MODERN PHYSICS

For Scientists and Engineers

Second Edition

STEPHEN T. THORNTON
University of Virginia

ANDREW REX
University of Puget Sound

SAUNDERS GOLDEN SUNBURST SERIES

SAUNDERS COLLEGE PUBLISHING
Harcourt Brace College Publishers
Fort Worth Philadelphia San Diego New York Orlando
Austin San Antonio Toronto Montreal London Sydney Tokyo

CHAPTER 3

The Experimental Basis of Quantum Theory

As far as I can see, our ideas are not in contradiction to the properties of the photoelectric effect observed by Mr. Lenard.

Max Planck, 1905

As was discussed in Chapter 1, during the final decades of the 1800s scientists discovered phenomena that could not always be explained by what we now call classical physics. Many scientists, however, were not concerned with these discrepancies. The level of experimentation was such that uncertainties were large, and the results of the experiments were often slow in being reported to other investigators. But perhaps more important was the confident attitude of physical scientists that Newton's laws and Maxwell's equations contained the fundamental description of nature.

In this atmosphere it is indeed surprising that the few exceptions to the classical laws discovered during the latter part of the nineteenth century led to the fabulous thirty-year period of 1900–1930, when our understanding of the laws of physics was dramatically changed. We have already discussed in Chapter 2 the first of these new developments, the special theory of relativity, which was introduced by Einstein in 1905 and successfully explained the null result of the Michelson-Morley experiment. The other great conceptual advance of 20th-century physics, the quantum theory, began in 1900 when Max Planck introduced his explanation of blackbody radiation.

We begin this chapter by learning of Röntgen's discovery of the x ray and Thomson's discovery of the electron. Millikan later determined the electron's charge. We shall see that, although it was necessary to assume that certain physical quantities may be quantized, scientists found this idea hard to accept. We will discuss the difficulties of explaining blackbody radiation with classical physics and how Planck's proposal solved the problem. Finally, we will see that Einstein's explanation of the photoelectric effect and Compton's understanding of x-ray scattering data made the quantum hypothesis difficult to refute. After many difficult and painstaking experiments, it became clear that quantization was not only necessary, it was also the correct description of nature.

3.1 Discovery of the X Ray and the Electron

In the late 1800s scientists and engineers were familiar with the "cathode rays" that could easily be generated from one of the metal plates in an evacuated tube across which a large electric potential had been established. The origin and constitution of these cathode rays were not known. The concept of an atomic substructure of matter was widely accepted because of its use in explaining the results of chemical experiments. Therefore, it was felt that cathode rays might have something to do with atoms. It was known, for example, that cathode rays could penetrate matter. Cathode rays were of great interest and under intense investigation in the late 1800s.

In 1895 Wilhelm Röntgen (1845–1923), who had received early training as a mechanical engineer but was at the time a professor of physics at the University of Würzburg in Germany, was studying the effects of cathode rays passing through various materials. During one such experiment he noticed that a nearby phosphorescent screen was glowing vividly in the darkened room. Röntgen soon realized he was observing a new kind of ray, one that, unlike cathode rays, was unaffected by magnetic fields and was far more penetrating than cathode rays. These **x rays,** as he called them, were apparently produced by the cathode rays bombarding the glass walls of his vacuum tube. Röntgen studied their transmission through many materials and even showed that he could obtain an image of the bones in a hand when the x rays were allowed to pass through as shown in Figure 3.1. This experiment created tremendous excitement, and medical applications of x rays were quickly developed. For this discovery, Röntgen received the first Nobel Prize award for physics in 1901.

New penetrating ray: x ray

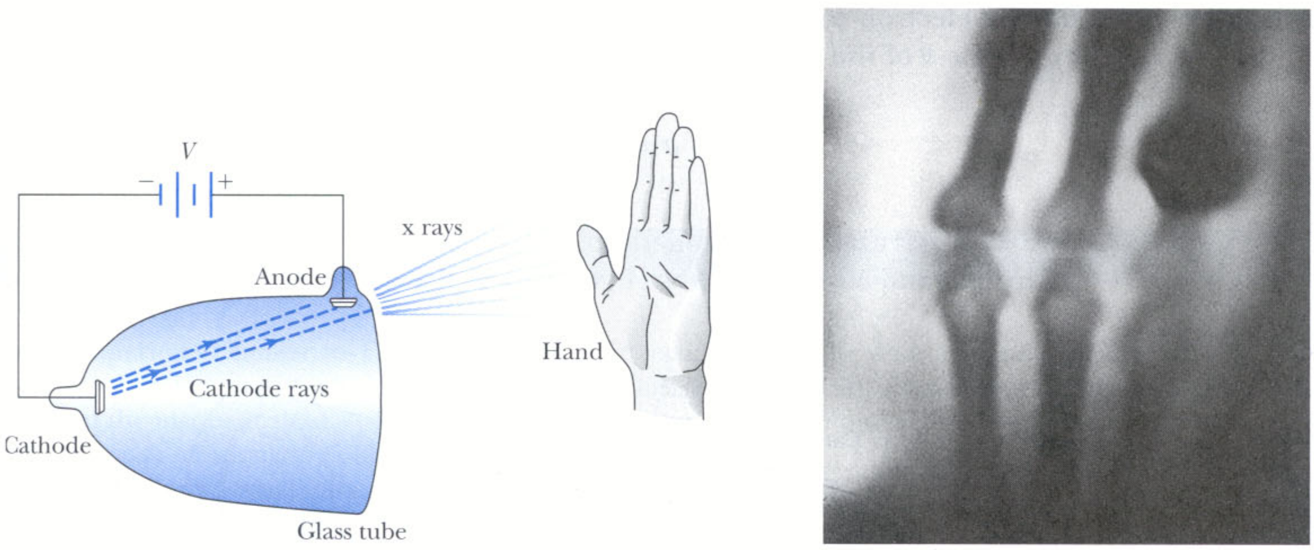


FIGURE 3.1 In Röntgen's experiment "x rays" were produced by cathode rays (electrons) hitting the glass near the anode. He studied the penetration of the x rays through several substances and even noted that if the hand was held between the glass tube and a screen, the darker shadow of the bones could be discriminated from the less dark shadow of the hand. *Photo courtesy of Deutsches Museum, München.*

Sir Joseph John Thomson, universally known as "J.J.," went to Cambridge University at age 20 and remained there for the rest of his life. Thomson's career with the Cavendish Laboratory spanned a period of over 50 years during which seven Nobel Prizes in physics were awarded. He served as Director from 1884 until 1918 when he stepped down in favor of Rutherford. *AIP Emilio Segrè Visual Archives.*

For several years before the discovery of x rays, J. J. Thomson (1856–1940), professor of experimental physics at Cambridge University, had been studying the properties of electrical discharges in gases. Thomson's apparatus was similar to that used by Röntgen and many other scientists because of its simplicity (see Figure 3.2). Thomson believed that cathode rays were particles, whereas several respected German scientists (such as H. Hertz) believed they were wave phenomena. Thomson was able to prove in 1897 that the charged particles emitted from a heated electrical cathode were in fact the same as cathode rays. The main features of Thomson's experiment are shown in the schematic apparatus of Figure 3.2. The rays from the cathode are attracted to the positive potential on aperture A (anode) and are further collimated by aperture B to travel in a straight line to impinge on the fluorescent screen in the rear of the tube, where they can be visually detected by a flash of light. A voltage across the deflection plates sets up an electrostatic field that can deflect charged particles. Previously, in a similar experiment, Hertz had observed no effect on the cathode rays due to the deflecting voltage. Thomson at first found the same result, but upon further evacuating the glass tube observed the deflection and proved that cathode rays had a negative charge. The previous experiment, in a poorer vacuum, had failed because the cathode rays had interacted with and ionized the residual gas. Thomson also studied the effects of a magnetic field upon the cathode rays by placing current coils outside the glass tube. He proved convincingly that the cathode rays acted as charged particles (electrons) in both electric and magnetic fields and received the Nobel Prize in 1906.

Measurement of electron's e/m

Thomson's method of measuring the ratio of the electron's charge to mass, e/m, is now a standard technique and generally studied as an example of charged particles passing through perpendicular electric and magnetic fields as shown schematically in Figure 3.3. With the magnetic field turned off, the electron entering the region between the plates is accelerated upward by the electric field

$$F_y = ma_y = qE \tag{3.1}$$

where m and q are, respectively, the mass and charge of the electron. The time for the electron to traverse the deflecting plates (length $= \ell$) is $t \approx \ell/v_0$. The exit angle θ of the electron is then given by

$$\tan\theta = \frac{v_y}{v_x} = \frac{a_y t}{v_0} = \frac{qE}{m}\frac{\ell}{{v_0}^2} \tag{3.2}$$

The ratio q/m can be determined if the velocity is known. By turning on the magnetic field and adjusting the strength of **B** so that no deflection of the electron

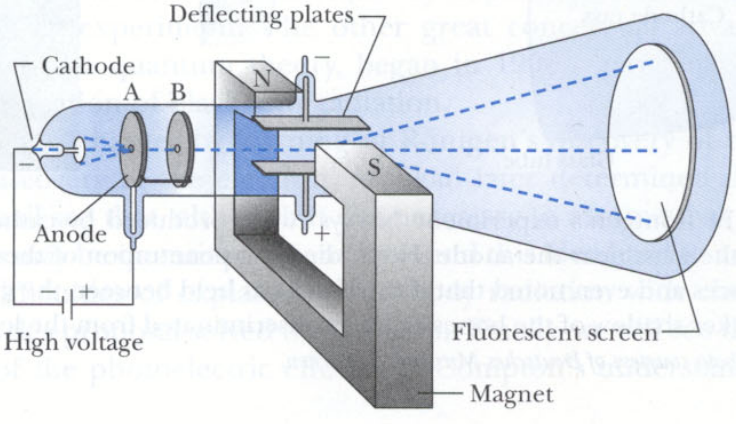


FIGURE 3.2 Apparatus of Thomson's cathode-ray experiment. Thomson proved that the rays emitted from the cathode were negatively charged particles (electrons) by deflecting them in electric and magnetic fields. The key to the experiment was to evacuate the glass tube.

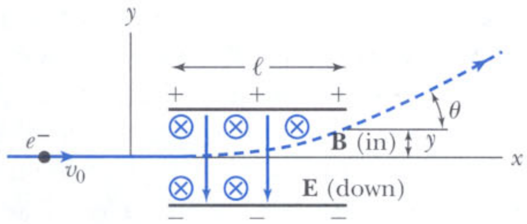


FIGURE 3.3 Thomson's method of measuring the electron's charge to mass ratio was to send electrons through a region containing a magnetic field (**B** into paper) perpendicular to an electric field (**E** down). The electrons having $v = E/B$ go through undeflected. Then, using the same energy electrons, the magnetic field is turned off and the electric field deflects the electrons, which exit at angle θ. The ratio of e/m can be determined from **B**, **E**, θ, and ℓ, where ℓ is the length of the field distance and θ is the emerging angle. See Equation (3.5).

occurs, the velocity can be determined. The condition for zero deflection is that the net force on the electron must be zero.

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} = 0 \tag{3.3}$$

Hence,

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B}$$

or because **v** and **B** are perpendicular, the electric and magnetic field strengths are related by

$$|\mathbf{E}| = |v_x||\mathbf{B}|$$

so that

$$v_x = \frac{E}{B} = v_0 \tag{3.4}$$

If we insert this value for v_0 into Equation (3.2), we extract the ratio of q/m.

$$\frac{q}{m} = \frac{v_0^2 \tan\theta}{E\ell} = \frac{E \tan\theta}{B^2\ell} \tag{3.5}$$

Example 3.1

In an experiment similar to Thomson's, we use deflecting plates 5 cm in length with an electric field of 1.0×10^4 V/m. Without the magnetic field we find an angular deflection of 30°, and with a magnetic field of 8×10^{-4} T (8 gauss) we find no deflection. What is the initial velocity of the electron and its q/m?

Solution: We find the electron's velocity v_0 from Equation (3.4).

$$v_0 = \frac{E}{B} = \frac{1.0 \times 10^4\ \mathrm{V/m}}{8.0 \times 10^{-4}\ \mathrm{T}} = 1.25 \times 10^7\ \mathrm{m/s}$$

Because we use all units for E and B in the international system (SI), the answer must be in meters/second.

Now we can determine q/m by using Equation (3.5):

$$\frac{q}{m} = \frac{E \tan\theta}{B^2\ell} = \frac{(1.0 \times 10^4\ \mathrm{V/m})(\tan 30°)}{(8 \times 10^{-4}\ \mathrm{T})^2(0.05\ \mathrm{m})}$$

$$= 1.80 \times 10^{11}\ \mathrm{C/kg}$$

Thomson's actual experiment, done in the manner of the previous example, obtained a result about 35% lower than the presently accepted value of 1.76×10^{11} C/kg for e/m. Thomson realized that the value of e/m (e = absolute value of electron charge) for an electron was much larger than had been anticipated and a factor of 1000 larger than any value of q/m that had been previously measured (for the hydrogen atom). He concluded that either m was small or e was large (or both), and the "carriers of the electricity" were quite penetrating as compared to atoms or molecules, which must be much larger in size.

3.2 Determination of Electron Charge

After Thomson's measurement of e/m and the confirmation of the cathode ray as the charge carrier (called *electron*), several investigators attempted to determine the electron's charge, which was poorly known in 1897. In 1911 the American physicist Robert A. Millikan (1865–1953) reported convincing evidence for an accurate determination of the electron's charge. Millikan's classic experiment began in 1907 at the University of Chicago. The experiment consisted of visual observation of the motion of uncharged and both positively and negatively charged oil drops moving under the influence of electrical and gravitational forces. The essential parts of the apparatus are shown in Figure 3.4. As the drops emerge from the nozzle, frictional forces sometimes cause them to be charged. Millikan's method consisted of balancing the upward force of the electric field between the plates against the downward force of the gravitational field.

Millikan's oil drop experiment

When an oil drop falls downward through the air, it experiences a frictional force $\mathbf{F}_f$ proportional to its velocity due to the air's viscosity:

$$\mathbf{F}_f = -b\mathbf{v} \tag{3.6}$$

The force has a minus sign because it always opposes the drop's velocity. The constant b is determined by Stokes's law and is proportional to the oil drop's radius. Millikan showed that Stokes's law for the motion of a small sphere through

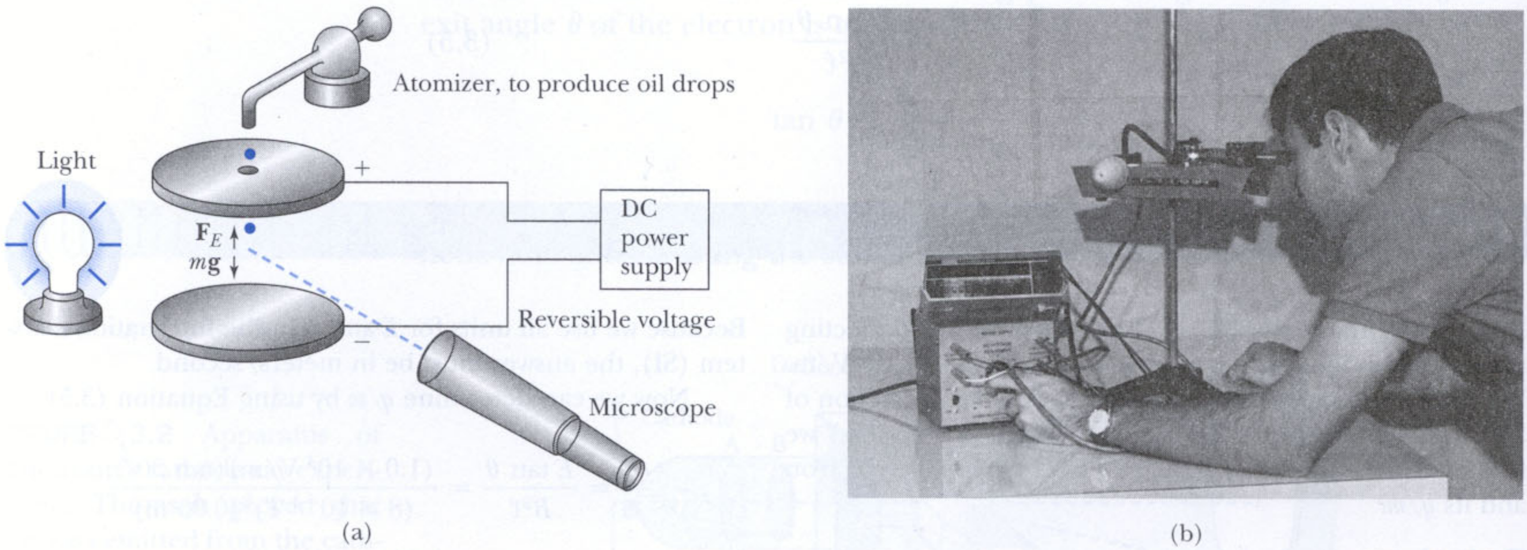
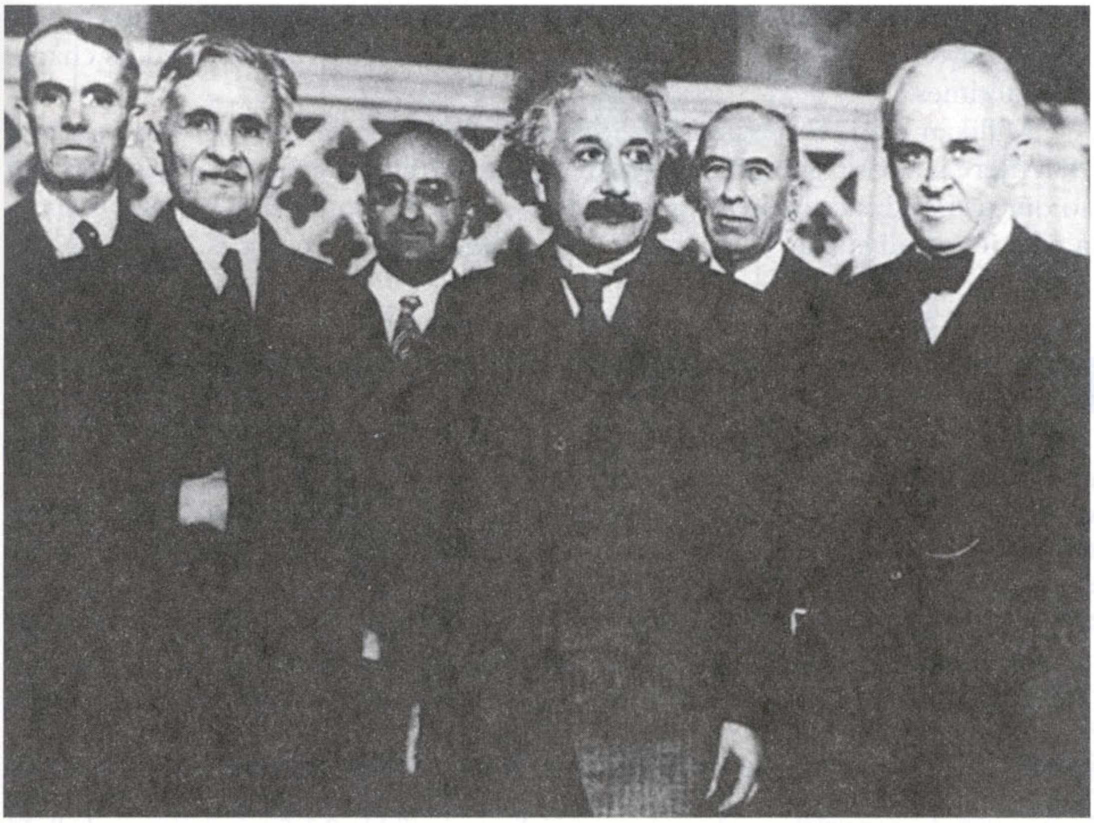


FIGURE 3.4 (a) Diagram of the Millikan oil-drop experiment to measure the charge of the electron. Some of the oil drops from the atomizer emerge charged, and the electric field (voltage) is varied to slow down or reverse the direction of the oil drops, which can have positive or negative charges. (b) A student looking through the microscope is adjusting the voltage between the plates to slow down a tiny plastic ball that serves as the oil drop.



Three great physicists (foreground), 1931: Michelson, Einstein, and Millikan. *Courtesy of California Institute of Technology.*

a resisting medium becomes incorrect for small-diameter spheres because of the atomic nature of the medium, and he found the appropriate correction. The buoyancy of the air produces an upward force on the drop, but we can neglect this effect for a first-order calculation.

To suspend the oil drop at rest between the plates, the upward electric force must equal the downward gravitational force. The frictional force is then zero because the velocity of the oil drop is zero.

$$\mathbf{F}_E = q\mathbf{E} = -m\mathbf{g} \qquad (\text{when } v = 0) \tag{3.7}$$

The magnitude of the electric field is $E = V/d$ and V is the voltage across large, flat plates separated by a small distance d. The magnitude of the electron charge q may then be extracted as

$$q = \frac{mgd}{V} \tag{3.8}$$

To calculate q we have to know the mass m of the oil drops. Millikan found he could determine m by turning off the electric field and measuring the terminal velocity of the oil drop. The radius of the oil drop is related to the terminal velocity by Stokes's law (see Problem 7). The mass of the drop can then be determined by knowing the radius r and density ρ of the type of oil used in the experiment:

$$m = \frac{4}{3}\pi r^3 \rho \tag{3.9}$$

If the power supply has a switch to reverse the polarity of the voltage and an adjustment for the voltage magnitude, the oil drop can be moved up and down in the apparatus at will. Millikan reported that in some cases he was able to observe a given oil drop for up to six hours and that the drop changed its charge several times.

Measurement of electron charge

Millikan made thousands of measurements using different oils and showed that there is a basic quantized electron charge. Millikan's value of e was very close to our presently accepted value of 1.602×10^{-19} C. Notice that we always quote a positive number for the charge e. The charge on an electron is then $-e$.

Example 3.2

For an undergraduate physics laboratory experiment we often make two changes in Millikan's procedure. First, we use plastic balls of about 1 micrometer (μm or micron) in diameter, for which we can measure the mass easily and accurately. This avoids the measurement of the oil drop's terminal velocity and the dependence on Stokes's law. The small plastic balls are still sprayed through an atomizer in liquid solution, but the liquid soon evaporates in air. The plastic balls are easily seen by a microscope. One other improvement is to bombard the region between the plates occasionally with ionizing radiation (such as x rays or α particles from radioactive sources). This radiation ionizes the air and makes it easier for the charge on a ball to change. By making many measurements we can determine whether the charges determined from Equation (3.8) are multiples of some basic charge unit.

One problem in the experiment is that occasionally one obtains fragments of broken balls or clusters of several balls. These can be eliminated by watching the flight of balls in free fall. The majority of balls will be single and fall faster than fragments, but slower than clusters. With a little experience one can select single unbroken balls.

In an actual undergraduate laboratory experiment the mass of the balls was $m = 5.7 \times 10^{-16}$ kg and the spacing eween the plates was $d = 4$ mm. Therefore q can be found from Equation (3.8).

$$q = \frac{mgd}{V} = \frac{(5.7 \times 10^{-16}\ \text{kg})(9.8\ \text{m/s}^2)(4 \times 10^{-3}\ \text{m})}{V}$$

$$q = \frac{(2.23 \times 10^{-17}\ \text{V})}{V}\text{C}$$

where V is the voltage between plates when the observed ball is stationary. Two students observed 30 balls and found the values of V shown in Table 3.1 for a stationary ball. In this experiment the voltage polarity can easily be changed, and a positive voltage represents a ball with a positive charge. Notice that charges of both signs are observed.

The values of $|q|$ are plotted on a histogram in units of $\Delta q = 0.2 \times 10^{-19}$ C. These are shown by the solid area in Figure 3.5. When 70 additional measurements from other students are added, a clear pattern of quantization develops with a charge $q = nq_0$, especially for the first three groups. The groups become increasingly smeared out for higher charges. The areas of the histogram can be separated for the various n values, and the value of q_0 found for each measurement is then averaged. For the histogram shown we find $q_0 = 1.7 \times 10^{-19}$ C for the first 30 measurements and $q_0 = 1.6 \times 10^{-19}$ C for all 100 observations.

TABLE 3.1
Student Measurements in Millikan Experiment

Particle	Voltage (V)	$q(\times 10^{-19}$ C)	Particle	Voltage	q	Particle	Voltage	q
1	−30.0	−7.43	11	−126.3	−1.77	21	−31.5	−7.08
2	+28.8	+7.74	12	−83.9	−2.66	22	−66.8	−3.34
3	−28.4	−7.85	13	−44.6	−5.00	23	+41.5	+5.37
4	+30.6	+7.29	14	−65.5	−3.40	24	−34.8	−6.41
5	−136.2	−1.64	15	−139.1	−1.60	25	−44.3	−5.03
6	−134.3	−1.66	16	−64.5	−3.46	26	−143.6	−1.55
7	+82.2	+2.71	17	−28.7	−7.77	27	+77.2	+2.89
8	+28.7	+7.77	18	−30.7	−7.26	28	−39.9	−5.59
9	−39.9	−5.59	19	+32.8	+6.80	29	−57.9	−3.85
10	+54.3	+4.11	20	−140.8	+1.58	30	+42.3	+5.27

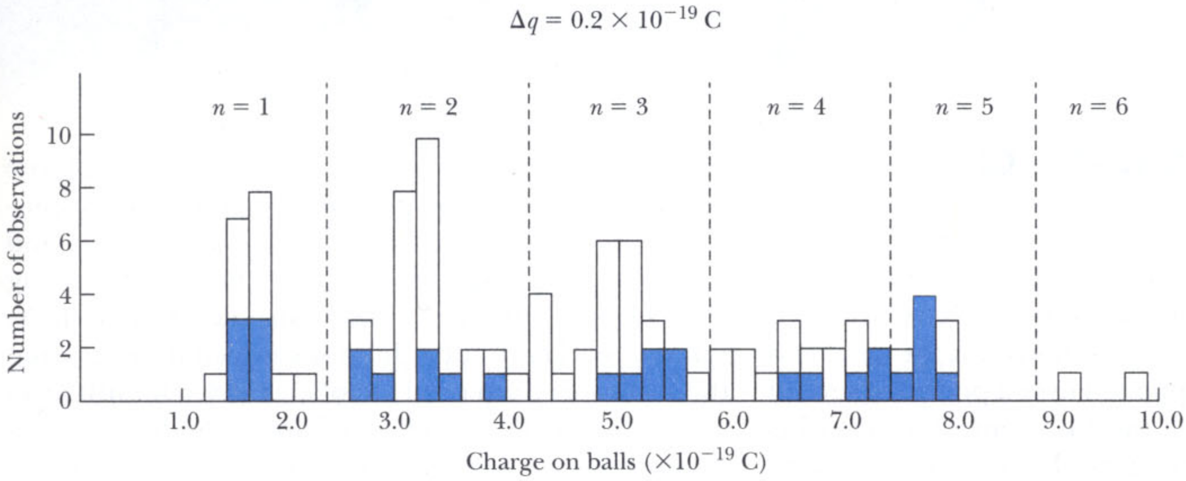


FIGURE 3.5 A histogram of the number of observations for the charge on a ball in a student Millikan experiment. The histogram is plotted for $\Delta q = 0.2 \times 10^{-19}$ C. The solid area refers to the first group's 30 measurements, and the open area to another 70 measurements. Notice the peaks, especially for the first three ($n = 1, 2, 3$) groups, indicating the electron charge quantization. When the basic charge q_0 is found from $q = nq_0$ (n = integer), $q_0 = 1.6 \times 10^{-19}$ C was determined in this experiment from all 100 observations.

3.3 Line Spectra

In the early 1800s optical spectroscopy became an important area of experimental physics, primarily because of the development of diffraction gratings. It had already been demonstrated that the chemical elements produced unique colors when burned in a flame or when excited in an electrical discharge. Prisms had been used to investigate these sources of spectra.

An example of a spectrometer used to observe optical spectra is shown in Figure 3.6. An electrical discharge excites atoms of a low-pressure gas contained in the tube. The collimated light passes through a diffraction grating with thousands of ruling lines per centimeter, and the diffracted light is separated at angle θ according to its wavelength λ. The equation expressing diffraction maxima is

$$d \sin \theta = n\lambda \tag{3.10}$$

where d is the distance between rulings, and n (an integer) is called the order number ($n = 1$ has the strongest scattered intensity). The resulting pattern of

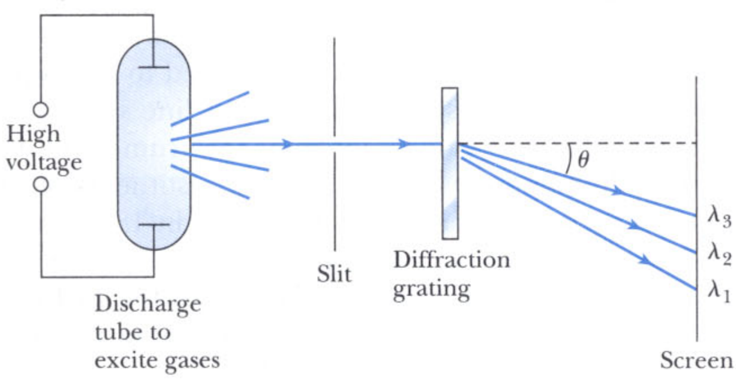


FIGURE 3.6 Schematic of an optical spectrometer. Light produced by a high-voltage discharge in the glass tube is collimated and passed through a diffraction grating, where it is deflected according to its wavelength. See Equation (3.10).

THE DISCOVERY OF HELIUM

It might seem that the discovery of helium, the second simplest of all elements, would have occurred centuries ago. As we shall see, this is not the case, and in fact the discovery happened over a period of several years in the latter part of the 19th century as scientists were scrambling to understand unexpected results. The account here is taken from *Helium*, by William H. Keesom.*

Spectroscopes, optical devices used to measure wavelengths of light, normally consist of a slit, a collimating lens, and a prism to refract the light. Their first use in a solar eclipse was on August 18, 1868, to investigate the sun's atmosphere. Several persons traveled to the total eclipse region in India and Malaysia (including P. J. C. Janssen, G. Rayet, C. T. Haig, and J. Herschel) and all reported, either directly or indirectly, to have observed an unusual yellow line in the spectra that would later be proven due to helium. It occurred to Janssen the day of the eclipse that it must be possible to see the sun's spectrum directly without the benefit of the eclipse, and he did so with a spectroscope on the days following the eclipse. The same idea had occurred to J. N. Lockyer earlier, but he did not succeed in measuring the sun's spectrum until October, 1868, a month or so after Janssen. This method of observing the sun's atmosphere at any time was considered to be an important discovery, and Janssen and Lockyer are prominently recognized not only for the evolution of helium's discovery but for the means of studying the sun's atmosphere as well.

The actual discovery of helium was delayed by the fact that the new yellow line seen in the sun's atmosphere was very close in wavelength to two well-known yellow lines of sodium. This is apparent in the atomic line spectra of both helium and sodium seen on the inside back cover of this text. Certainly the line spectra of many elements were known by 1898, and scientists were busy cataloguing each element's characteris-

---

light bands and dark areas on the screen is called a *line spectrum*. By 1860 Bunsen and Kirchhoff realized the usefulness of the wavelengths of these line spectra in allowing identification of the chemical elements and the composition of materials. It was discovered that each element had its own characteristic wavelengths as shown on the inside back cover. The field of spectroscopy flourished because finer and more evenly ruled gratings became available, and improved experimental techniques allowed more spectral lines to be observed and catalogued. Particular interest was paid to the sun's spectrum in hopes of understanding the origin of sunlight. The helium atom was actually "discovered" by its line spectra from the sun before it was recognized on Earth (see Special Topic).

Characteristic line spectra of elements

Many scientists believed that the lines in the spectra somehow reflected the complicated internal structure of the atom, and that by carefully investigating the wavelengths for many elements, the structure of atoms and matter could be understood. That belief was eventually partially realized.

Balmer's empirical result

For much of the 19th century scientists attempted to find some simple underlying order for the characteristic wavelengths of line spectra. Hydrogen appeared to have an especially simple-looking spectrum, and because some chemists thought hydrogen atoms might be the constituents of heavier atoms, hydrogen was singled out for intensive study. Finally, in 1885, Johann Balmer, a Swiss schoolteacher, succeeded in obtaining a simple empirical formula that fit the wavelengths of the fourteen lines then known in the hydrogen spectrum. Four lines were in the visible region, and the remaining ultraviolet lines had tic spectra. By December, 1868, Lockyer, A. Secchi, and Janssen each independently recognized that the yellow line was different than that of sodium.

Another difficulty was to prove that the new yellow line, called D_3, was not due to some other known element, especially hydrogen. For many years Lockyer thought that D_3 was related to hydrogen and he and E. Frankland performed several experiments that were not able to prove his thesis. Lockyer wrote as late as 1887 that D_3 was a form of hydrogen. However, in contradiction, Lord Kelvin reported in 1871 during his presidential address to the British Association that Frankland and Lockyer could not find the D_3 line to be related to any terrestrial (from Earth) flame. Kelvin reported that it seemed to represent a new substance, which Frankland and Lockyer proposed to call helium (from the Greek word for "sun").

It was not until 1895 that helium was finally clearly observed on Earth by Sir William Ramsay, who had received a letter reporting that W. F. Hillebrand had produced nitrogen gas by boiling uranium ores *(pitchblende)* in dilute sulphuric acid. Ramsay was skeptical of the report and proceeded to reproduce it. He was astounded, after finding a small amount of nitrogen and the expected argon gas, to see a brilliant yellow line that he compared with those from sodium, finding the wavelengths to be different. Sir William Crookes measured the wavelength and reported the following day that it was the D_3 line, proving the terrestrial existence of helium. Later in 1895 H. Kayser found the helium line in spectra taken from a gas that had evolved from a spring in Germany's Black Forest. Eventually, in 1898, helium was confirmed in the Earth's atmosphere by E. C. Baly. No one person can be credited for the discovery of helium.

The remarkable properties of *liquid* helium are discussed in Section 9.7.

*W. H. Keesom, *Helium.* Amsterdam, London, and New York: Elsevier, 1942.

---

been identified in the spectra of white stars. This series of lines, called the *Balmer series,* is shown in Figure 3.7. Balmer found that the expression

$$\lambda = 364.56 \frac{k^2}{k^2 - 4} \text{ nm} \tag{3.11}$$

(where $k = 3, 4, 5, \ldots; k > 2$) fit all the visible hydrogen lines. Wavelengths are normally given in units of nanometers* (nm). It is more convenient to take the inverse of Equation (3.11) and write Balmer's formula in the form

$$\frac{1}{\lambda} = \frac{1}{364.56 \text{ nm}} \frac{k^2 - 4}{k^2} = \frac{4}{364.56 \text{ nm}}\left(\frac{1}{2^2} - \frac{1}{k^2}\right) = R_H\left(\frac{1}{2^2} - \frac{1}{k^2}\right) \tag{3.12}$$

*Wavelengths were formerly listed in units of angstroms (one angstrom (Å) = 10^{-10} m), named after Ångstrom who was one of the first persons to observe and measure the wavelengths of the four visible lines of hydrogen.

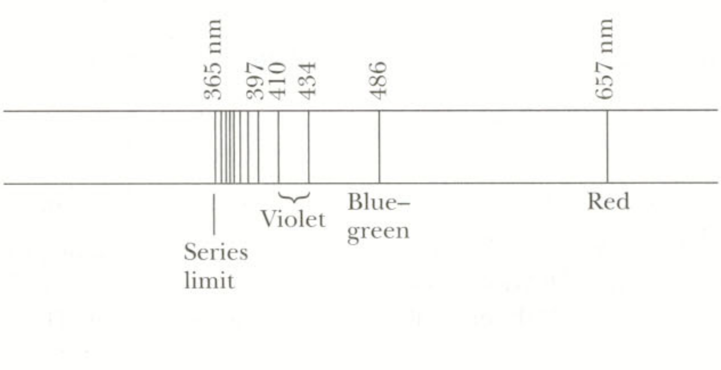


FIGURE 3.7 The Balmer series of line spectra of the hydrogen atom with wavelengths indicated in nm. The four visible lines are noted as well as the lower limit of the series.

TABLE 3.2
Hydrogen Series of Spectral Lines

Discoverer (year)	Wavelength	n	k
Lyman (1916)	Ultraviolet	1	>1
Balmer (1885)	Visible, ultraviolet	2	>2
Paschen (1908)	Infrared	3	>3
Brackett (1922)	Infrared	4	>4
Pfund (1924)	Infrared	5	>5

where R_H is called the *Rydberg constant* (for hydrogen) and has the more accurate value 1.096776×10^7 m^{-1}, and k is an integer greater than two ($k > 2$).

Efforts by Johannes Rydberg and particularly Walther Ritz eventually resulted in 1890 in a more general empirical equation for calculating the wavelengths called the *Rydberg equation.*

Rydberg equation

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{k^2}\right) \tag{3.13}$$

where $n = 2$ corresponds to the Balmer series and $k > n$ always. Some 20 years after Balmer's contribution, other series of the hydrogen atom's spectral lines were discovered in the early 1900s. By 1925 five series had been discovered, each having a different integer n (see Table 3.2). The understanding of the Rydberg equation (3.13) and the discrete spectrum of hydrogen were important research topics early in the 20th century.

Example 3.3

The visible lines of the Balmer series were observed first because they are most easily seen. Show that the wavelengths of spectral lines in the *Lyman* ($n = 1$) and *Paschen* ($n = 3$) series are not in the visible region. Find the wavelengths of the four visible atomic hydrogen lines. Assume the visible wavelength region is $\lambda = 400$ to 700 nm.

Solution: We use Equation (3.13) first to examine the Lyman series ($n = 1$)

$$\frac{1}{\lambda} = R_H\left(1 - \frac{1}{k^2}\right)$$

$$= 1.0968 \times 10^7\left(1 - \frac{1}{k^2}\right)\text{m}^{-1}$$

$k = 2$ $\quad \frac{1}{\lambda} = 1.0968 \times 10^7\left(1 - \frac{1}{4}\right)\text{m}^{-1}$

$\lambda = 1.216 \times 10^{-7}$ m $= 121.6$ nm (Ultraviolet)

$k = 3$ $\quad \frac{1}{\lambda} = 1.0968 \times 10^7\left(1 - \frac{1}{9}\right)\text{m}^{-1}$

$\lambda = 1.026 \times 10^{-7}$ m $= 102.6$ nm (Ultraviolet)

Because the wavelengths are decreasing for higher k values, all the wavelengths in the Lyman series are in the ultraviolet region and not visible by eye.

For the Balmer series ($n = 2$) we find

$k = 3$ $\quad \frac{1}{\lambda} = 1.0968 \times 10^7\left(\frac{1}{4} - \frac{1}{9}\right)\text{m}^{-1}$

$\lambda = 6.565 \times 10^{-7}$ m $= 656.5$ nm (Red)

$k = 4$ $\quad \frac{1}{\lambda} = 1.0968 \times 10^7\left(\frac{1}{4} - \frac{1}{16}\right)\text{m}^{-1}$

$\lambda = 4.863 \times 10^{-7}$ m $= 486.3$ nm (Blue-green)

$k = 5$ $\quad \frac{1}{\lambda} = 1.0968 \times 10^7\left(\frac{1}{4} - \frac{1}{25}\right)\text{m}^{-1}$

$\lambda = 4.342 \times 10^{-7}$ m $= 434.2$ nm (Violet)

$k = 6$ $\quad \frac{1}{\lambda} = 1.0968 \times 10^7\left(\frac{1}{4} - \frac{1}{36}\right)\text{m}^{-1}$

$\lambda = 4.103 \times 10^{-7}$ m $= 410.3$ nm (Violet)

$k = 7$ $\quad \frac{1}{\lambda} = 1.0968 \times 10^7\left(\frac{1}{4} - \frac{1}{49}\right)\text{m}^{-1}$

$\lambda = 3.971 \times 10^{-7}$ m $= 397.1$ nm (Ultraviolet)

Therefore $k = 7$ and higher k values will be in the ultraviolet region. The four lines $k = 3$, 4, 5, and 6 of the Balmer series are visible, although the 410 nm ($k = 6$) line is difficult to see because it is barely in the visible region and is weak in intensity.

The next series, $n = 3$, named after Paschen, has wavelengths of

$$k = 4 \qquad \frac{1}{\lambda} = 1.0968 \times 10^7 \left(\frac{1}{9} - \frac{1}{16}\right) \text{m}^{-1}$$

$$\lambda = 1.876 \times 10^{-6}\ \text{m} = 1876\ \text{nm} \qquad \text{(Infrared)}$$

$$k = 5 \qquad \frac{1}{\lambda} = 1.0968 \times 10^7 \left(\frac{1}{9} - \frac{1}{25}\right) \text{m}^{-1}$$

$$\lambda = 1.282 \times 10^{-6}\ \text{m} = 1282\ \text{nm} \qquad \text{(Infrared)}$$

$$k = \infty \qquad \frac{1}{\lambda} = 1.0968 \times 10^7 \left(\frac{1}{9} - \frac{1}{\infty}\right) \text{m}^{-1}$$

$$\lambda = 8.206 \times 10^{-7}\ \text{m} = 820.6\ \text{nm} \qquad \text{(Infrared)}$$

Thus the Paschen series has wavelengths entirely in the infrared region. Notice that the series limit is found for $k = \infty$. The higher series, $n \geq 4$, will all have wavelengths longer than the visible region.

3.4 Quantization

As we discussed in Chapter 1, some early Greek philosophers believed that matter must be composed of fundamental units that could not be divided further. The word "atom" means "not further divisible." Today some scientists believe, as these ancient philosophers did, that matter must eventually be indivisible. However, as we have encountered new experimental facts, our ideas about the fundamental, indivisible "building blocks" of matter have changed. More will be said about the "elementary" particles in Chapter 14.

Is matter indivisible?

Whatever the elementary units of matter may eventually turn out to be, we suppose there are some basic units of mass-energy of which matter is composed. This idea is hardly foreign to us: we have seen already that Millikan's oil drop experiment showed the quantization of electric charge. Current theories predict that charges are quantized in units (called **quarks**) of $\pm e/3$ and $\pm 2e/3$, but quarks can not be directly observed experimentally. The charges of particles that have been directly observed are quantized in units of $\pm e$.

Electric charge is quantized

In nature we see other examples of quantization. The measured atomic weights are not continuous—they have only discrete values which are close to integral multiples of a unit mass. Molecules are formed from an integral number of atoms. The water molecule is made up of exactly two atoms of hydrogen and one of oxygen. The fact that an organ pipe produces one fundamental musical note with overtones is a form of quantization arising from fitting a precise number (or fractions) of sound waves into the pipe.

Quantization occurs often in nature

The line spectra of atoms discussed in the previous section again show that characteristic wavelengths have precise values and are not distributed continuously. By the end of the 19th century, radiation spectra had been well studied. There certainly didn't appear to be any quantization effects observed in blackbody radiation spectra emitted by hot bodies. However, these radiation spectra were to have a tremendous influence on the discovery of quantum physics.

3.5 Blackbody Radiation

It has been known for many centuries that when matter is heated, it emits radiation. We can feel the heat radiation emitted by the heating element of an electric stove as it warms up. As the heating element reaches 550°C, its color becomes dark red, turning to bright red around 700°C. If the temperature were increased still further, the color would progress through orange, yellow, and finally white. We can determine experimentally that a broad spectrum of wavelengths is emitted when matter is heated. This process was of great interest to physicists of the nineteenth century. They measured the intensity of radiation being emitted as a function of material, temperature, and wavelength.

Radiation emission and absorption

All bodies simultaneously emit and absorb radiation. When a body's temperature is constant in time, the body is said to be in *thermal equilibrium* with its surroundings. In order for the temperature to be constant, the body must absorb thermal energy at the same rate as it emits it. This implies that a good thermal emitter is also a good absorber.

Physicists generally try to study the simplest or most idealized case of a problem first in order to gain the insight that is needed to analyze more complex situations. For thermal radiation the simplest case is a **blackbody**, which has the ideal property that it absorbs all the radiation falling on it and reflects none. The simplest way to construct a blackbody is to drill a small hole in the wall of a hollow container as shown in Figure 3.8. Radiation entering the hole will be reflected around inside the container and then finally absorbed. Only a small fraction of the entering rays will be re-emitted through the hole. If the blackbody is in thermal equilibrium, then it must also be an excellent emitter of radiation as well.

Blackbody radiation is unique

Blackbody radiation is theoretically interesting because of its universal character: the radiation properties of the blackbody (that is, the cavity) are independent of the particular material of which the container is made. Physicists could study the previously mentioned properties of intensity vs. wavelength (called *spectral distribution*) at fixed temperatures without having to understand the details of emission or absorption by a particular kind of atom. The question of precisely what the thermal radiation actually consisted of was also of interest, although it was assumed, for lack of evidence to the contrary (and correctly, it turned out!), to be electromagnetic radiation.

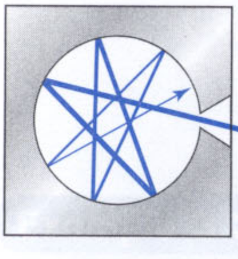


FIGURE 3.8 Blackbody radiation. Electromagnetic radiation (for example, light) entering a small hole reflects around inside the container before being eventually absorbed.

The intensity $\mathcal{I}(\lambda, T)$ is the total power radiated per unit area per unit wavelength at a given temperature. Measurements of $\mathcal{I}(\lambda, T)$ for a blackbody are displayed in Figure 3.9. Two important observations should be noted:

1. The maximum of the distribution shifts to smaller wavelengths as the temperature is increased.
2. The total power radiated increases with the temperature.

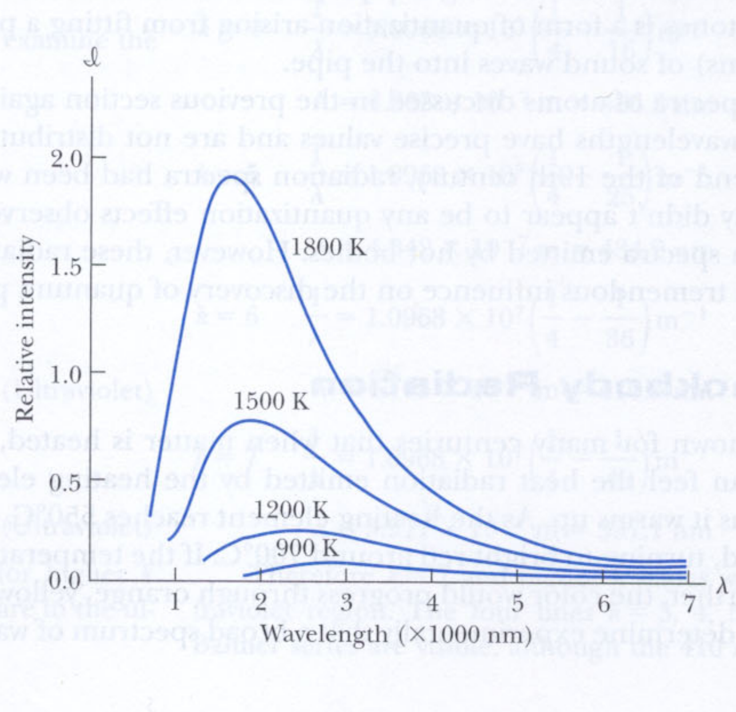


FIGURE 3.9 Spectral distribution of radiation emitted from a blackbody for different blackbody temperatures.

The first observation is commonly referred to as **Wien's displacement law,**

$$\lambda_{max} T = 2.898 \times 10^{-3} \text{ m}\cdot\text{K} \tag{3.14}$$

Wien's displacement law

where λ_{max} is the wavelength of the peak of the spectral distribution at a given temperature. Wilhelm Wien received the Nobel Prize in 1911 for his discoveries concerning radiation. We can quantify the second observation by integrating the quantity $\mathcal{I}(\lambda, T)$ over all wavelengths to find the power per unit area at T.

$$R(T) = \int_0^\infty \mathcal{I}(\lambda, T)\, d\lambda \tag{3.15}$$

Josef Stefan found empirically in 1879, and Boltzmann demonstrated theoretically several years later, that $R(T)$ is related to the temperature by

$$R(T) = \epsilon\sigma T^4 \tag{3.16}$$

Stefan-Boltzmann law

This is known as the **Stefan-Boltzmann law,** with the constant σ experimentally measured to be 5.6705×10^{-8} W/(m$^2\cdot$K^4). The Stefan-Boltzmann law equation can be applied to any material for which the emissivity is known. The **emissivity** ϵ ($\epsilon = 1$ for an idealized blackbody) is simply the ratio of the emissive power of an object to that of an ideal blackbody and is always less than 1. Thus, Equation (3.16) is a useful and valuable relation for practical scientific and engineering work.

Example 3.4

A furnace has walls of temperature 1600°C. What is the wavelength of maximum intensity emitted when a small door is opened?

Solution: If we assume blackbody radiation, we determine λ_{max} from Equation (3.14).

$$T = (1600 + 273)\text{K} = 1873 \text{ K}$$

$$\lambda_{max}(1873 \text{ K}) = 2.898 \times 10^{-3} \text{ m}\cdot\text{K}$$

$$\lambda_{max} = 1.55 \times 10^{-6} \text{ m} = 1550 \text{ nm}$$

Example 3.5

The wavelength of maximum intensity of the sun's radiation is observed to be near 500 nm. Assume the sun to be a blackbody and calculate (a) the sun's surface temperature, (b) the power per unit area $R(T)$ emitted from the sun's surface, and (c) the energy received by the Earth each day from the sun's radiation.

Solution: From Equation (3.14) we calculate the sun's surface temperature with $\lambda_{max} = 500$ nm.

$$(500 \text{ nm})\, T_{sun} = 2.898 \times 10^{-3} \text{ m}\cdot\text{K}\frac{10^9 \text{ nm}}{\text{m}}$$

$$T_{sun} = \frac{2.898 \times 10^6}{500} \text{ K} = 5800 \text{ K} \tag{3.17}$$

The power per unit area $R(T)$ at this temperature can be found by again assuming a blackbody:

$$R(T) = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2\cdot\text{K}^4} (5800 \text{ K})^4$$

$$= 6.42 \times 10^7 \text{ W/m}^2 \tag{3.18}$$

Because this is the power per unit surface area, we need to multiply this by $4\pi r^2$, the surface area of the sun. The radius of the sun is 6.96×10^5 km.

$$\text{Surface area (sun)} = 4\pi(6.96 \times 10^8 \text{ m})^2 = 6.09 \times 10^{18} \text{ m}^2$$

Thus the total power, P_{sun}, radiated from the sun's surface is

$$P_{sun} = 6.42 \times 10^7 \frac{\text{W}}{\text{m}^2}(6.09 \times 10^{18} \text{ m}^2) = 3.91 \times 10^{26} \text{ W} \tag{3.19}$$

The fraction F of the sun's radiation received by Earth is given by the fraction of the total area over which the radiation is spread.

$$F = \frac{\pi r_e^2}{4\pi R_{es}^2}$$

where r_e = radius of Earth = 6.37×10^6 m, and R_{es} = mean Earth-sun distance = 1.49×10^{11} m. Then

$$F = \frac{\pi r_e^2}{4\pi R_{es}^2} = \frac{(6.37 \times 10^6 \text{ m})^2}{4(1.49 \times 10^{11} \text{ m})^2} = 4.57 \times 10^{-10}$$

Thus the radiation received by the Earth from the sun is

$$P_{\text{Earth}} \text{ (received)} = 4.57 \times 10^{-10}\,(3.91 \times 10^{26} \text{ W}) = 1.79 \times 10^{17} \text{ W} \tag{3.20}$$

and in one day the Earth receives

$$U_{\text{Earth}} = 1.79 \times 10^{17} \frac{\text{J}}{\text{s}} \frac{60 \text{ s}}{\text{min}} \frac{60 \text{ min}}{\text{h}} \frac{24 \text{ h}}{\text{day}} = 1.55 \times 10^{22} \text{ J} \tag{3.21}$$

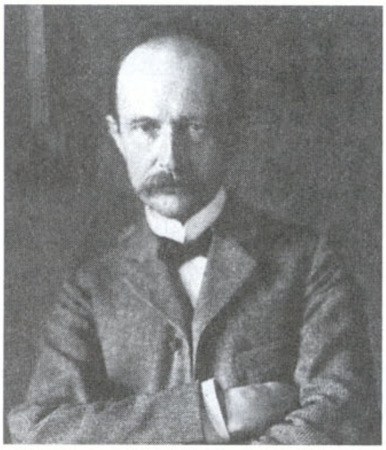
The power per unit exposed area received by the Earth is

$$R_{\text{Earth}} = \frac{1.79 \times 10^{17} \text{ W}}{\pi (6.37 \times 10^6 \text{ m})^2} = 1400 \text{ W/m}^2 \tag{3.22}$$

Needless to say, this is the source of most of our energy on Earth. Measurements of the sun's radiation outside the Earth's atmosphere give a value near 1400 W/m^2, so our calculation is fairly accurate. Apparently the sun does act as a blackbody, and most of the energy received by the Earth comes primarily from the surface of the sun.

Blackbody radiation problem

Attempts to understand and derive from basic principles the shape of the blackbody spectral distribution (Figure 3.9) were unsuccessful throughout the 1890s despite the persistent effort of some of the best scientists of the day. Blackbody radiation was one of the outstanding problems of the late nineteenth century because it presented physicists with a real dilemma. The nature of the dilemma can be understood from classical electromagnetic theory, together with statistical thermodynamics. The radiation emitted from the blackbody can be expressed as a superposition of electromagnetic waves of different frequencies within the cavity. That is, radiation of a given frequency is represented by a standing wave inside the cavity. The equipartition theorem of thermodynamics assigns equal average energy kT to each possible wave configuration. For long wavelengths λ there are only few configurations whereby a standing wave can form inside the cavity. However, as the wavelength becomes shorter the number of standing wave possibilities increases, and as $\lambda \rightarrow 0$ the number of possible configurations increases without limit. This means the total energy of all configurations is infinite, because each standing wave configuration has the nonzero energy kT. This problem for small wavelengths became known as "the ultraviolet catastrophe."



Max Planck (1858–1947) spent most of his productive years as a professor at the University of Berlin (1889–1928). His theory of the *quantum of action* was slow to be accepted because of its contradiction with the heat radiation law of Wilhelm Wien. Finally, after Einstein's photoelectric effect explanation and Rutherford and Bohr's atomic model, Planck's contribution became widely acclaimed. *AIP Emilio Segrè Visual Archives.*

In the late 1890s the German theoretical physicist Max Planck (1858–1947) became interested in this problem. By this time, different empirical expressions for the blackbody spectrum had been separately fit to the data for both short wavelengths and long wavelengths, but no one had explained the whole spectrum. Planck tried various functions of wavelength and temperature until he found a single formula that fit the measurements of $\mathcal{I}(\lambda, T)$ over the entire wavelength range. He announced this result in October of 1900, and immediately his equation was compared with recent data of Rubens and Kurlbaum. The result was that Planck's formula was even more accurate over the entire spectrum than the previous empirical ones, which were only valid for either short or long wavelengths.

Planck became quite excited and started working to find a sound theoretical basis for his empirical equation. He was an expert in thermodynamics and statistical mechanics. Following Hertz's work using oscillators to confirm the existence of Maxwell's electromagnetic waves, and lacking detailed information about the atomic composition of the cavity walls, Planck assumed that the radiation in the cavity was emitted (and absorbed) by some sort of "oscillators" that were contained in the walls. Whereas we would now refer to the radiation of the electromagnetic field in the cavity, Planck referred to the radiation produced by the "oscillators," a term we will briefly continue to use. When adding up the energies of the oscillators, he assumed (for convenience) that each one had an energy that was an integral multiple of $h\nu$, where ν is the frequency of the oscillating wave. He was applying a technique invented by Boltzmann and ultimately expected to take the limit $h \to 0$, in order to include all the possibilities. However, he noticed that by keeping h finite he arrived at the equation needed for $\mathcal{I}(\lambda, T)$,

$$\mathcal{I}(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \tag{3.23}$$

Planck's radiation law

Equation (3.23) is **Planck's radiation law.** (The derivation of Equation (3.23) is sufficiently complicated that we have omitted it here.) No matter what he tried, he could only arrive at the correct result by making two important modifications of classical theory:

1. The oscillators (of electromagnetic origin) can only have certain discrete energies determined by $E_n = nh\nu$, where n is an integer, ν is the frequency, and h is called **Planck's constant** and has the value

$$h = 6.6261 \times 10^{-34}\ \text{J} \cdot \text{s} \tag{3.24}$$

Planck's constant h

2. The oscillators can absorb or emit energy in discrete multiples of the fundamental quantum of energy given by

$$\Delta E = h\nu \tag{3.25}$$

Planck himself found these results quite disturbing and spent several years trying to find a way to keep the agreement with experiment while letting $h \to 0$. Each attempt failed.

Example 3.6

Show that Planck's radiation law avoids the *ultraviolet catastrophe.*

Solution: The ultraviolet catastrophe occurs because the number of configurations (~ intensity) in the classical calculation becomes infinite as $\lambda \to 0$. If we let $\lambda \to 0$ in Equation (3.23), the value of $e^{hc/\lambda kT} \to \infty$. The exponential term dominates the λ^5 term as $\lambda \to 0$, so the denominator in Equation (3.23) is infinite, and the value of $\mathcal{I}(\lambda, T) \to 0$. Note that as the wavelength gets smaller, the frequency becomes larger ($\lambda\nu = c$), and $h\nu \gg kT$. Few oscillators will be able to obtain such large energies, partly because of the large energy necessary to take the energy step from 0 to $h\nu$. The probability of occupying the states with small wavelengths (large frequency and high energy) is vanishingly small, so the total energy of the system remains finite. The *ultraviolet catastrophe* is avoided.

Example 3.7

Show that Wien's displacement law follows from Planck's radiation law.

Solution: Wien's law, Equation (3.14), refers to the wavelength λ for which $\mathcal{I}(\lambda, T)$ is a maximum for a given temperature. Therefore, to find this maximum we let $d\mathcal{I}/d\lambda = 0$ and solve for λ.

$$\frac{d\mathcal{I}(\lambda, T)}{d\lambda} = 0 \qquad \text{for } \lambda = \lambda_{\max}$$

$$2\pi c^2 h \frac{d}{d\lambda}\left[\lambda^{-5}\left(e^{hc/\lambda kT} - 1\right)^{-1}\right] = 0$$

$$-5\lambda_{\max}^{-6}\left(e^{hc/\lambda_{\max}kT} - 1\right)^{-1} - \lambda_{\max}^{-5}\left(e^{hc/\lambda_{\max}kT} - 1\right)^{-2} \cdot \left(\frac{-hc}{kT\lambda_{\max}^2}\right) e^{hc/\lambda_{\max}kT} = 0$$

Multiplying by $\lambda_{\max}^6\left(e^{hc/\lambda_{\max}kT} - 1\right)$ results in

$$-5 + \frac{hc}{\lambda_{\max}kT}\left(\frac{e^{hc/\lambda_{\max}kT}}{e^{hc/\lambda_{\max}kT} - 1}\right) = 0$$

Let

$$x = \frac{hc}{\lambda_{\max}kT}$$

then

$$-5 + \frac{xe^x}{(e^x - 1)} = 0$$

and

$$xe^x = 5(e^x - 1)$$

This is a transcendental equation and can be solved numerically (try it!) with the result, $x = 4.966$, and, therefore,

$$\frac{hc}{\lambda_{\max}kT} = 4.966$$

$$\lambda_{\max}T = \frac{hc}{4.966\,k} = \frac{1240\ \text{eV}\cdot\text{nm}}{4.966\left(8.617\times 10^{-5}\dfrac{\text{eV}}{\text{K}}\right)}\,\frac{10^{-9}\ \text{m}}{\text{nm}}$$

and finally,

$$\lambda_{\max}T = 2.898 \times 10^{-3}\ \text{m}\cdot\text{K}$$

which is Wien's displacement law.

Example 3.8

Use Planck's radiation law to derive the Stefan-Boltzmann law.

Solution: To determine $R(T)$ we integrate $\mathcal{I}(\lambda, T)$ over all wavelengths

$$R(T) = \int_0^\infty \mathcal{I}(\lambda, T)\,d\lambda$$

$$= 2\pi c^2 h \int_0^\infty \frac{1}{\lambda^5}\,\frac{1}{e^{hc/\lambda kT} - 1}\,d\lambda$$

Let $x = \dfrac{hc}{\lambda kT}$, then $dx = -\dfrac{hc}{kT}\dfrac{d\lambda}{\lambda^2}$. Then we have

$$R(T) = -2\pi c^2 h \int_\infty^0 \left(\frac{kT}{hc}\right)^6 x^5\,\frac{1}{e^x - 1}\,\frac{1}{x^2}\left(\frac{hc}{kT}\right)^2 dx$$

$$= +2\pi c^2 h\left(\frac{kT}{hc}\right)^4 \int_0^\infty \frac{x^3}{e^x - 1}\,dx$$

We look up this integral in Appendix 7 and find it to be $\pi^4/15$.

$$R(T) = 2\pi c^2 h\left(\frac{kT}{hc}\right)^4 \frac{\pi^4}{15}$$

$$R(T) = \frac{2\pi^5 k^4}{15h^3c^2}\,T^4$$

Putting in the values for the constants k, h, and c results in

$$R(T) = 5.67\times 10^{-8}\ T^4\,\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}$$

3.6 Photoelectric Effect

Perhaps the most compelling, and certainly the simplest, evidence for the quantization of radiation energy comes from the only acceptable explanation of the **photoelectric effect.** The photoelectric effect, discovered by Hertz in 1887 as he confirmed Maxwell's electromagnetic wave theory of light, is one of several ways in which electrons can be emitted by materials. By the early 1900s it was known that electrons are bound to matter. In metals the valence electrons are "free"—they are able to move easily from atom to atom, but are not able to leave the surface of the material. The methods known now by which electrons can be made to completely leave the material include

Methods of electron emission

1. Thermionic emission—application of heat allows electrons to gain enough energy to escape.
2. Secondary emission—the electron gains enough energy by transfer from another high-speed particle that strikes the material from outside.
3. Field emission—a strong external electric field pulls the electron out of the material.
4. Photoelectric effect—incident light (electromagnetic radiation) shining on the material transfers energy to the electrons allowing them to escape.

It is not surprising that electromagnetic radiation acts on electrons within metals giving the electrons increased kinetic energy. Because electrons in metals are weakly bound, we expect that light can give electrons enough extra kinetic energy to allow them to escape. We call the ejected electrons **photoelectrons.** The minimum extra kinetic energy that allows electrons to escape the material is called the **work function** ϕ. The work function is the minimum binding energy of the electron to the material. The work functions of alkali metals are smaller than those of other metals. We shall see why this is so in Chapter 8.

Photoelectrons

Work function

Experimental Results of Photoelectric Effect

Experiments carried out around 1900 showed that photoelectrons were produced when visible and/or ultraviolet light falls on clean metal surfaces. Photoelectricity was studied using an experimental apparatus shown schematically in Figure 3.10. Incident light falling on the **emitter** (also called the **photocathode** or **cathode**) ejects electrons. Some of the electrons travel toward the **collector**

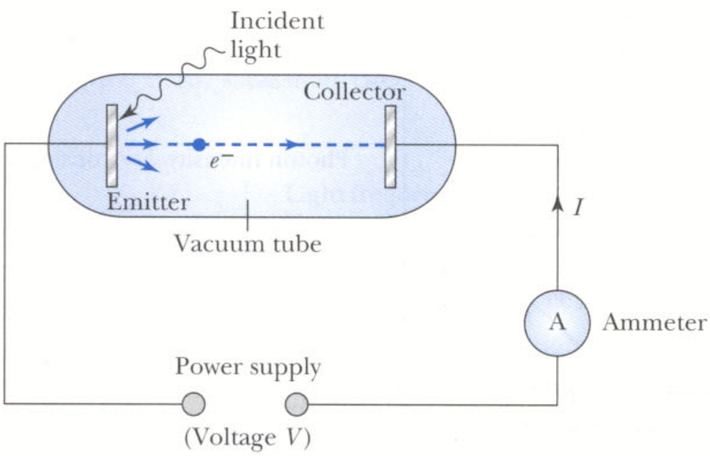


FIGURE 3.10 Photoelectric effect. Electrons, emitted when light shines on a surface, are collected, and the photocurrent I is measured. A negative voltage, relative to that of the emitter, can be applied to the collector. When this retarding voltage is sufficiently large, the emitted electrons are repelled, and the current to the collector drops to zero.

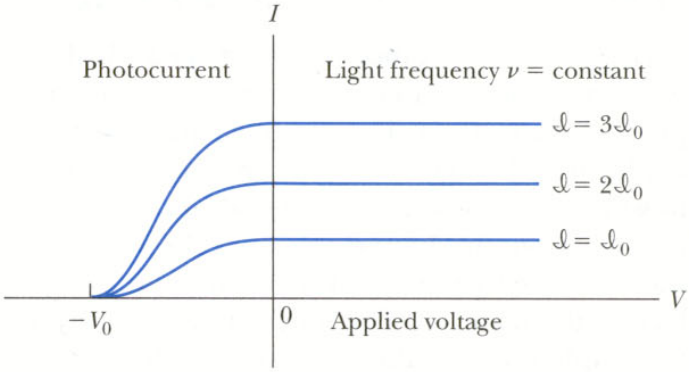


FIGURE 3.11 The photoelectric current I is shown as a function of the voltage V applied between the emitter and collector for a given frequency ν of light for three different light intensities. Notice that no current flows for a retarding potential more negative than $-V_0$ and that the photocurrent is constant for potentials near or above zero (this assumes that the emitter and collector are closely spaced or in spherical geometry to avoid loss of photoelectrons).

(also called the **anode**), where either a negative (retarding) or positive (accelerating) applied voltage V is imposed by the power supply. The current I measured in the ammeter (photocurrent) arises from the flow of photoelectrons from emitter to collector.

The pertinent experimental facts about the photoelectric effect are these:

1. The kinetic energies of the photoelectrons are independent of the light intensity. In other words, a stopping potential (applied voltage) of $-V_0$ is sufficient to stop all photoelectrons, *no matter what the light intensity,* as shown in Figure 3.11. For a given light intensity there is a maximum photocurrent, which is reached as the applied voltage increases from negative to positive values.
2. The maximum kinetic energy of the photoelectrons, for a given emitting material, depends only on the frequency of the light. In other words, for light of different frequency (Figure 3.12) a different retarding potential $-V_0$ is required to stop the most energetic photoelectrons. The value of V_0 depends on the frequency ν but not on the intensity (see Figure 3.11).
3. The smaller the work function ϕ of the emitter material, the smaller is the threshold frequency of the light that can eject photoelectrons. No photoelectrons are produced for frequencies below this threshold frequency, no matter what the intensity. Data similar to Millikan's results (discussed later) are shown in Figure 3.13, where the threshold frequencies ν_0 are measured for three different metals.
4. When the photoelectrons are produced, however, their number is proportional to the intensity of light as shown in Figure 3.14. That is, the maximum photocurrent is proportional to the light intensity.

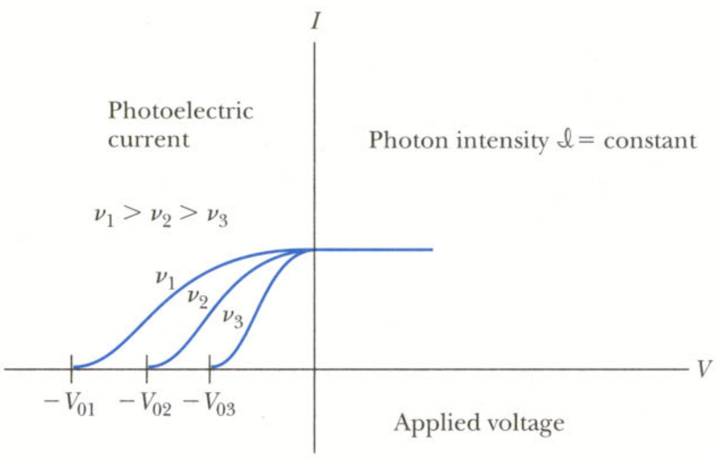


FIGURE 3.12 The photoelectric current I is shown as a function of applied voltage for three different light frequencies. The retarding potential $-V_0$ is different for each ν and is more negative for larger ν.

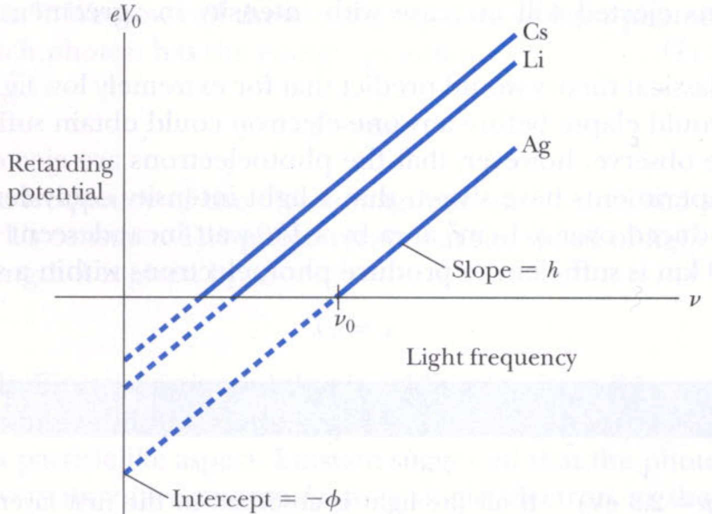


FIGURE 3.13 The retarding potential eV_0 (maximum electron kinetic energy) is plotted vs. light frequency for three different emitter materials.

5. The photoelectrons are emitted almost instantly ($\leq 3 \times 10^{-9}$ s) following illumination of the photocathode, independent of the intensity of the light.

Except for (5), these experimental facts were known in rudimentary form by 1902, primarily due to the work of Philipp Lenard, a German experimental physicist who won the Nobel Prize in 1905 for this and other research on the identification and behavior of electrons.

Classical Interpretation

As stated previously, we can understand from classical theory that electromagnetic radiation should be able to eject photoelectrons from matter. However, the classical theory predicts that the total amount of energy in a light wave increases as the light intensity increases. Therefore, classically the electrons should have more kinetic energy if the light intensity is increased. However, according to result (1) earlier and Figure 3.11, a characteristic retarding potential $-V_0$ is sufficient to stop all photoelectrons for a given light frequency ν, no matter what the intensity. Classical electromagnetic theory is unable to explain this result. Similarly, classical theory cannot explain result (2), because the maximum kinetic energy of the photoelectrons depends on the value of the light frequency ν and not on the intensity.

The existence of a threshold frequency, as shown in experimental result (3) is completely inexplicable in classical theory. Classical theory cannot predict the results shown in Figure 3.13. Classical theory does predict that the number of

Difficulties of classical theory

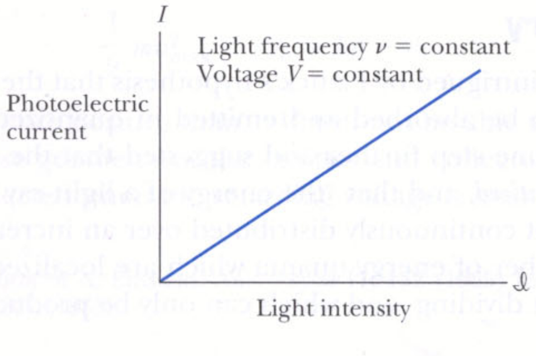


FIGURE 3.14 The photoelectric current I is a linear function of the light intensity for a constant ν and V.

photoelectrons ejected will increase with intensity in agreement with experimental result (4).

Finally, classical theory would predict that for extremely low light intensities, a long time would elapse before any one electron could obtain sufficient energy to escape. We observe, however, that the photoelectrons are ejected almost immediately. Experiments have shown that a light intensity equivalent to the illumination produced over a 1-cm^2 area by a 100-watt incandescent bulb at a distance of 1000 km is sufficient to produce photoelectrons within a second.

Example 3.9

Photoelectrons may be emitted from sodium ($\phi = 2.3$ eV) even for light intensities as low as 10^{-8} W/m^2. Calculate classically how long the light must shine in order to produce a photoelectron of kinetic energy 1 eV.

Solution: Let's assume that all of the light is absorbed in the first layer of atoms in the surface. First calculate the number of sodium atoms per unit area in a layer one atom thick.

$$\frac{\text{Avogadro's number}}{\text{Na gram molecular weight}} \times \text{Density} = \frac{\text{Number of Na atoms}}{\text{Volume}}$$

$$\frac{6.02 \times 10^{23}\ \text{atoms/mole}}{23\ \text{g/mole}} \times 0.97 \frac{\text{g}}{\text{cm}^3} =$$

$$2.54 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3} = 2.54 \times 10^{28} \frac{\text{atoms}}{\text{m}^3} \tag{3.26}$$

To estimate the thickness of one layer of atoms, we assume a cubic structure.

$$\frac{1\ \text{atom}}{d^3} = 2.54 \times 10^{28} \frac{\text{atoms}}{\text{m}^3}$$

$$d = 3.40 \times 10^{-10}\ \text{m}$$

= thickness of one layer of sodium atoms

If all the light is absorbed in the first layer of atoms, the number of exposed atoms per m^2 is

$$2.54 \times 10^{28} \frac{\text{atoms}}{\text{m}^3} \times 3.40 \times 10^{-10}\ \text{m} = 8.64 \times 10^{18} \frac{\text{atoms}}{\text{m}^2}$$

With the intensity of 10^{-8} W/m^2, each atom will receive energy at the rate of

$$10^{-8} \frac{\text{W}}{\text{m}^2} \times \frac{1}{8.64 \times 10^{18}\ \text{atoms/m}^2} = 1.16 \times 10^{-27}\ \text{W}$$

$$= 1.16 \times 10^{-27} \frac{\text{J}}{\text{s}} \times \frac{1}{1.6 \times 10^{-19}\ \text{J/eV}}$$

$$= 7.25 \times 10^{-9}\ \text{eV/s}$$

We have assumed that each atom absorbs, on the average, the same energy and that a single electron in the atom absorbs all the energy. The energy needed to eject the photoelectron is 2.3 eV for the work function and 1 eV for the kinetic energy, for a total of 3.3 eV. Using the rate of energy absorption of 7.25×10^{-9} eV/s, we can calculate the time T needed to absorb 3.3 eV:

$$T = 3.3\ \text{eV} \frac{1}{7.25 \times 10^{-9}\ \text{eV/s}}$$

$$= 4.55 \times 10^{8}\ \text{s}$$

$$= 14\ \text{years}$$

The time calculated classically to eject a photoelectron is 14 years!

Einstein's Theory

Albert Einstein was intrigued by Planck's hypothesis that the electromagnetic radiation field had to be absorbed and emitted in quantized amounts. Einstein took Planck's idea one step further and suggested that the *electromagnetic radiation field itself is quantized*, and that "the energy of a light ray spreading out from a point source is not continuously distributed over an increasing space but consists of a finite number of energy quanta which are localized at points in space, which move without dividing, and which can only be produced and absorbed as complete units."* We now call these energy quanta of light **photons.** According to Einstein each photon has the energy quantum,

$$E = h\nu \tag{3.27}$$

Energy quantum

where ν is the frequency of the electromagnetic wave associated with the light and h is Planck's constant. The photon travels at the speed of light in a vacuum, and its wavelength λ is given by

$$\lambda\nu = c \tag{3.28}$$

In other words, Einstein proposed that in addition to its well-known wavelike aspect, amply exhibited in interference phenomena, light should also be considered to have a particlelike aspect. Einstein suggested that the photon (quantum of light) delivers its entire energy $h\nu$ to a single electron in the material. In order to leave the material, the struck electron must give up an amount of energy ϕ to overcome its binding in the material. The electron may lose some additional energy by interacting with other electrons on its way to the surface. Whatever energy remains will then appear as kinetic energy of the electron as it leaves the emitter. The conservation of energy requires that

$$\text{Energy before (photon)} = \text{Energy after (electron)}$$

$$h\nu = \phi + \text{K.E. (electron)} \tag{3.29}$$

Because the energies involved here are on the order of eV, we are safe in using the nonrelativistic form of the electron's kinetic energy, $\frac{1}{2}mv^2$. The electron's energy will be degraded as it passes through the emitter material, so, strictly speaking, we want to experimentally detect the maximum value of the kinetic energy.

$$h\nu = \phi + \frac{1}{2}mv_{\text{max}}^2 \tag{3.30}$$

The retarding potentials measured in the photoelectric effect are thus the opposing potentials needed to stop the most energetic electrons.

$$eV_0 = \frac{1}{2}mv_{\text{max}}^2 \tag{3.31}$$

Quantum Interpretation

We should now re-examine the experimental results of the photoelectric effect to see whether Einstein's quantum interpretation can explain all the data. The first and second experimental results are easily explained because the kinetic energy of the electrons does not depend on the light intensity at all, but only on the light frequency and the work function of the material.

$$\frac{1}{2}mv_{\text{max}}^2 = eV_0 = h\nu - \phi \tag{3.32}$$

A potential slightly more positive than $-V_0$ will not be able to repel all the electrons, and, for a close geometry of the emitter and collector, practically all the electrons will be collected when the retarding voltage is near zero. For very large

*See the English translation of A. Einstein, *Ann. Physik* **17,** 132 (1905) by A. B. Arons and M. B. Peppard, *Am. J. Phys.* **33,** 367 (1965).

positive potentials all the electrons will be collected, and the photocurrent will level off as shown in Figure 3.11. If the light intensity increases, there will be more photons per unit area, more electrons ejected, and therefore a higher photocurrent, as displayed in Figure 3.11.

If a different light frequency is used, say ν_2, then a different stopping potential is required to stop the most energetic electrons [see Equation (3.32)], $eV_{02} = h\nu_2 - \phi$. For a constant light intensity (more precisely, a constant number of photons/area/time), a different stopping potential V_0 is required for each ν, but the maximum photocurrent will not change, because the number of photoelectrons ejected is constant (see Figure 3.12). The quantum theory easily explains Figure 3.14, because the number of photons increases linearly with the light intensity, producing more photoelectrons and hence more photocurrent.

Equation (3.32), proposed by Einstein in 1905, predicts that the stopping potential will be linearly proportional to the light frequency, with a slope *h, the same constant found by Planck.* The slope is independent of the metal used to construct the photocathode. The data available in 1905 were not sufficiently accurate either to prove or to disprove Einstein's theory, and the theory was received with skepticism, even by Planck himself. R. A. Millikan, then at the University of Chicago, tried to show Einstein was wrong by undertaking a series of elegant experiments that required almost ten years to complete. In 1916 he reported data confirming Einstein's prediction. From data similar to that shown in Figure 3.13, Millikan found the value of h to be in almost exact agreement with the one determined for blackbody radiation by Planck. Equation (3.32) can be rewritten

Millikan believed Einstein was wrong

$$eV_0 = \frac{1}{2}mv_{\text{max}}^2 = h\nu - h\nu_0 \tag{3.33}$$

where $\phi = h\nu_0$ represents the negative of the y intercept. The frequency ν_0 represents the threshold frequency for the photoelectric effect (when the kinetic energy of the electron is precisely zero). Einstein's theory of the photoelectric effect was gradually accepted after 1916; finally in 1922 he received the Nobel Prize for the year 1921, primarily for his explanation of the photoelectric effect.*

Quantization of electromagnetic radiation field

We should summarize what we have learned about the quantization of the electromagnetic radiation field. First, electromagnetic radiation consists of photons which are particlelike (or corpuscular), each consisting of energy

$$E = h\nu = \frac{hc}{\lambda} \tag{3.34}$$

where ν and λ are the frequency and wavelength of the light, respectively. The total energy of a beam of light is the sum total of the energy of all the photons and for monochromatic light is an integral multiple of $h\nu$ (generally the integer is very large).

This representation of the photon picture must be true over the entire electromagnetic spectrum from radio waves to visible light, x rays, and even high-energy gamma rays. This must be true because, as we saw in Chapter 2, a photon of given frequency, observed from a moving system, can be redshifted or blueshifted by an arbitrarily large amount, depending on the system's speed and

*R. A. Millikan also received the Nobel Prize in 1923, partly for his precise study of the photoelectric effect and partly for measuring the charge of the electron. Millikan's award was the last in a series of Nobel Prizes spanning 18 years that honored the fundamental efforts to measure and understand the photoelectric effect: Lenard, Einstein, and Millikan.

direction of motion. We will examine these possibilities later. During emission or absorption of any form of electromagnetic radiation (light, x rays, gamma rays, etc.), photons must be created or absorbed. The photons have only one speed: the speed of light (= c in vacuum).

Example 3.10

Light of wavelength 400 nm is incident upon lithium (ϕ = 2.9 eV). Calculate (a) the photon energy and (b) the stopping potential V_0.

Solution: (a) Light is normally described by wavelengths in nm, so it is useful to have an equation to calculate the energy in terms of λ.

$$E = h\nu = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^{8}\ \text{m/s})}{\lambda(1.602 \times 10^{-19}\ \text{J/eV})(10^{-9}\ \text{m/nm})}$$

$$E = \frac{1.240 \times 10^{3}\ \text{eV}\cdot\text{nm}}{\lambda} \tag{3.35}$$

For a wavelength of λ = 400 nm we have for the photon's energy:

$$E = \frac{1.240 \times 10^{3}\ \text{eV}\cdot\text{nm}}{400\ \text{nm}} = 3.1\ \text{eV}$$

(b) We determine the stopping potential from Equation (3.32).

$$eV_0 = h\nu - \phi = E - \phi$$
$$= 3.1\ \text{eV} - 2.9\ \text{eV} = 0.2\ \text{eV}$$
$$V_0 = 0.2\ \text{V}$$

A retarding potential of 0.2 V will stop all photoelectrons.

Example 3.11

What frequency of light is needed to produce electrons of kinetic energy 3 eV from illumination of lithium?

Solution: We determine the photon energy from Equation (3.30).

$$h\nu = \phi + \frac{1}{2}mv^2_{\text{max}}$$
$$= 2.9\ \text{eV} + 3.0\ \text{eV} = 5.9\ \text{eV}$$

The photon frequency is now found to be

$$\nu = \frac{E}{h} = \frac{(5.9\ \text{eV})(1.6 \times 10^{-19}\ \text{J/eV})}{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})}$$
$$= 1.42 \times 10^{15}\ \text{s}^{-1} = 1.42 \times 10^{15}\ \text{Hz}$$

Example 3.12

For the light intensity of Example 3.9, $\mathcal{I} = 10^{-8}\ \text{W/m}^2$, a wavelength of 350 nm is used. What is the number of photons/area/s in the light beam?

Solution: From Equation (3.35) we have

$$E_\gamma = \frac{1.24 \times 10^{3}\ \text{eV}\cdot\text{nm}}{350\ \text{nm}} = 3.5\ \text{eV}$$

where E_γ represents the photon's energy. Because

$$\text{Intensity } \mathcal{I} = \left[N \frac{\text{photons}}{\text{area time}}\right]\left[E_\gamma \frac{\text{energy}}{\text{photon}}\right]$$
$$= NE_\gamma \frac{\text{energy}}{\text{area time}}$$

then

$$N = \frac{\mathcal{I}}{E_\gamma} = \frac{10^{-8}\ \text{J}\cdot\text{s}^{-1}\text{m}^{-2}}{(1.6 \times 10^{-19}\ \text{J/eV})(3.5\ \text{eV/photon})}$$
$$= 1.8 \times 10^{10}\ \frac{\text{photons}}{\text{m}^2\cdot\text{s}}$$

Thus even a low-intensity light beam has a large flux of photons, and even a few photons can produce a photocurrent (albeit a very small one!).

3.7 X-Ray Production

In the photoelectric effect, a photon gives up all of its energy to an electron, which may then escape from the material to which it was bound. Can the inverse process occur? Can an electron (or any charged particle) give up its energy and create a photon? The answer is yes, but the process must be consistent with other laws of physics. Recall that photons must be created or absorbed as whole units. A photon cannot give up half its energy. Rather, it must give up all its energy. If in some physical process only part of the photon's energy were required, then a **new** photon would be created to carry away the remaining energy.

However, electrons do not act as photons. An electron may give up part or all of its kinetic energy and still be the same electron. As we now know, a photon is electromagnetic radiation. When an electron interacts with the strong electric field of the atomic nucleus and is consequently accelerated, the electron will radiate electromagnetic energy. According to classical electromagnetic theory, it would do so continuously. In the quantum picture we must think of the electron as emitting a series of photons with varying energies; this is the only way that the inverse photoelectric effect can occur. An energetic electron passing through matter will radiate photons and lose kinetic energy. The process by which photons are emitted by an electron slowing down is called **bremsstrahlung,** from the German word for "braking radiation." The process is shown schematically in Figure 3.15 where an electron (energy E_i) passing through the electric field of a nucleus slows down and produces a photon ($E = h\nu$). The final energy of the electron is then

Bremsstrahlung process

$$E_f = E_i - h\nu \tag{3.36}$$

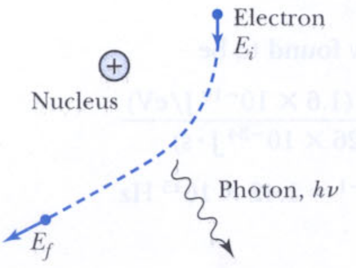


FIGURE 3.15 *Bremsstrahlung* is a process by which an electron is accelerated while under the influence of the nucleus. The accelerated electron emits a photon.

from the conservation of energy. The nucleus absorbs very little energy in order to conserve linear momentum. One or more photons may be created in this way as electrons pass through matter.

In Section 3.1 we mentioned Röntgen's discovery of x rays. The x rays are produced by the bremsstrahlung effect in apparatus shown schematically in Figure 3.16. Current passing through a filament produces copious numbers of electrons by thermionic emission. These electrons are focused by the cathode structure into a beam and are accelerated by voltages of thousands of volts until they impinge on a metal anode surface, producing x rays by bremsstrahlung (and other processes) as they stop in the anode material. Much of the electron's kinetic energy is lost by heating the anode and not by bremsstrahlung. The x-ray tube is evacuated so that the air between the filament and anode will not scatter

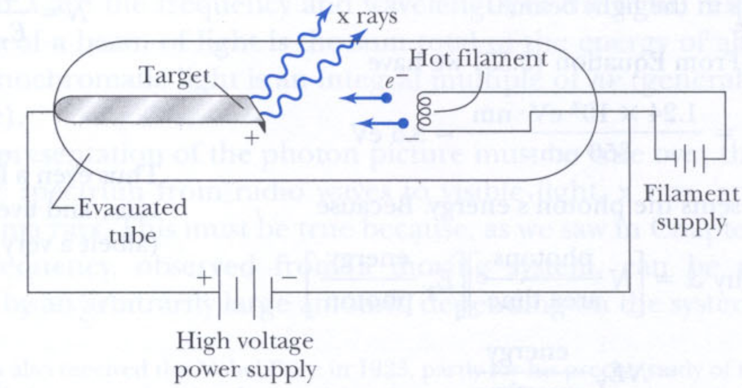


FIGURE 3.16 Schematic of x-ray tube where x rays are produced by the bremsstrahlung process of energetic electrons.

the electrons. The x rays produced pass through the sides of the tube and can be used for a large number of applications, including medical diagnosis and therapy, fundamental research in crystal and liquid structure, and, in engineering, the diagnosis of flaws in large welds and castings. X rays from a standard tube include photons of many wavelengths. By scattering x rays from crystals we can produce strongly collimated monochromatic (single wavelength) x-ray beams. Early x-ray spectra produced by x-ray tubes of accelerating potential 35 kV are shown in Figure 3.17. These particular tubes had targets of tungsten, molybdenum, and chromium. The smooth, continuous x-ray spectra are those produced by bremsstrahlung, and the sharp "characteristic x rays" are produced by atomic excitations and will be explained in Section 4.6. X-ray wavelengths typically range from 0.01 to 1 nm. However, high-energy accelerators can produce x rays with wavelengths as short as 10^{-6} nm.

Notice that in Figure 3.17 the minimum wavelength λ_{min} for all three targets is the same. The minimum wavelength λ_{min} corresponds to the maximum frequency. If the electrons are accelerated through a voltage V_0, then their kinetic energy is eV_0. The maximum photon energy therefore occurs when the electron

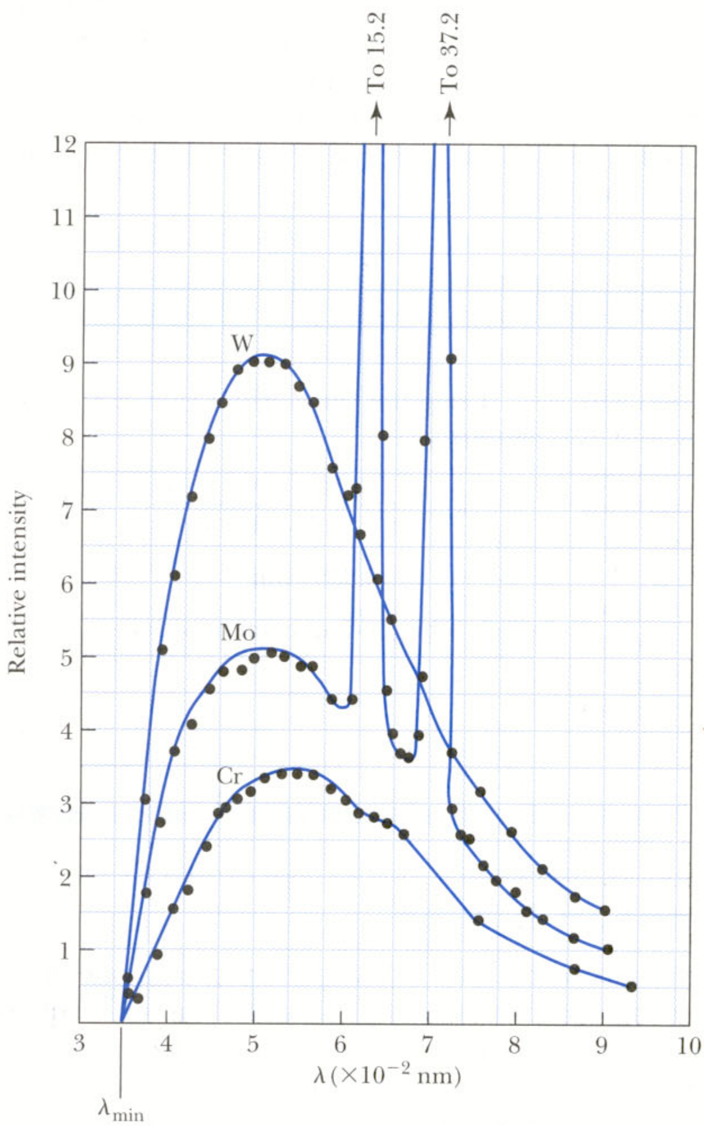


FIGURE 3.17 The relative intensity of x rays produced in an x-ray tube is shown for an accelerating voltage of 35 kV. Notice that λ_{min} is the same for all three targets. *From C. T. Ulrey, Physical Review* ***11****, 405 (1918).*

gives up all of its kinetic energy and creates one photon (this is relatively unlikely, however). This process is the **inverse photoelectric effect.** The conservation of energy requires that the electron kinetic energy equals the maximum photon energy (where we neglect the work function ϕ because it is normally so small compared to eV_0).

$$eV_0 = h\nu_{\max} = \frac{hc}{\lambda_{\min}}$$

or

Duane-Hunt rule

$$\lambda_{\min} = \frac{hc}{e}\frac{1}{V_0} = \frac{1.24 \times 10^{-6}\ \text{V}\cdot\text{m}}{V_0} \tag{3.37}$$

The relation Equation (3.37) was first found experimentally and is known as the **Duane-Hunt rule.** Its explanation in 1915 by the quantum theory is now considered further evidence of Einstein's photon concept. The value $\lambda_{\min}$ depends only on the accelerating voltage and is the same for all targets.

Only the quantum hypothesis explains all of the data. Because the heavier elements have stronger nuclear electric fields, they are more effective in decelerating electrons and making them radiate. The intensity of the x rays increases with the square of the atomic number of the target. The intensity is also approximately proportional to the square of the voltage used to accelerate the electrons. This is why high voltages and tungsten anodes are so often used in x-ray machines. Tungsten also has a very high melting temperature and can withstand high electron-beam currents.

Example 3.13

If we have a tungsten anode (work function $\phi = 4.5$ eV) and electron acceleration voltage of 35 kV, why do we ignore in Equation (3.36) the initial kinetic energy of the electrons from the filament and the work functions of the filaments and anodes? What is the minimum wavelength of the x rays?

Solution: The initial kinetic energies and work functions are on the order of a few electron volts (eV), whereas the kinetic energy of the electrons due to the accelerating voltage is 35,000 eV. The error in neglecting everything but eV_0 is small.

Using the Duane-Hunt rule of Equation (3.37) we determine

$$\lambda_{\min} = \frac{1.24 \times 10^{-6}\ \text{V}\cdot\text{m}}{35 \times 10^3\ \text{V}} = 3.54 \times 10^{-11}\ \text{m}$$

$$= 0.0354\ \text{nm}$$

which is in good agreement with the data of Figure 3.17.

3.8 Compton Effect

When a photon enters matter, it is likely to interact with one of the atomic electrons. Classically, the electrons will oscillate at the photon frequency because of the interaction of the electron with the electric and magnetic field of the photon and will reradiate electromagnetic radiation (photons) at this same frequency. This is called **Thomson scattering.** However, in the early 1920s Arthur Compton experimentally confirmed an earlier observation by J. A. Gray that, especially at backward-scattering angles, there appeared to be a component of the emitted radiation (called a **modified** wave) that had a longer wavelength than

Thomson scattering

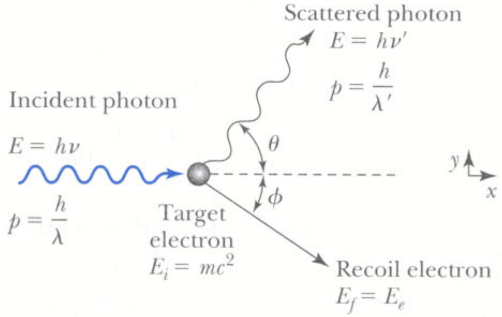


FIGURE 3.18 Compton scattering of a photon by an electron essentially at rest.

Professor Arthur Compton of the University of Chicago is shown here in 1931 looking into an ionization chamber that he designed to study cosmic rays in the atmosphere. These complex detectors had to be quite sturdy and were carefully tested and calibrated on Earth before sending up with balloons. *UPI/Corbis-Bettmann.*

the original primary (**unmodified**) wave. Classical electromagnetic theory cannot explain this modified wave. Compton then attempted to understand theoretically such a process and could only find one explanation: Einstein's photon particle concept must be correct. The scattering process is shown in Figure 3.18.

Compton proposed in 1923 that the photon is scattered from only one electron, rather than from all the electrons in the material, and that the laws of the conservation of energy and momentum apply as in any elastic collision between two particles. We recall from Chapter 2 that the momentum of a particle moving at the speed of light (photon) is given by

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda} \tag{3.38}$$

We treat the photon as a particle with a definite energy and momentum. Scattering takes place in a plane, which we take to be the *xy* plane in Figure 3.18. Because momentum is a vector, both *x* and *y* components must be conserved. The energy and momentum before and after the collision are given below (treated relativistically).

Compton scattering

	Initial System	**Final System**
Photon energy	$h\nu$	$h\nu'$
Photon momentum in x direction (p_x)	$\frac{h}{\lambda}$	$\frac{h}{\lambda'}\cos\theta$
Photon momentum in y direction (p_y)	0	$\frac{h}{\lambda'}\sin\theta$
Electron energy	mc^2	$E_e = mc^2 + \text{K.E.}$
Electron momentum in x direction (p_x)	0	$p_e\cos\phi$
Electron momentum in y direction (p_y)	0	$-p_e\sin\phi$

In the final system the electron's total energy is related to its momentum by

$$E_e^2 = (mc^2)^2 + p_e^2c^2 \tag{3.39}$$

We can write the conservation laws now as

$$\text{Energy:} \quad h\nu + mc^2 = h\nu' + E_e \tag{3.40a}$$

$$p_x: \quad \frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + p_e\cos\phi \tag{3.40b}$$

$$p_y: \quad \frac{h}{\lambda'}\sin\theta = p_e\sin\phi \tag{3.40c}$$

We will relate the change in wavelength $\Delta\lambda = \lambda' - \lambda$ to the scattering angle θ of the photon. We first eliminate the recoil angle ϕ by squaring Equations (3.40b) and (3.40c) and adding them together, resulting in

$$p_e^2 = \left(\frac{h}{\lambda}\right)^2 + \left(\frac{h}{\lambda'}\right)^2 - 2\left(\frac{h}{\lambda}\right)\left(\frac{h}{\lambda'}\right)\cos\theta \tag{3.41}$$

Then we substitute E_e from Equation (3.40a) and p_e from Equation (3.41) into Equation (3.39) (setting $\lambda = c/\nu$).

$$[h(\nu - \nu') + mc^2]^2 = m^2c^4 + (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu')\cos\theta$$

Squaring the left-hand side and canceling terms leaves

$$mc^2(\nu - \nu') = h\nu\nu'(1 - \cos\theta)$$

Rearranging terms gives

$$\frac{h}{mc^2}(1 - \cos\theta) = \frac{\nu - \nu'}{\nu\nu'} = \frac{\dfrac{c}{\lambda} - \dfrac{c}{\lambda'}}{\dfrac{c^2}{\lambda\lambda'}} = \frac{1}{c}(\lambda' - \lambda)$$

or

Compton effect

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta) \tag{3.42}$$

which is the result Compton found in 1923 for the increase in wavelength of the scattered photon.

Compton then proceeded to check the validity of his theoretical result by performing a very careful experiment in which he scattered x rays of wavelength 0.071 nm from carbon at several angles and showed that the modified wavelength was observed in good agreement with his prediction.* A part of his data is shown in Figure 3.19 where both the modified (λ') and unmodified scattered waves (λ) are seen.

The kinetic energy and scattering angle of the recoiling electron can also be predicted. Experiments in which the recoiling electrons were detected were soon carried out, thus confirming Compton's theory completely. The process of elastic photon scattering from electrons is now called the **Compton effect.** Note that the difference in wavelength, $\Delta\lambda = \lambda' - \lambda$, only depends on the constants h, c, and m_e in addition to the scattering angle θ. The quantity $\lambda_c = h/m_ec =$ 2.43×10^{-3} nm is called the **Compton wavelength** of the electron. Only for wavelengths on the same order as λ_c (or shorter) will the fractional shift $\Delta\lambda/\lambda$ be large. For visible light, for example with $\lambda = 500$ nm, the maximum $\Delta\lambda/\lambda$ is of the order of 10^{-5}, and $\Delta\lambda$ would be difficult to detect. The probability of the occurrence of the Compton effect for visible light is also quite small. However, for the x rays of wavelength 0.071 nm used by Compton, the ratio of $\Delta\lambda/\lambda$ is ~0.03 and could easily be observed. Thus, the Compton effect is important only for x rays or γ-ray photons and is small for visible light.

Compton wavelength

The physical process of the Compton effect can be described as follows. The photon elastically scatters from an essentially free electron in the material. (The

*An interesting self-account of Compton's discovery can be found in A. H. Compton, *Am. J. Phys.* **29,** 817–820 (1961).

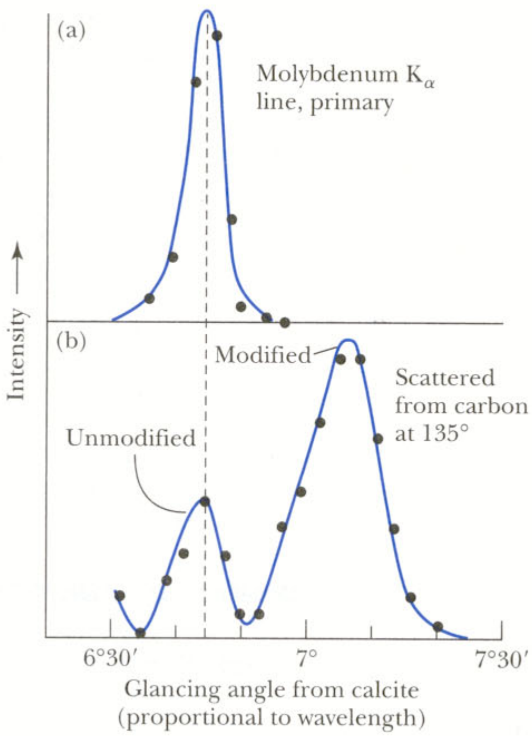


FIGURE 3.19 Compton's original data showing the primary x-ray beam from Mo unscattered in (a), and the scattered spectrum from carbon at 135° showing both the modified and unmodified wave in (b). *Adapted from Arthur H. Compton, Physical Review* **22**, *409 (1923).*

photon's energy is so much larger than the binding energy of the almost free electron that the binding energy can be neglected.) The newly created scattered photon then has a modified, longer wavelength. What happens if the photon scatters from one of the tightly bound inner electrons? Then the binding energy is not negligible, and the electron may not be able to be dislodged. The scattering would then effectively be from a much heavier system (nucleus + electrons). Then the mass in Equation (3.42) will be several thousand times larger than m_e, and $\Delta\lambda$ would be correspondingly smaller. Scattering from tightly bound electrons results in the unmodified photon scattering ($\lambda \approx \lambda'$), which also is observed in Figure 3.19. Thus, the quantum picture also explains the existence of the unmodified wavelength predicted by the classical theory (Thomson scattering) alluded to earlier.

The success of the Compton theory convincingly demonstrated the correctness of both the quantum concept and the particle nature of the photon. The use of the laws of the conservation of energy and momentum applied relativistically to pointlike scattering of the photon from the electron finally convinced the great majority of scientists of the validity of the new modern physics. Compton received the Nobel Prize in 1927.

Example 3.14

An x ray of wavelength 0.05 nm scatters from a gold target. (a) Can the x ray be Compton-scattered from an electron bound by as much as 62,000 eV? (b) What is the largest wavelength of scattered photon that can be observed? (c) What is the kinetic energy of the most energetic recoil electron and at what angle does it occur?

Solution: From Equation (3.35) the x-ray energy is

$$E_{\text{x ray}} = \frac{1.24 \times 10^3 \text{ eV}\cdot\text{nm}}{0.05 \text{ nm}} = 24{,}800 \text{ eV} = 24.8 \text{ keV}$$

Therefore, the x ray does not have enough energy to dislodge the inner electron, which is bound by 62 keV. In this case we have to use the atomic mass in Equation (3.42), which results in little change in the wavelength (Thomson scattering). Scattering may still occur from outer electrons.

The longest wavelength $\lambda' = \lambda + \Delta\lambda$ occurs when $\Delta\lambda$ is a maximum or when $\theta = 180°$.

$$\lambda' = \lambda + \frac{h}{m_e c}(1 - \cos 180°) = \lambda + \frac{2h}{m_e c}$$

$$= 0.05 \text{ nm} + 2(0.00243 \text{ nm}) = 0.055 \text{ nm}$$

The energy of the scattered photon is then a minimum and has the value

$$E'_{\text{x ray}} = \frac{1.24 \times 10^3 \text{ eV}\cdot\text{nm}}{0.055 \text{ nm}} = 2.25 \times 10^4 \text{ eV} = 22.5 \text{ keV}$$

The difference in energy of the initial and final photon must equal the kinetic energy of the electron (neglecting binding energies). The recoil electron must scatter in the forward direction at $\phi = 0°$ when the final photon is in the backward direction ($\theta = 180°$) in order to conserve momentum. The kinetic energy of the electron is then a maximum.

$$E_{\text{x ray}} = E'_{\text{x ray}} + \text{K.E. (electron)}$$

$$\text{K.E. (electron)} = E_{\text{x ray}} - E'_{\text{x ray}}$$

$$= 24.8 \text{ keV} - 22.5 \text{ keV} = 2.3 \text{ keV}$$

Because $\Delta\lambda$ does not depend on λ or λ', we can determine the wavelength (and energy) of the incident photon by merely observing the kinetic energy of the electron at forward angles (see Problem 50).

3.9 Pair Production and Annihilation

A general rule of nature is that if some process is not absolutely forbidden (by some law like conservation of energy, momentum, or charge) it will eventually occur. In the photoelectric effect, bremsstrahlung, and the Compton effect, we have studied exchanges of energy between photons and electrons. Have we covered all possible exchanges? For example, can the kinetic energy of a photon be converted into particle mass and vice versa? It would appear that if none of the conservation laws are violated, then such a process should be possible.

First, let us consider the conversion of photon energy into mass. The electron, which has a mass, $m = 0.51 \text{ MeV}/c^2$, is the lightest particle within an atom. Because an electron has negative charge, we must also create a positive charge to balance charge conservation. However, in 1932, C. D. Anderson (Nobel Prize, 1936) observed a positively charged electron (e^+) in cosmic radiation. This particle, called a **positron,** had been predicted to exist several years earlier by P. A. M. Dirac (Nobel Prize, 1933). It has the same mass as the electron but an opposite charge. Positrons are also observed when high-energy gamma rays (photons) pass through matter. Experiments show that a photon's energy can be converted entirely into an electron and a positron in the reaction

Positron

Pair production

$$\gamma \rightarrow e^+ + e^- \tag{3.43}$$

However, this process only occurs when the photon passes through matter, because energy and momentum are not conserved when the reaction takes place in isolation: the missing momentum must be supplied by interaction with a massive object such as a nucleus.

Example 3.15

Show that a photon cannot produce an electron–positron pair in free space as shown in Figure 3.20a.

Solution: Let the total energy and momentum of the electron and positron be E_-, p_- and E_+, p_+, respectively. The conservation laws are then

Energy
$$h\nu = E_+ + E_- \tag{3.44a}$$

Momentum, p_x
$$\frac{h\nu}{c} = p_- \cos\theta_- + p_+ \cos\theta_+ \tag{3.44b}$$

Momentum, p_y
$$0 = p_- \sin\theta_- - p_+ \sin\theta_+ \tag{3.44c}$$

Equation (3.44b) can be written as

$$h\nu = p_- c \cos\theta_- + p_+ c \cos\theta_+ \qquad (3.45)$$

Show that a photon cannot produce an electron–positron pair in free space as shown in Figure 3.20a.

Solution: Let the total energy and momentum of the electron and positron be E_-, p_- and E_+, p_+, respectively. The conservation laws are then

Energy $\qquad h\nu = E_+ + E_- \qquad (3.44a)$

Momentum, $p_x \qquad \frac{h\nu}{c} = p_- \cos\theta_- + p_+ \cos\theta_+ \qquad (3.44b)$

Momentum, $p_y \qquad 0 = p_- \sin\theta_- - p_+ \sin\theta_+ \qquad (3.44c)$

Equation (3.44b) can be written as

$$h\nu = p_- c \cos\theta_- + p_+ c \cos\theta_+ \qquad (3.45)$$

If we insert $E_\pm{}^2 = p_\pm{}^2 c^2 + m^2 c^4$ into Equation (3.44a), we have

$$h\nu = \sqrt{p_+{}^2 c^2 + m^2 c^4} + \sqrt{p_-{}^2 c^2 + m^2 c^4} \qquad (3.46)$$

The maximum value of $h\nu$ is, from Equation (3.45),

$$h\nu_{\text{max}} = p_- c + p_+ c$$

But from Equation (3.46), we also have

$$h\nu > p_- c + p_+ c$$

Equations (3.45) and (3.46) are inconsistent and cannot simultaneously be valid. Equations (3.44), therefore, do not describe a possible reaction. The reaction displayed in Figure 3.20a is not possible, because energy and momentum are not simultaneously conserved.

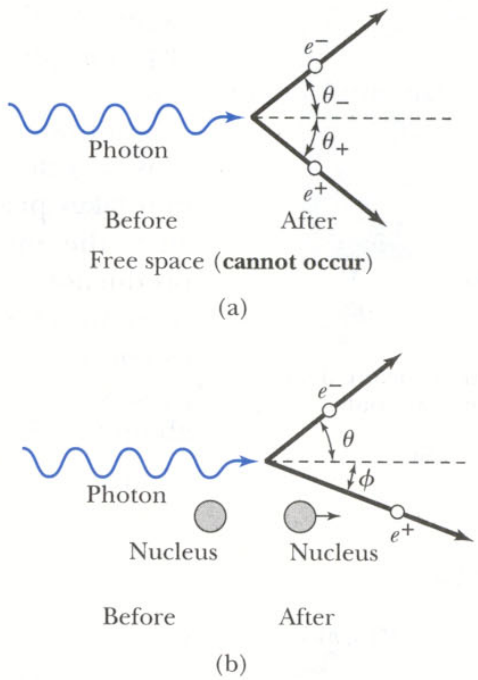


FIGURE 3.20 (a) A photon cannot decay into an electron–positron pair in free space, but (b) near a nucleus, the nucleus can absorb sufficient momentum to allow the process to proceed.

Consider the conversion of a photon into an electron and positron (called **pair production**) that takes place inside an atom where the electric field of a nucleus is large. The nucleus recoils and takes away a negligible amount of energy but a considerable amount of momentum. The conservation of energy will now be

$$h\nu = E_+ + E_- + \text{K.E. (nucleus)} \qquad (3.47)$$

A diagram of the process is shown in Figure 3.20b. The photon energy must be at least equal to $2m_e c^2$ in order to create the rest masses.

$$h\nu > 2m_e c^2 = 1.02 \text{ MeV} \qquad \text{(for pair production)} \qquad (3.48)$$

The probability of pair production increases dramatically both with higher photon energy and with higher atomic number Z of the nearby nucleus because of the correspondingly higher electric field which mediates the process.

The next question concerns the new particle, the positron. Why is it not commonly found in nature? We need to answer also the question posed earlier. Can mass be converted to pure kinetic energy?

Positrons are found in nature. They are detected in cosmic radiation and as products of radioactivity from a few radioactive elements. However, their lives are doomed because of their interaction with electrons. When positrons and electrons are in near proximity for even a short period of time they annihilate each other, producing photons. A positron passing through matter will quickly lose its kinetic energy through atomic collisions and with some probability will **annihilate** with an electron. After a positron slows down, it is drawn to an electron by their mutual electric attraction, and the electron and positron may then form an atomlike configuration called **positronium,** where they rotate around each other. Eventually the electron and positron come together and annihilate each other (typically in 10^{-10} s) producing electromagnetic radiation (photons). The process $e^+ + e^- \longrightarrow \gamma + \gamma$ is called **pair annihilation.**

Pair annihilation

Consider a positronium "atom" in free space. It must emit at least two photons in order to conserve energy and momentum. If the positronium annihilation takes place near a nucleus, it is possible that only one photon will be created, the missing momentum being supplied by nucleus recoil as in pair production, and under certain conditions three photons may be produced. Because the emission of two photons is by far the most likely annihilation mode, let us consider this mode, displayed in Figure 3.21. The conservation laws for the process $(e^+e^-)_{\text{atom}} \longrightarrow \gamma + \gamma$ will be (we neglect the atomic binding energy of about 6.8 eV)

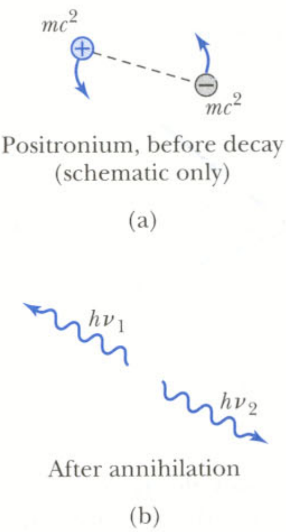


FIGURE 3.21 Annihilation of positronium atom (consisting of an electron and positron), producing two photons.

$$\text{Energy} \qquad 2m_ec^2 \approx h\nu_1 + h\nu_2 \tag{3.49a}$$

$$\text{Momentum} \qquad 0 = \frac{h\nu_1}{c} - \frac{h\nu_2}{c} \tag{3.49b}$$

where the photons obviously emerge in precisely opposite directions with equal energies, because the initial momentum is assumed to be zero (positronium at rest). Hence $\nu_1 = \nu_2 = \nu$. Thus Equation (3.49a) becomes

$$2m_ec^2 = 2h\nu$$

or

$$h\nu = m_ec^2 = 0.511 \text{ MeV} \tag{3.50}$$

In other words, the two photons from positronium annihilation will move in opposite directions, each with energy 0.511 MeV. This is exactly what is observed experimentally.

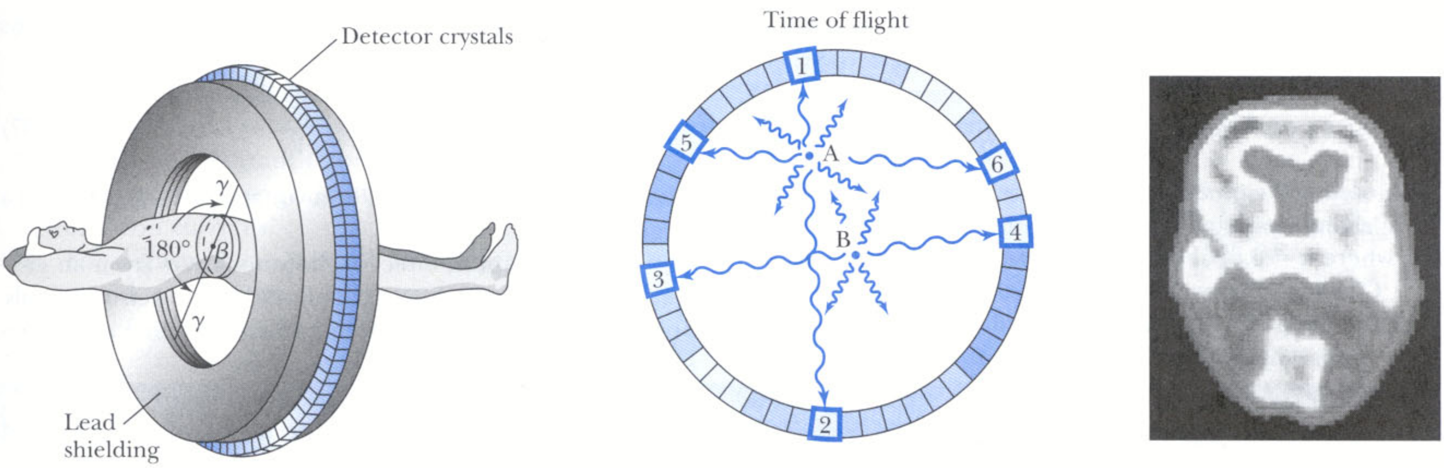
The production of two photons in opposite directions with energies of about $\frac{1}{2}$ MeV is so characteristic a signal of the presence of a positron that it has useful applications. Positron Emission Tomography (PET) scanning has become a standard diagnostic technique in medicine. A positron-emitting radioactive chemical (containing a nucleus such as ^{15}O, ^{11}C, ^{13}N, or ^{18}F) injected into the body causes two characteristic annihilation photons to be emitted from the points where the chemical has been concentrated by physiological processes. The location in the body where the photons originate is identified by measuring the directions of two gamma-ray photons of the correct energy that are detected in coincidence, as shown in Figure 3.22. Measurement of blood flow in the brain is an example of a diagnostic tool used in the evaluation of strokes, brain tumors, and other brain lesions.

PET scan

Before leaving the subject of positrons we should pursue briefly the idea of **antiparticles.** The positron is the antiparticle of the electron, having the opposite charge but the same mass.* In 1955 the antiproton was discovered by E. G. Segrè and O. Chamberlain (Nobel Prize, 1959), and by now, many antiparticles have been found. Physicists love to find symmetry in nature. We now

Antiparticles

*There are other particle properties (for example, spin) that will be described later (particularly in Chapters 7 and 14) and also need to be considered.



(a) (b) (c)

FIGURE 3.22 Positron emission tomography is a useful medical diagnostic to study the path and location of a positron-emitting radiopharmaceutical in the human body. (a) Appropriate radiopharmaceuticals are chosen to concentrate by physiological processes in the region to be examined. (b) The positron travels only a few mm before annihilation, which produces two photons that, after detection, give the positron position. (c) PET scan of a normal brain. *(a) and (b) are after G. L. Brownell, et al., Science **215**, 619 (1982); (c) National Institute of Health/Science Photo Library.*

believe that every particle has an antiparticle. In some cases, as for photons or neutral pi mesons, the particle and antiparticle are the same, but for most other particles (for example, the neutron and proton), particle and antiparticle are distinct.

We know that matter and antimatter cannot exist together in our world, because their ultimate fate would be annihilation. However, we may let our speculation run rampant! If we believe in symmetry, might there not be another world, perhaps in a distant galaxy, that is made of antimatter? Because galaxies are so far apart in space, annihilation would be infrequent. Modern cosmology predicts that the universe should be made up almost entirely of real particle matter and explains the obvious asymmetry this involves. However, if a large chunk of antimatter ever struck the Earth, it would tend to restore the picture of a symmetric universe. As we see from Problem 49, however, in such an event there would be no one left to receive the appropriate Nobel Prize.

Summary

In 1895 Röntgen discovered x rays, and in 1897 Thomson proved the existence of electrons and measured their charge to mass ratio. Finally, in 1911 Millikan reported an accurate determination of the electron's charge. Experimental studies resulted in the empirical Rydberg equation to calculate the wavelengths of the hydrogen atom's spectrum:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{k^2}\right) \qquad k > n \tag{3.13}$$

where $R_H = 1.096776 \times 10^7\ \text{m}^{-1}$.

In order to explain blackbody radiation Planck proposed his quantum theory of radiation in 1900 to signal the era of modern physics. From Planck's theory we can derive Wien's displacement law

$$\lambda_{max}\, T = 2.898 \times 10^{-3}\ \text{m} \cdot \text{K} \tag{3.14}$$

and the Stefan-Boltzmann law,

$$R(T) = \epsilon\sigma T^4 \tag{3.16}$$

Planck's radiation law gives the power radiated per unit area per unit wavelength from a blackbody.

$$\mathcal{I}(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \qquad (3.23)$$

The oscillators of the electromagnetic radiation field can only change energy by quantized amounts given by $\Delta E = h\nu$, where $h = 6.6261 \times 10^{-34}$ J·s is called *Planck's constant.*

Classical theory could not explain the photoelectric effect, but in 1905, Einstein proposed that the electromagnetic radiation field itself is quantized. We call these particlelike quanta of light *photons,* and they each have energy $E = h\nu$ and momentum $p = h/\lambda$. The photoelectric effect is easily explained by the photons each interacting with only one electron. The conservation of energy gives

$$h\nu = \phi + \frac{1}{2} mv_{\text{max}}^2 \qquad (3.30)$$

where ϕ is the work function of the emitter. The retarding potential required to stop all electrons depends only on the photon's frequency

$$eV_0 = \frac{1}{2} mv_{\text{max}}^2 = h\nu - h\nu_0 \qquad (3.33)$$

where $\phi = h\nu_0$. Millikan proved experimentally in 1916 that Einstein's theory was correct.

Bremsstrahlung radiation (x rays) is emitted when charged particles (for example, electrons) pass through matter and are accelerated by the nuclear field. These x rays have a minimum wavelength

$$\lambda_{\text{min}} = \frac{hc}{eV_0} \qquad (3.37)$$

where electrons accelerated by a voltage of V_0 impinge on a target.

In the Compton effect a photon scatters from an electron with a new photon created, and the electron recoils. For an incident and exit photon of wavelength λ and λ', respectively, the change in wavelength is

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta) \qquad (3.42)$$

when the exit photon emerges at angle θ to the original photon direction. The Compton wavelength of the electron is $\lambda_c = h/m_e c = 2.43 \times 10^{-3}$ nm. The success of the Compton theory in 1923 convincingly demonstrated the particle-like nature of the photon.

Finally, photon energy can be converted into mass in pair production

$$\gamma \rightarrow e^+ + e^- \qquad (3.43)$$

where e^+ is the positron, the antiparticle of the electron. Similarly a particle and antiparticle annihilate catastrophically in the process called *pair annihilation.*

$$e^+ + e^- \rightarrow \gamma + \gamma$$

Questions

1. How did the ionization of gas by cathode rays prevent H. Hertz from discovering the true character of electrons?
2. Why do television tubes generally deflect electrons with magnetic fields rather than with electric fields, as is done in cathode-ray-tube oscilloscopes?
3. In Thomson's *e/m* experiment, does it matter whether the electron passing through interacts first with the electric field or with the magnetic field? Explain.
4. Women in the late 1890s were terrified about the possible misuse of the new Röntgen x rays. Why do you think this fear occurred? Why was it no problem?
5. In Example 3.2, why would you be concerned about observing a cluster of several balls in the Millikan electron charge experiment?
6. In Figure 3.5 why does the histogram start smearing out for balls with multiple electron charges?
7. How is it possible for the plastic balls in Example 3.2 to have both positive and negative charges? What is happening?
8. Why do you suppose Millikan tried several different kinds of oil, as well as H_2O and Hg, for his oil drop experiment?
9. In the experiment of Example 3.2, how could you explain an experimental value of $q = 0.8 \times 10^{-19}$C?
10. Why do you suppose scientists worked so hard to develop better diffraction gratings?
11. Why was helium discovered in the sun's spectrum before being observed on Earth? Why was hydrogen observed on Earth first?
12. Do you believe there is any relation between the wavelengths of the Paschen (1908) and Pfund (1924) series and the respective dates they were discovered? Explain.
13. It is said that no two snowflakes look exactly alike, but we know that snowflakes have a quite regular, although complex, crystal structure. Discuss how this could be due to quantized behavior.
14. Why do we say that the elementary units of matter or "building blocks" must be some basic unit of mass-energy rather than of only mass?
15. Why is a red-hot object cooler than a white-hot one of the same material?
16. Why did scientists choose to study blackbody radiation from something as complicated as a hollow container rather than the radiation from something simple like a solid cylindrical thin disk (like a dime)?

17. Why does the sun apparently act as a blackbody?
18. In a typical photoelectric effect experiment, consider replacing the metal photocathode by a gas. What difference would you expect?
19. Why is it important to produce x-ray tubes with high accelerating voltages that are also able to withstand electron currents?
20. For a given beam current and target thickness, why would you expect a tungsten target to produce a higher x-ray intensity than targets of molybdenum or chromium?
21. List all possible known interactions between photons and electrons discussed in this chapter. Can you think of any more?
22. What do you believe to be an optimum lifetime for a positron-emitting radioactive nuclide used in brain tumor diagnostics? Explain.

Problems

3.1 Discovery of the X Ray and the Electron

1. Design an apparatus that will produce the correct magnetic field needed in Figure 3.2.
2. For an electric field of 2×10^5 V/m, what is the strength of the magnetic field needed to pass an electron of speed 2×10^6 m/s with no deflection? Draw **v**, **E**, and **B** directions for this to occur.
3. Across what potential difference does an electron have to be accelerated in order to reach the speed $v = 2 \times 10^7$ m/s? Should you use relativistic calculations?
4. An electron entering Thomson's e/m apparatus (Figure 3.2) has an initial velocity (in horizontal direction only) of 0.5×10^7 m/s. Lying around the lab is a permanent horseshoe magnet of strength 1.3×10^{-2} T, which you would like to use. What electric field will you need in order to produce zero deflection of the electrons as they travel through the apparatus? When the magnetic field is turned off, but the same electric field remains, how large a deflection will occur if the region of nonzero **E** and **B** fields is 2 cm long?

3.2 Determination of Electron Charge

5. Consider the following possible forces on an oil drop in Millikan's experiment: gravitational, electrical, frictional, and buoyant. Draw a diagram indicating the forces on the oil drop (a) when the electric field is turned off and the droplet is falling freely, and (b) when the electric field causes the droplet to rise.
6. Neglecting the buoyancy force on an oil droplet, show that the terminal speed of the droplet is $v_t = mg/f$, where f is the coefficient of friction when the droplet is in free fall. (Remember that the frictional force $\mathbf{F}_f$ is given by $\mathbf{F}_f = -f\mathbf{v}$ where velocity is a vector).
7. Stokes's law relates the coefficient of friction f to the radius r of the oil drop and the viscosity η of the medium the droplet is passing through: $f = 6\pi\eta r$. Show that the radius of the oil drop is given in terms of the terminal velocity v_t (see previous problem), η, g, and the density of the oil ρ by $r = 3\sqrt{\eta v_t/2g\rho}$.
8. In a Millikan oil drop experiment the terminal velocity of the droplet is observed to be 1.3 mm/s. The density of the oil is $\rho = 900$ kg/m^3 and the viscosity of air is $\eta = 1.82 \times 10^{-5}$ kg/m·s. Using the results of the two previous problems, calculate (a) the droplet radius, (b) the mass of the droplet, and (c) the coefficient of friction.

3.3 Line Spectra

9. What is the series limit (that is, the smallest wavelength) for the Lyman series? For the Balmer series?
10. Light from a slit passes through a transmission diffraction grating of 400 lines/mm, which is located 2.5 m from a screen. What are the distances on the screen (from the unscattered slit image) of the three brightest visible (first order) hydrogen lines?
11. A transmission diffraction grating of 420 lines/mm is used to study the light intensity of different orders (n). A screen is located 2.5 m from the grating. What is the separation on the screen between the three brightest red lines for a hydrogen source?

3.4 Quantization

12. Quarks have charges $\pm e/3$ and $\pm 2e/3$. What combination of three quarks could yield (a) a proton, (b) a neutron?

3.5 Blackbody Radiation

13. Calculate λ_{max} for blackbody radiation for (a) liquid helium (4.2 K), (b) room temperature (293 K), and (c) a steel furnace (2500 K).
14. Calculate the temperature of a blackbody if the spectral distribution peaks at (a) gamma rays, $\lambda = 10^{-14}$ m, (b) x rays, 1 nm, (c) red light, 670 nm, (d) broadcast television waves, 1 m, and (e) AM radio waves, 204 m.
15. A blackbody's temperature is increased from 900 K to 1900 K. By what factor does the total power radiated per unit area increase?
16. For a blackbody at a given temperature T, what is the long-wavelength limit ($\lambda \gg hc/kT$) of Planck's radiation law? (This is the Rayleigh-Jeans result known to Planck in 1895).

17. A tungsten filament of a typical incandescent light bulb operates at a temperature near 3000 K. At what wavelength is the intensity a maximum?

18. Use a computer to calculate Planck's radiation law for a temperature of 3000 K, which is the temperature of a typical tungsten filament in an incandescent light bulb. Plot the intensity versus wavelength. (a) How much of the power is in the visible region (400–700 nm) compared with the ultraviolet and infrared? (b) What is the ratio of the intensity at 400 nm and 700 nm to the maximum?

19. Show that the ultraviolet catastrophe is avoided for short wavelengths ($\lambda \to 0$) with Planck's radiation law by calculating the limiting intensity $\mathcal{I}(\lambda, T)$ as $\lambda \to 0$.

20. Estimate the power radiated by (a) a basketball at 20°C, (b) the human body (assume a temperature of 37°C).

21. At what wavelength is the radiation emitted by the human body a maximum? Assume a temperature of 37°C.

22. If we have waves in a one-dimensional box, such that the wave displacement $\Psi(x, t) = 0$ for $x = 0$ and $x = L$, where L is the length of the box, and

$$\frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} - \frac{\partial^2\Psi}{\partial x^2} = 0 \qquad \text{(wave equation)}$$

show that the solutions are of the form

$$\Psi(x, t) = a(t)\sin\left(\frac{n\pi x}{L}\right) \qquad (n = 1, 2, 3, \ldots)$$

and $a(t)$ satisfies the (harmonic-oscillator) equation

$$\frac{d^2a(t)}{dt^2} + \Omega_n{}^2 a(t) = 0$$

where $\Omega_n = \dfrac{n\pi c}{L}$ is the angular frequency, $2\pi\nu$.

23. If the angular frequencies of waves in a three-dimensional box of sides L generalize to

$$\Omega = \frac{\pi c}{L}(n_x^2 + n_y^2 + n_z^2)^{1/2}$$

where all n are integers, show that the number of distinct states in the frequency interval ν to $\nu + d\nu (\nu = \Omega/2\pi)$ is given by (where ν is large)

$$dN = 4\pi \frac{L^3}{c^3}\nu^2 d\nu.$$

24. Let the energy density in the frequency interval ν to $\nu + d\nu$ within a blackbody at temperature T be $dU(\nu, T)$. Show that the power emitted through a small hole of area ΔA in the container is

$$\frac{c}{4}dU(\nu, T)\,\Delta A$$

25. Derive the Planck radiation law emitted by a blackbody. Remember that light has two directions of polarization and treat the waves as an ensemble of harmonic oscillators.

3.6 Photoelectric Effect

26. An FM radio station of frequency 107.7 MHz puts out a signal of 50,000 W. How many photons/s are emitted?

27. How many photons/s are contained in a beam of electromagnetic radiation of total power 150 W if the source is (a) an AM radio station of 1100 kHz, (b) 8-nm x rays, and (c) 4-MeV gamma rays?

28. What is the threshold frequency for the photoelectric effect on lithium ($\phi = 2.9$ eV)? What is the stopping potential if the wavelength of the incident light is 400 nm?

29. What is the maximum wavelength of incident light that can produce photoelectrons from silver ($\phi = 4.7$ eV)? What will be the maximum kinetic energy of the photoelectrons if the wavelength is halved?

30. A 2-mW laser ($\lambda = 530$ nm) shines on a cesium photocathode ($\phi = 1.9$ eV). Assuming an efficiency of 10^{-5} for producing photoelectrons (that is, one photoelectron produced for every 10^5 incident photons), what is the photoelectric current?

31. An experimenter finds that no photoelectrons are emitted from tungsten unless the wavelength of light is less than 230 nm. Her experiment will require photoelectrons of maximum kinetic energy 2.0 eV. What frequency light should be used to illuminate the tungsten?

32. The human eye is sensitive to a pulse of light containing as few as 100 photons. For yellow light of wavelength 580 nm how much energy is contained in the pulse?

33. In a photoelectric experiment it is found that a stopping potential of 1.0 V is needed to stop all the electrons when incident light of wavelength 260 nm is used and 2.3 V is needed for light of wavelength 207 nm. From these data determine Planck's constant and the work function of the metal.

34. What is the limit of energies and frequencies for visible light of wavelengths 400–700 nm?

3.7 X-Ray Production

35. What is the minimum x-ray wavelength produced for an x-ray machine operated at 30 kV?

36. The Stanford Linear Accelerator can accelerate electrons to 50 GeV (50×10^9 eV). What is the minimum wavelength photons it can produce by bremsstrahlung? Is this photon still called an x ray?

37. A television tube operates at 20,000 V. What is λ_{min} for the continuous x-ray spectrum produced when the electrons hit the phosphor?

3.8 Compton Effect

38. Calculate the maximum $\Delta\lambda/\lambda$ of Compton scattering for green light ($\lambda = 530$ nm). Could this be easily observed?

39. A photon having 40 keV scatters from a free electron at rest. What is the maximum energy that the electron can obtain?

40. If a 6 keV photon scatters from a free proton at rest, what is the change in the photon's wavelength if the photon recoils at 90°?

41. Is it possible to have a scattering similar to Compton scattering from a proton in H_2 gas? What would be the Compton wavelength for a proton? What energy photon would have this wavelength?

42. An instrument has resolution $\Delta\lambda/\lambda = 0.4\%$. What wavelength incident photons should be used in order to resolve the modified and unmodified scattered photons for scattering angles of (a) 30°, (b) 90°, and (c) 170°?

43. Derive the relation for the recoil kinetic energy of the electron and its recoil angle ϕ in Compton scattering. Show that

$$\text{K.E. (electron)} = \frac{\Delta\lambda/\lambda}{1 + \dfrac{\Delta\lambda}{\lambda}} h\nu$$

$$\cot\phi = \left(1 + \frac{h\nu}{mc^2}\right)\tan\frac{\theta}{2}$$

44. A gamma ray of 700 keV energy Compton scatters from an electron. Find the energy of the photon scattered at 110°, the energy of the scattered electron, and the recoil angle of the electron.

45. A photon of wavelength 2 nm Compton scatters from an electron at an angle of 90°. What is the modified wavelength and the percentage change, $\Delta\lambda/\lambda$?

3.9 Pair Production and Annihilation

46. How much photon energy would be required to produce a proton–antiproton pair? Where could such a high-energy photon come from?

47. What is the minimum photon energy needed to create an e^-–e^+ pair when a photon collides (a) with a free electron at rest and (b) with a free proton at rest?

General Problems

48. What wavelength photons are needed to produce 30 keV electrons in a Compton scattering?

49. The gravitational energy of the Earth is approximately $\frac{1}{2}(GM_E^2/R_E)$ where M_E is the mass of the Earth. This is approximately the energy needed to blow the Earth into small fragments (the size of asteroids). How large would an antimatter meteorite the density of nickel-iron ($\rho \sim 5 \times 10^3$ kg/m^3) have to be in order to blow up the Earth when it strikes? Compute the energy involved in the particle–antiparticle annihilation and compare it with the total energy in all the nuclear arsenals of the world (~2000 megaton (MT), where 1 MT = 4.2×10^{15} J).

50. Show that the maximum kinetic energy of the recoil electron in Compton scattering is given by

$$\text{K.E.}_{\text{max}}\text{ (electron)} = h\nu \frac{\dfrac{2h\nu}{mc^2}}{1 + \dfrac{2h\nu}{mc^2}}$$

At what angles θ and ϕ does this occur? If we detect a scattered electron at $\phi = 0°$ of 100 keV, what energy photon was scattered?

51. Using the Wien displacement law, make a log–log plot of λ_{max} (from 10^{-8} m to 10^{-2} m) versus temperature (from 10^0 K to 10^5 K). Mark on the plot the regions of visible, ultraviolet, infrared, and microwave wavelengths. Put the following points on the line: sun (5800 K), furnace (1900 K), room temperature (300 K), and the background radiation of the universe (2.7 K). Discuss the electromagnetic radiation that is emitted from each of these sources. Does it make sense?

52. (a) What is the maximum possible energy for a Compton-backscattered x ray ($\theta = 180°$)? Express your answer in terms of λ, the wavelength of the incoming photon. (b) Evaluate numerically if the incoming photon's energy is 100 keV.