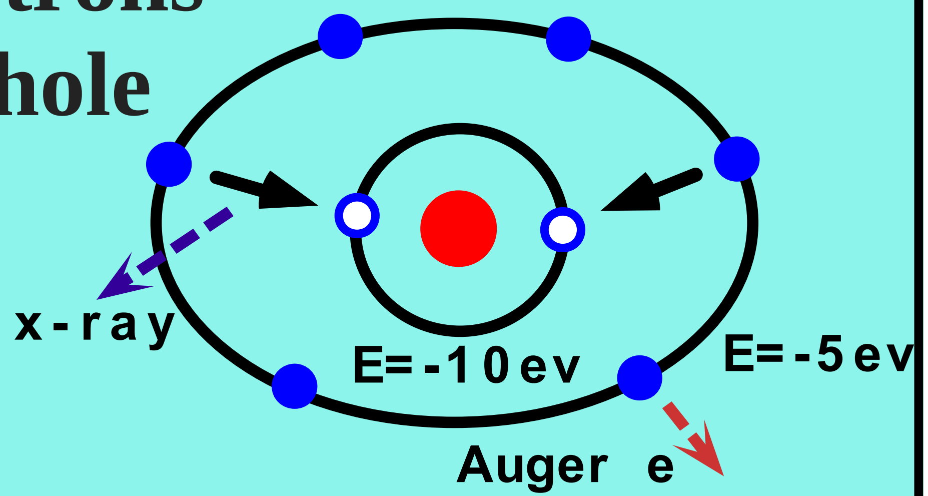
Atomic force and energy

- **How does an atom maintain its stability? electric force: attraction between the nucleus (protons) and electrons**
- **Electric force → binding energy**
  - **a few eV to keV**
  - **decreases with decreasing nuclear charge**
  - **decreases with increasing distance (orbits)**



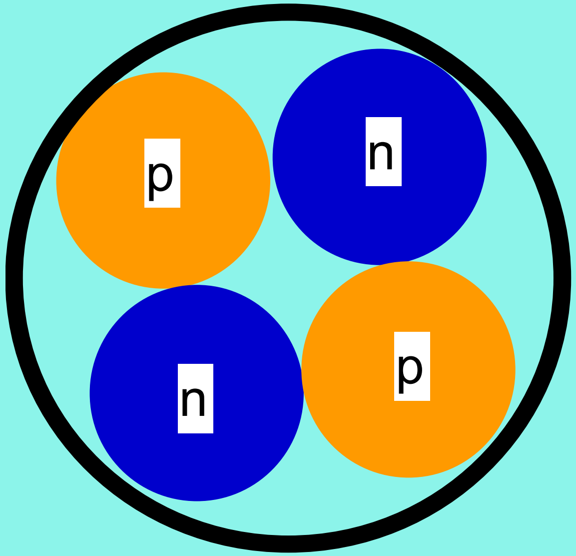
Transition of orbital electrons

- **usually stable: orbital electrons in the low orbits: minimum energy (ground state)**
- **unstable: vacancies on the inner orbits due to some process (excited state)**
- **back to stable: outer electrons jumping to fill the inner hole**
  - **characteristic x-rays**
  - **Auger electron**



Nuclear structure

- **nucleus size ≈ 10^{-13} cm**
- **nucleus mass ≈ 1 u (1 GeV)**
- **nucleus composed of protons (Z, atomic number) & neutrons (N)**
- **mass number A = Z + N**
- **proton charge: +1**
  neutron charge: 0
- **both spin 1/2**



Nuclear structure

$^{A}X_{Z}$ (Tc-99m)

- isotopes: same Z but different N

  $^{1}H_{1}$ and $^{2}H_{1}$, $^{15}O_{8}$ and $^{16}O_{8}$

- isobars: same A but different Z

  $^{14}C_{6}$ and $^{14}N_{7}$

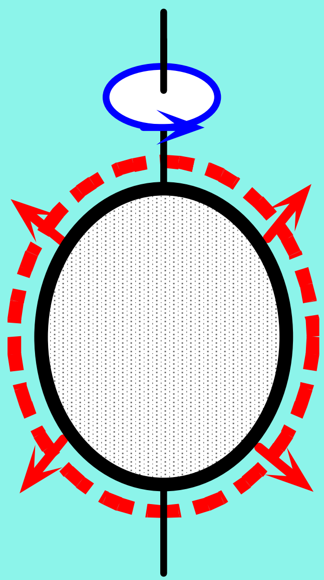
- isotones: same N but different Z

  $^{3}H_{1}$ and $^{4}He_{2}$

- isomers: same A and Z but different energy $^{99m}Tc_{43}$ and $^{99}Tc_{43}$

Nuclear model

- **shell model:**
  similar to the atom
- **liquid drop model:**
  rotation and vibration
- **both can only explain**
  part of experimental data



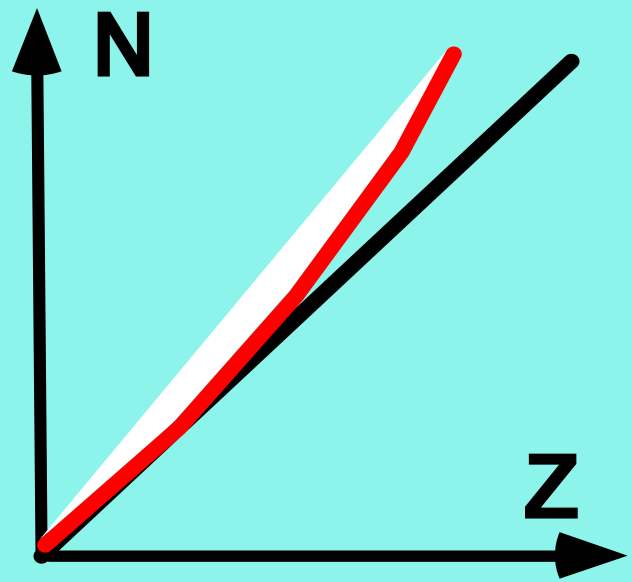
Nuclear force and energy

- **nuclear force: strong and shortly ranged attraction mediated by pions between nucleons**
- **binding energy for a nucleon: ~ 8 MeV**
- **transition of nucleons between excited and ground states (< 8 MeV)**

Stability of nucleus

- **for light nuclei (A ≤ 40), N = Z**
- **for heavy nuclei (A > 40), N > Z**

The repulsive electric forces between protons increase with increasing number of protons so that stronger nuclear forces provided by more neutrons are needed to cancel out the electric forces.



Nuclear decay

- **to achieve the stable N/Z ratio by**
  1. emitting charged particles (β^-, β^+, α)
  2. capturing orbital electrons
  3. fission
- **to release extra energy by**
  1. γ **(isomeric) decay**
  2. internal conversion
- **conservation of energy, mass, charge, momentum, etc.**

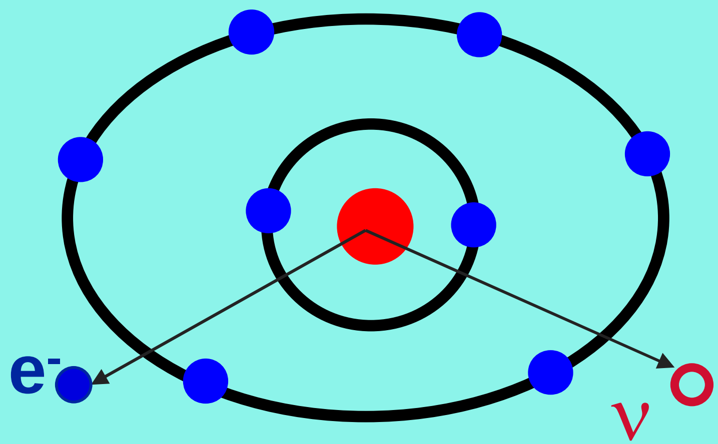
β^{-} decay

- **β^{-} decay for neutron rich isotopes**

  A neutron decays to a proton, electron and anti-neutrino: $n \longrightarrow p + e^{-} + \bar{\nu}$

  $$^{99}Mo_{42} \longrightarrow {}^{99m}Tc_{43} + e^{-} + \bar{\nu}$$

  $$^{131}I_{53} \longrightarrow {}^{131}Xe_{54} + e^{-} + \bar{\nu}$$



- **γ-ray emission possible if the child nucleus is in an excited state.**
- **continuous energy for electrons – energy shared with neutrinos**

β⁺ decay

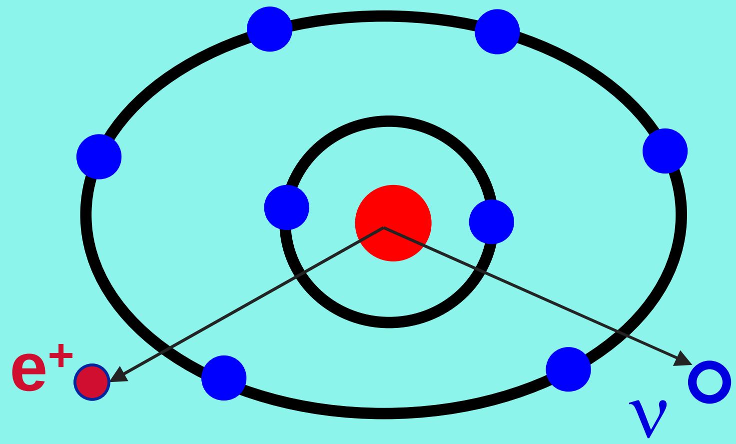
β^+ decay for proton rich isotopes

A proton decays to a neutron, positron and neutrino:

$p \rightarrow n + e^+ + \nu$

$^{18}F_9 \longrightarrow {}^{18}O_8 + e^+ + \nu$

$^{15}O_8 \longrightarrow {}^{15}N_7 + e^+ + \nu$



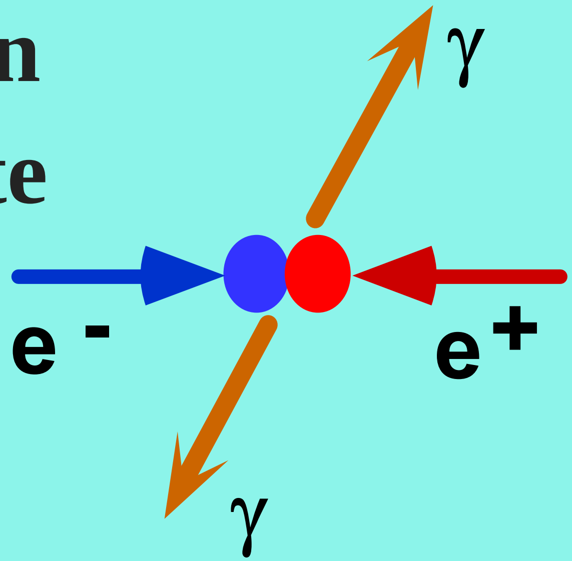
- **γ-ray emission possible if the child nucleus is in an excited state.**
- **continuous energy for electrons – energy shared with neutrinos**

Annihilation

$e^- + e^+ = 2\gamma$

- **each γ photon energy: 511 keV due to energy-mass conservation**
- **2 γ's always traveling in opposite directions due to momentum conversation**

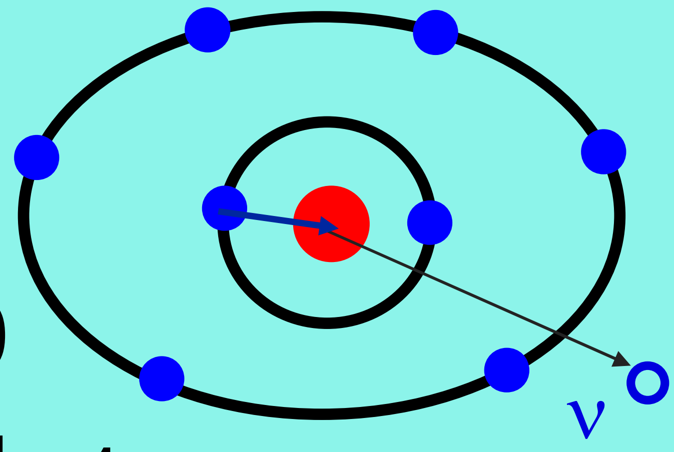
→ PET imaging



Electron capture

- **proton rich nuclei:** **$p + e^- \rightarrow n + \nu$**
- **to absorb orbital electron into nucleus**

  $^7Be_4 + e^- \rightarrow {}^7Li_3 + \nu$



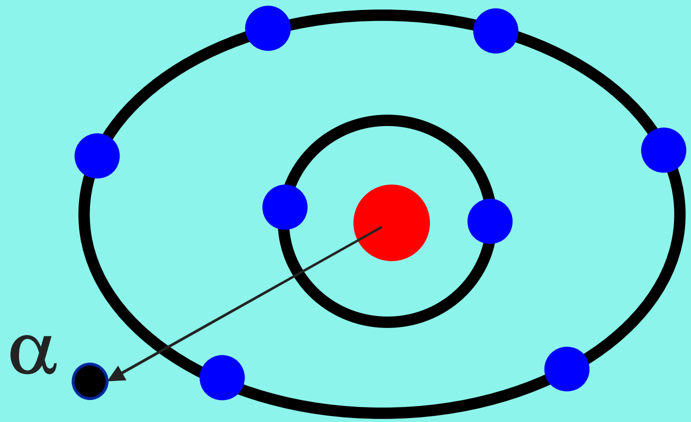
- **competing process with β^+ decay (same product of z-1)**
- **successive x-rays and Auger electrons**
- **probability increasing with larger Z (closer electron shells).**

α decay

- to emit a helium nucleus (α)

$$^{226}Ra_{88} \rightarrow {}^{222}Rn_{86} + {}^{4}He_{2}$$

$$^{235}U_{92} \rightarrow {}^{231}Th_{90} + {}^{4}He_{2}$$



- usually occurring for heavy nuclei
- often followed by γ-rays
- short range ~ 10^{-6} cm, can be stopped by paper
- Unstable Rn-222 gas may cause lung cancer.

Radon gas

- **relatively short half life: $T_{1/2}$= 3.825 d**
- **α, β and γ decays, decay rate high over the first hour and eventually to stable Pb-210 ($T_{1/2}$= 22 y)**

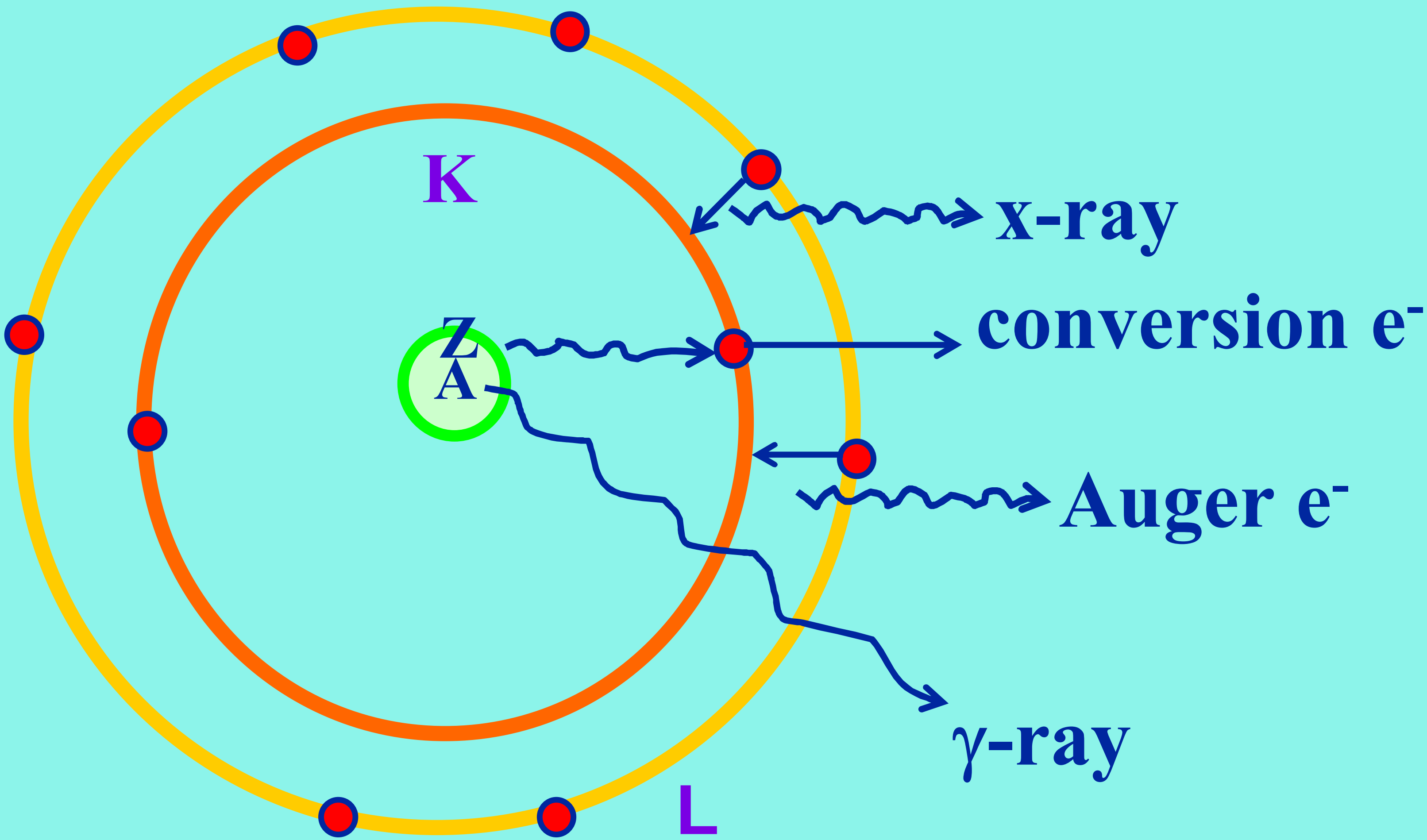
Isomeric transition (I.T.)

- **emitting γ photons to release extra energy of the nucleus**

$$^{99m}Tc_{43} \rightarrow {}^{99}Tc_{43} + \gamma$$

- **occurs when a nucleus transfers from an excited state (isomeric state) to ground state**
- **energy difference passed to γ photons, similar to emission of characteristic x-rays**
- **often follows other decays**

I.T. and I.C.



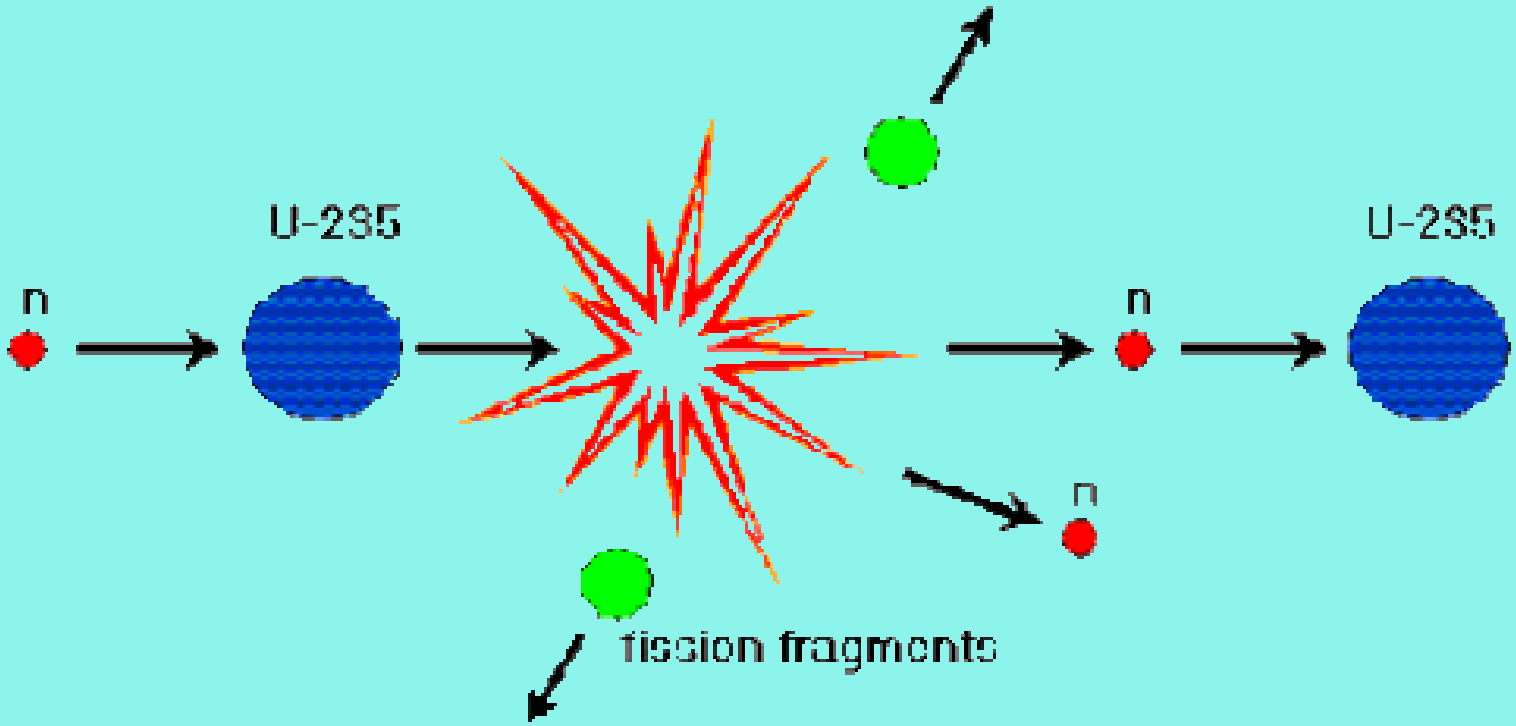
Internal conversion (I.C.)

- **ejecting an orbital electron to release extra energy in the nucleus**
- **highest probability for k-shell electrons but other shells possible**
- **successive x-rays and Auger electrons**
- **competing process with IT**

Fission

breaking a heavy nucleus into two fragments (usual mass ratio: 40:60)

$$^{235}U_{92} + n \longrightarrow {}^{92}Kr_{36} + {}^{141}Ba_{56} + 3n + 200\ \text{MeV}$$



Fission

- **a neutron needed to initiate fission but 2 or 3 neutrons are created (chain reaction)**
- **most energy released as heat, also kinetic energy for fragments and n (~ 1.5 MeV)**
- **low probability (e.g. $T_{1/2} = 2 \times 10^{17}$y for ^{235}U)**
- **controllable in reactor, not controllable in atomic bomb**

Fusion

combining two light nuclei into one heavier nucleus

$^{2}H_{1} + ^{3}H_{1} \longrightarrow ^{4}He_{2} + n + \text{energy}$

- **naturally occurred in the sun**
- **can be initiated artificially but currently uncontrollable in a hydrogen bomb**

Nuclear decay rules

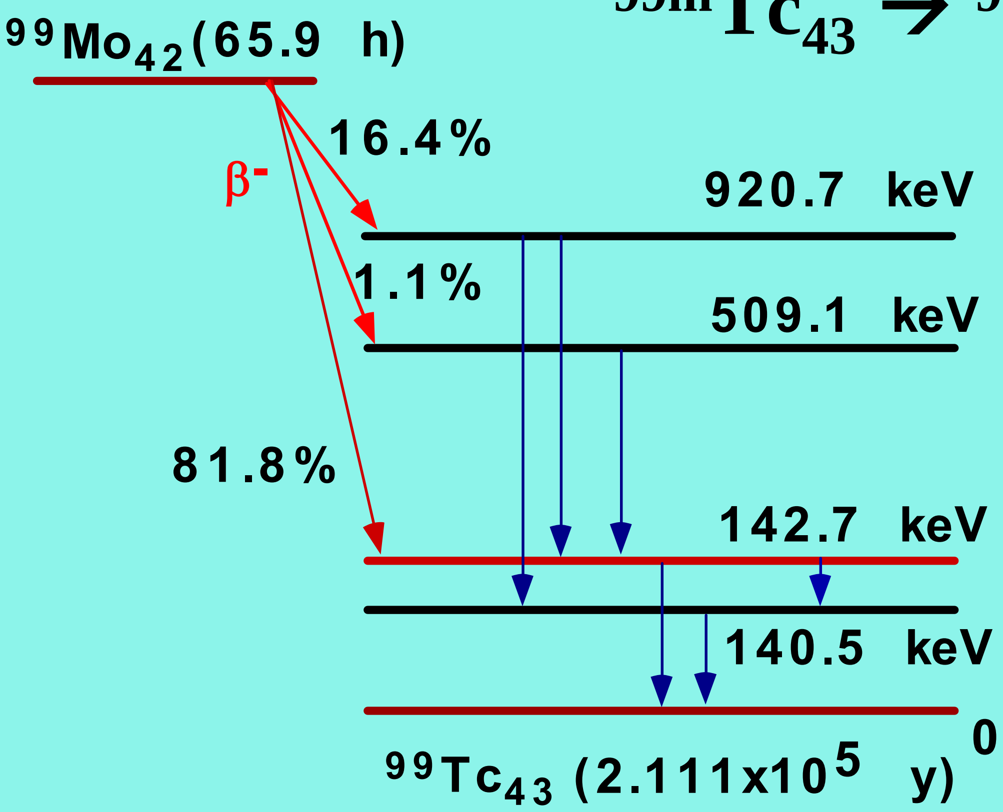
based on conservation laws

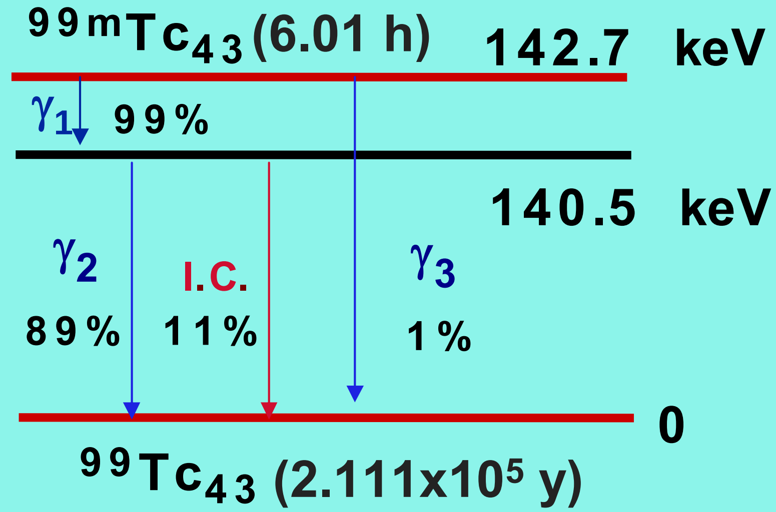
- **γ-decay (I.T.) and internal conversion: no changes for A & Z**
- **β⁻-decay:** $^{A}X_{Z} \rightarrow {}^{A}Y_{Z+1} + e^{-} + \nu$
- **β⁺-decay:** $^{A}X_{Z} \rightarrow {}^{A}Y_{Z-1} + e^{+} + \nu$
- **e-capture:** $^{A}X_{Z} + e^{-} \rightarrow {}^{A}Y_{Z-1} + \nu$
- **α-decay:** $^{A}X_{Z} \rightarrow {}^{A-4}Y_{Z-2} + \alpha$

Nuclear decay scheme

$$^{99}Mo_{42} \rightarrow {}^{99m}Tc_{43} + e^{-} + \nu$$

$$^{99m}Tc_{43} \rightarrow {}^{99}Tc_{43} + \gamma$$





Activity

- $$A(t) = -\frac{\Delta N(t)}{\Delta t} = \lambda \cdot N(t)$$

  λ - decay constant (1/sec, 1/min, 1/hr)

- **decay equation: $N(t) = N_0 \exp(-\ln 2 \; t/T_{1/2})$**

  N: number of survived nuclei, ln2 = 0.693

- **half life $T_{1/2} = \ln 2/\lambda$**

- **tradition unit: 1 Ci = 3.7×10^{10} dps (1 g Ra-226)**

  SI: 1 Bq = 1 dps **1 mCi = 37 MBq**

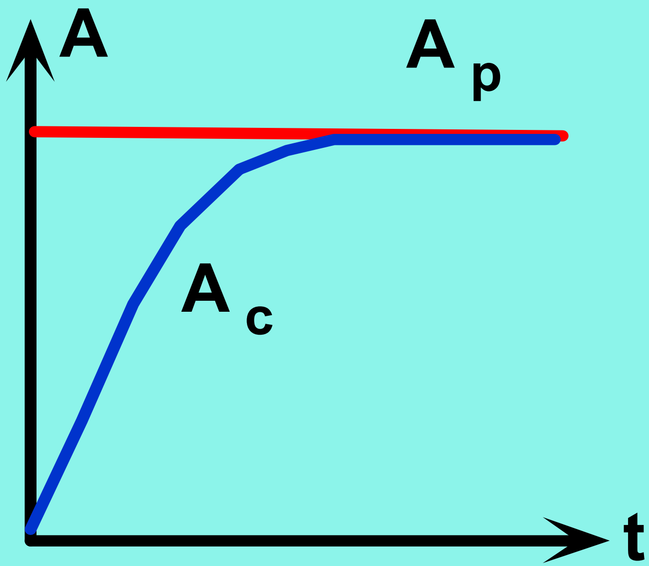
Parent-child-grandchild decay

secular equilibrium: $T_c << T_p$

$^{226}Ra \longrightarrow {}^{222}Rn + \alpha$ (1600 y)

$^{222}Rn \longrightarrow {}^{218}Po + \alpha$ (3.8 d)

$$A_c(t) = A_{0p}\left(1 - e^{-\frac{0.693t}{T_c}}\right)$$



$A_c(t) = A_{0p}$ for large $t > 5\,T_c$

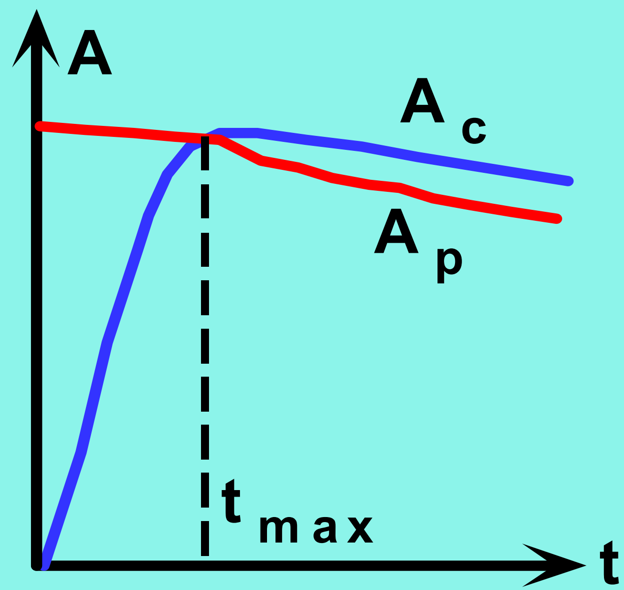
Parent-child-grandchild decay

transient equilibrium: $T_c = 0.1\ T_p$

$^{99}Mo_{42} \rightarrow {}^{99m}Tc_{43} + e^- + \nu$ (66 h)

$^{99m}Tc_{43} \rightarrow {}^{99}Tc_{43} + \gamma$ (6 h)

$$\frac{A_c(t)}{A_p(t)} = \frac{T_p}{T_p - T_c}$$



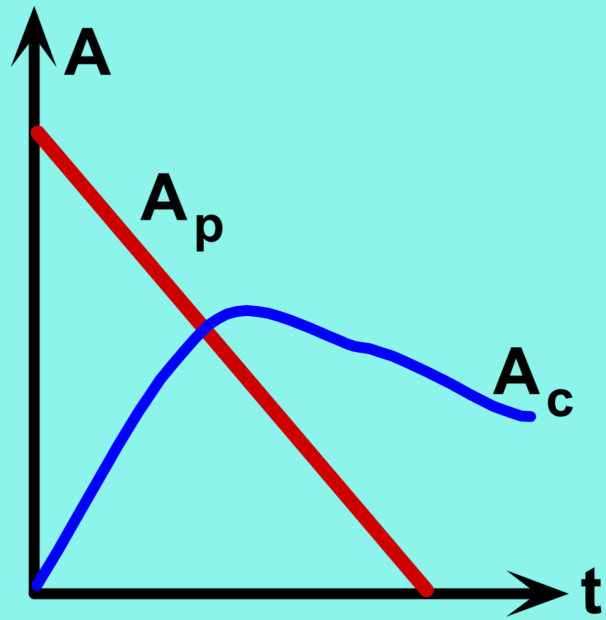
→ **decay with the same halflife after t_{max}: equilibrium**

t_{max} = 24 hr for Mo-Tc equilibrium

Parent-child-grandchild decay

no equilibrium: $T_c > T_p$

^{131m}Te (30 h) → ^{131}I (8 d)



Effective half life

$$\frac{1}{T_{eff}} = \frac{1}{T_{phy}} + \frac{1}{T_{bio}}$$

$$T_{eff} = \frac{T_{phy} \times T_{bio}}{T_{phy} + T_{bio}}$$

$$\text{if } T_{ph} >> T_{bi}, \; T_{eff} \approx T_{bi}$$

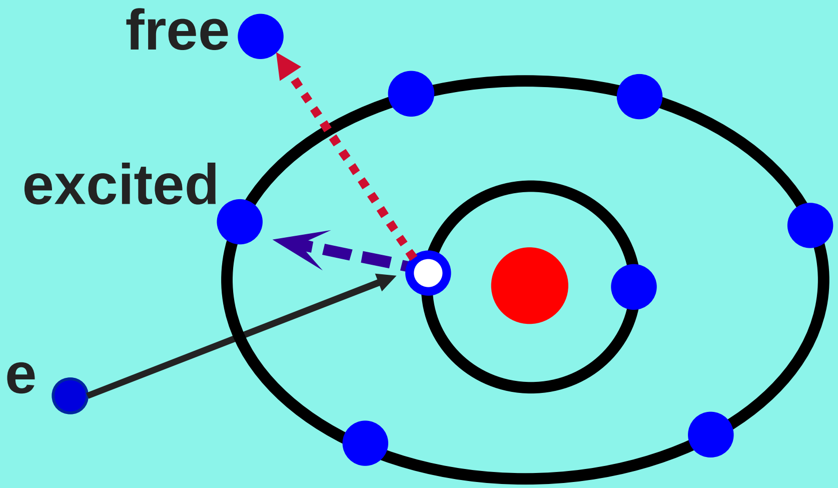
$$\text{if } T_{ph} << T_{bi}, \; T_{eff} \approx T_{ph}$$

Interaction between particles and matter

- **charged particles (electron) with matter (atoms)**
- **neutrons with matter**
- **photons with matter**

Electron with matter

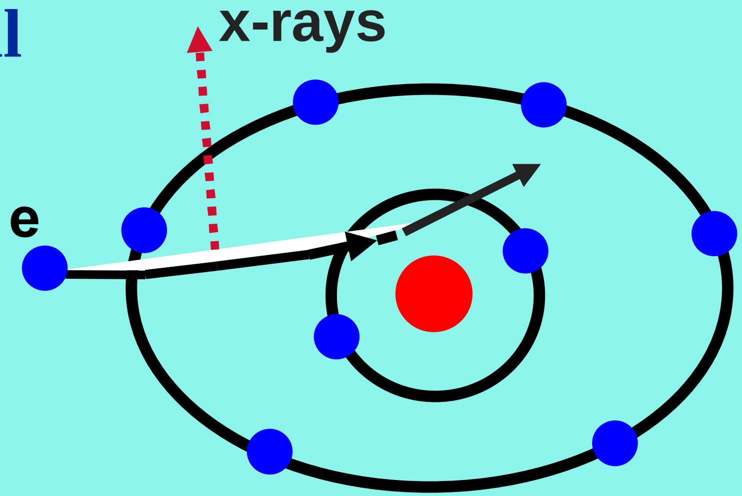
- **mainly interact with orbital electrons due to the same mass**
  - **ionization**
    → free electron and ion
  - **excitation**
    → excited atom



Bremsstrahlung

- **bremsstrahlung (braking radiation) x-rays**

  The electron penetrates the orbital electrons and is defected and decelerated by the nucleus.



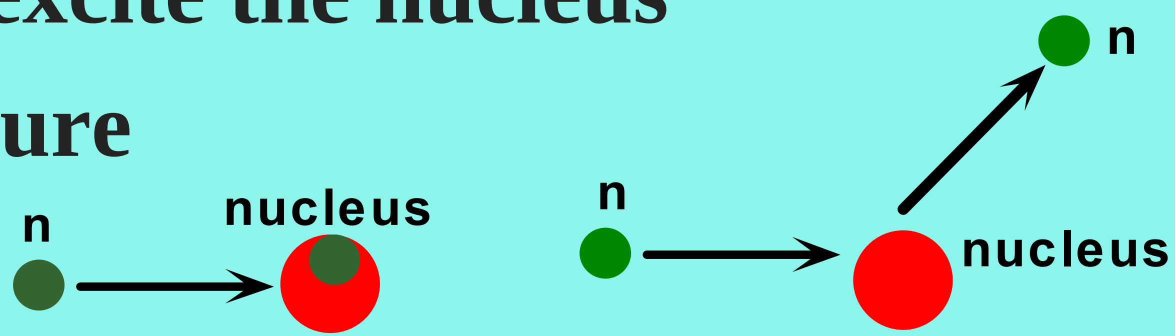
- **Probability increases with greater e⁻ energy and larger atomic number of the medium.**

  e.g. in W, 10 MeV electrons losing 50% energy
  100 MeV electrons losing 90% energy

Neutron with matter

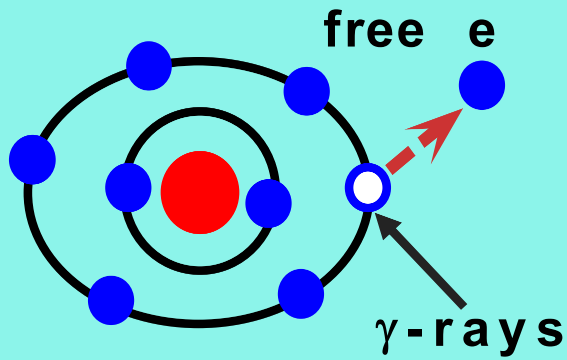
mainly interact with nucleus due to the similar mass

- **elastic scattering: kinetic energy unchanged before and after scattering**
- **inelastic scattering: kinetic energy decreased to excite the nucleus**
- **neutron capture**

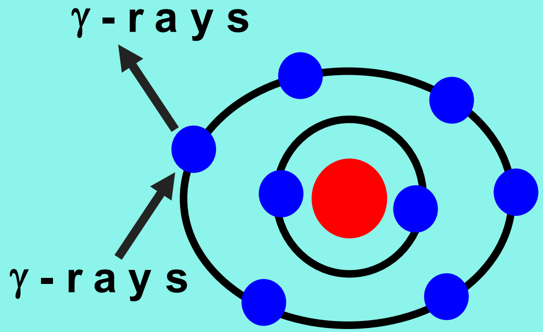


Photon with matter

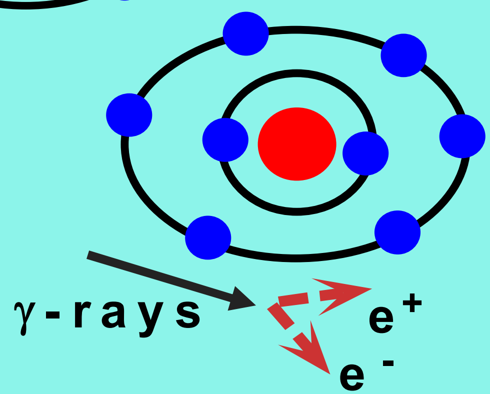
- **photoelectric process**
  low photon energy < 60 keV



- **Compton scatter**
  medium photon energy



- **pair production**
  high photon energy > 1.02 MeV



Photoelectric process

- **no photon after the process (absorption)**
- **probability decreasing with**

greater photon energy (proportional to $1/E^3$)
lower atomic number (proportional to Z^3)

probability → 0 when E > 60 keV in tissue

- **followed by characteristic x-ray and free electron**

Compton scatter

- photon changing direction and decreasing energy (minimum E at 180°), but not absorbed
- 0° < recoiled electron < 90°
- dominating at medium energies (60 keV – several MeV)
- almost same probability for all media (except for H_2) → proportional to Z/A
- followed possibly by characteristic x-rays or free electron

Pair production

- E $\geqslant$ 1.02 MeV
- no photons after the process
- probability increasing with

  higher photon energy slowly

  larger atomic number: proportional to Z

  dominating when E > 10 MeV in tissue
- energy deposited locally: E – 1.02 MeV

Range of radiation in matter

- **type of radiation:**

  charged < n < photon

- **radiation energy:**

  higher → longer range

- **composition and density of the medium:**

  heavier, denser → shorter range

Range of electron and α in matter

- **electron range:**

  $R_{air}(m) = 4\ E_{max}(MeV)$

  $R_{H_2O}(cm) = 0.5\ E_{max}(MeV)$

- **α-particle (4 - 8 MeV) range:**

  $R_{air}\ (cm) = 0.325\ E^{3/2}\ (MeV)$ **in air**

  $R = R_{air}\ (\rho_{air} A^{1/2} / \rho A_{air}^{1/2})$ **in others**

Photon attenuation in medium

- **photon attenuation in medium (absorption and scatter): number of photons (I) decreases with depth (d) inside the medium according to the exponential function:**

$$I = I_0 e^{-\mu d}$$

linear attenuation coefficient: μ

mass attenuation coefficient: $\mu_m = \mu/\text{density}$

- **useful in radiation protection, particularly for x-ray**

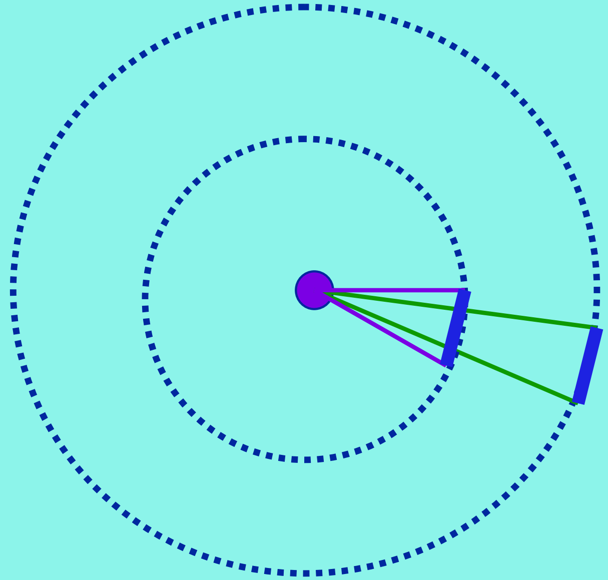
Photon attenuation in medium

- **transmission factor: $\exp(-\mu\ell)$**
- **attenuation factor: 1 - $\exp(-\mu\ell)$**
- **HVL ($\ell_{1/2}$) : I = 0.5 I_0 → $\ell_{1/2}$ = ln2/μ = 0.693/μ**

  TVL: I = 0.1 I_0 → $\ell_{1/10}$ = ln10/μ = 2.30/μ
- **mean free path: ℓ_m = 1/μ**
- **μ depending on E_γ and medium composition**

Inverse square law

- $I_d = I_0/d^2$ for an isotropic emitting point source



- longer distance →
  smaller solid angle
  → fewer particles
  useful in radiation protection, particularly for PET
- exposure rate constant for a point source Γ ($R \cdot cm^2/(mCi \cdot hr)$) →
  exposure rate (R/hr) = $\Gamma \cdot A/d^2$

Photon detection

- **gas-filled detectors**

  only detecting number of photons (activity)

- **Scintillation detectors**

  detecting both activity and photon energy

- **Semi-conductor detectors**

  detecting both activity and photon energy

Gas-filled detectors

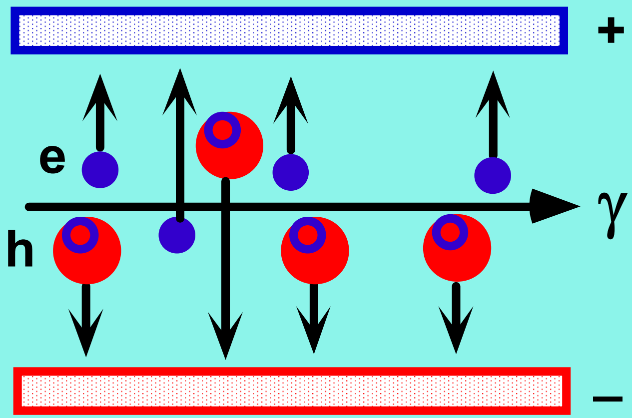
- **ionization chamber (12 a.p. Ar or air)**

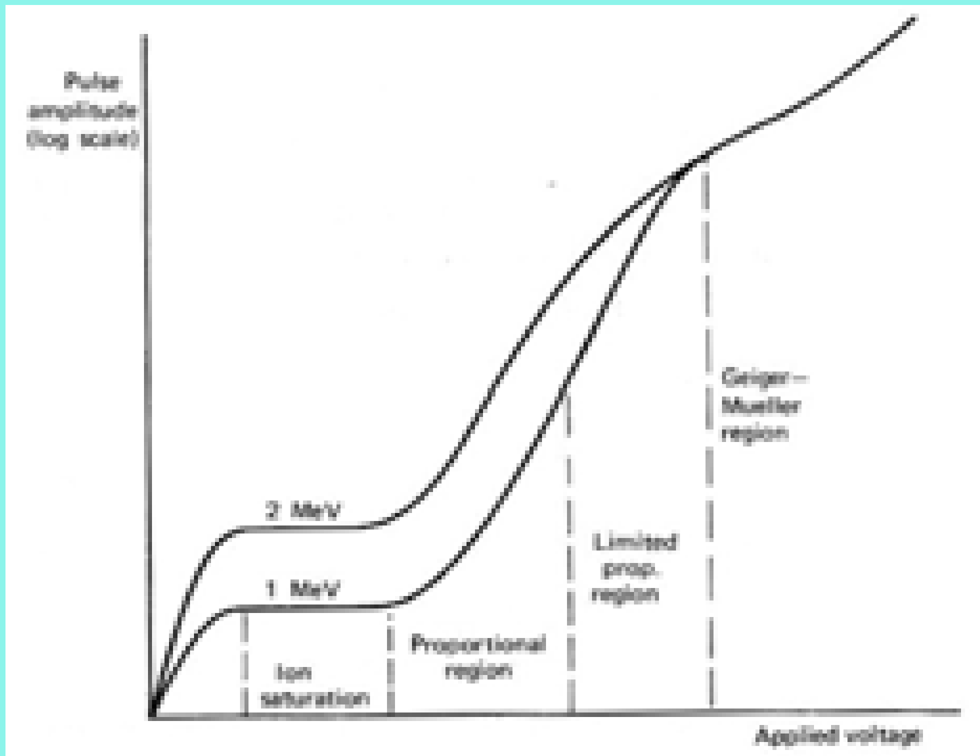
  for dose calibrator and accurate survey meter

- **Geiger-Muller counter**

  filled with quenching gas

  for sensitive survey meter and area monitor





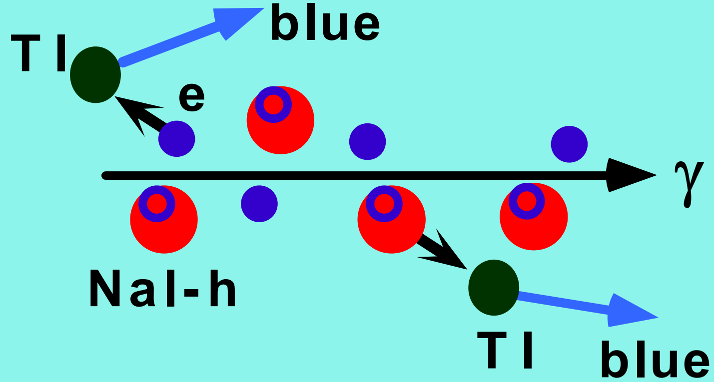
Personnel dosimeter badges

- **thermoluminescent dosimeter (TLD): To measure cumulative dose over weeks or months**
- **LiF chip: exposed to radiation – heated – emitting light – measured using PMT**
- **Al_2O_3 chip: exposed to radiation – optically stimulated – emitting light**
- **pocket dosimeter: ionization chamber, to measure dose for a short period of time**

Scintillation detectors

- **NaI(Tl) for well counter and gamma camera**
- **BGO, LSO, GSO, LYSO for dedicated PET scanner**

one γ photon → e-h pairs → exciting Tl → Tl back to g.s. → blue light (~ 3 eV photons) → e^- → one electrical pulse



Scintillation detectors

- **activity: number of the pulses**
- **energy: number of the blue photons**
- **linearity of scintillation: The number of blue photons, so the pulse height, is linearly proportional to the γ photon energy.**

Production of radioactive nuclides

- **nuclear reactor for producing neutron-rich nuclides (β^- emitters)**
- **particle accelerator (cyclotron) for producing proton-rich nuclides (β^+ emitters)**
- **generator for producing Tc-99m and Rb-82**

Nuclear reactor

- **produce neutron-rich nuclides (β^- emitters)**

$^{14}N(n,p)^{14}C$: $^{14}N + n = {}^{14}C + p$

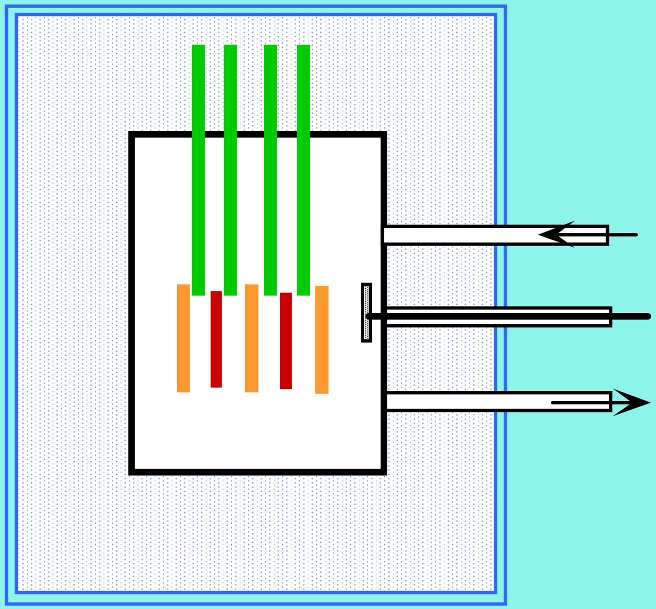
$^{23}Na(n,\gamma)^{24}Na$

$^{31}P(n,\gamma)^{32}P$

$^{32}S(n,p)^{32}P$

$^{130}Te(n,\gamma)^{131}Te \rightarrow {}^{131}I$

$^{99}Mo_{42}$



- **components: uranium, control rods (Cd), moderator (D_2O or graphite), coolant**

Cyclotron

produce proton-rich nuclides (β^+ emitters)

$^{11}B(p,n)^{11}C$: $^{11}B + p = {}^{11}C + n$

$^{12}C(d,n)^{13}N$

$^{14}N(d,n)^{15}O$

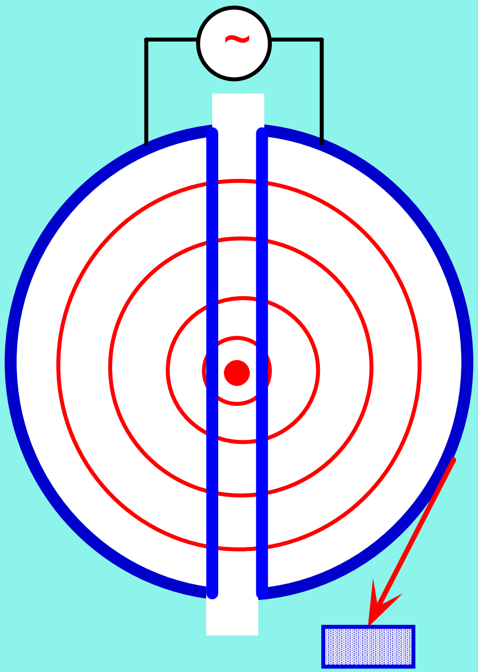
$^{18}O(p,n)^{18}F$, $^{20}Ne(d,\alpha)^{18}F$

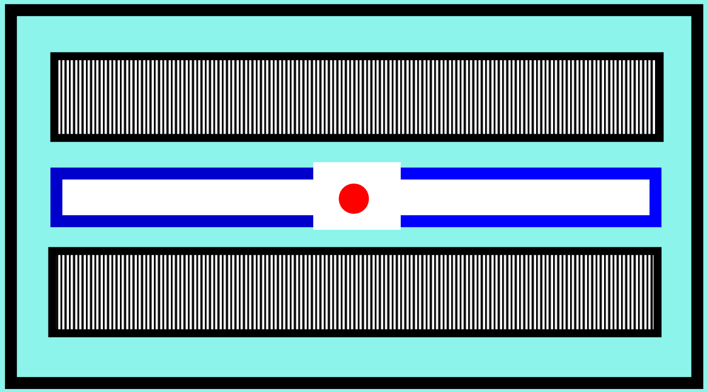
$^{111}Cd(p,n)^{111}In$

$^{201}Hg(d,2n)^{201}Tl$

Cyclotron

- **accelerate charged particle beam (E_p ~ 10 MeV) to increase the probability of the (p,n) reaction**
- **components: magnets, high-frequency AC voltage, Dees, ion source, target chambers**

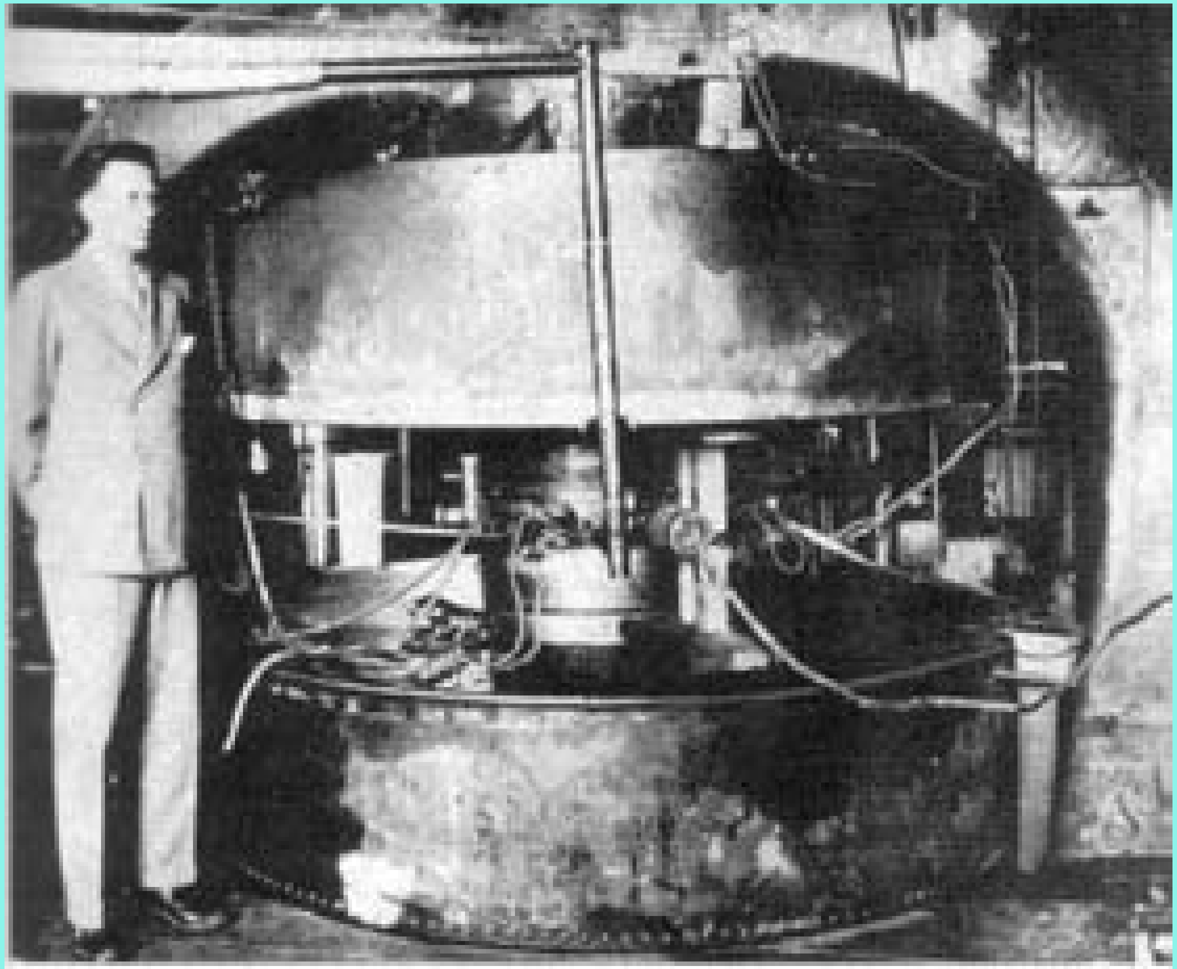




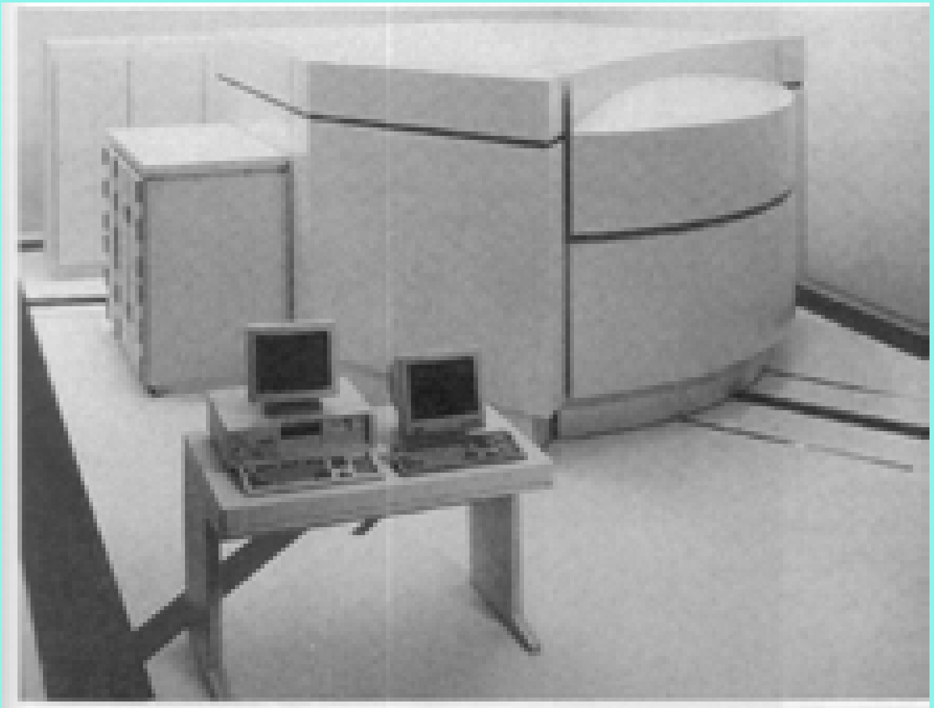
Cyclotron

- **magnetic field: keeping charged particle rotating**
- **AC: increasing energy of particle in the gap**
- **ion source: producing charged particles**
- **target chamber: nuclear reaction**

The first cyclotron (1930s)

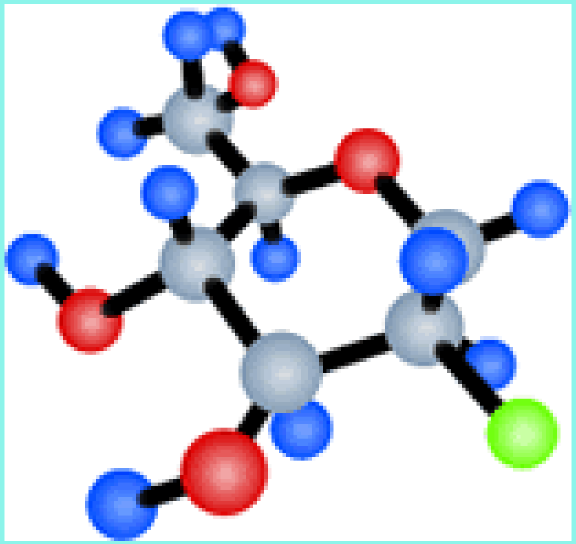


Medical cyclotron



Biosynthesizer (hot cell)

attach the cyclotron-produced radionuclide to a biological compound, e.g. F-18 to FDG (2-deoxy-2 [18F]fluoro-D-glucose)



Generator

produce Tc-99m or Rb-82

parent	$T_{1/2}$	child	decay	$T_{1/2}$
^{82}Sr	25 d	^{82}Rb	β^+,EC	1.3 m
^{99}Mo	66 h	^{99m}Tc	IT	6 h
^{68}Ge	275 d	^{68}Ga	β^+,EC	68 m
^{113}Sn	120 d	^{113m}In	IT	100 m

Generator

- **components: alumina column, lead shield, air filter, saline eluent**
- **MoO_4^{--} bound to alumina (Al_2O_3) but $^{99m}TcO_4^-$ not strongly bound**
- **eluted with 5 to 25 ml saline: ~ 80% ^{99m}Tc extracted in one elution**