EL582/BE620 --- Medical Imaging - I

Physics of MRI

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Based on J. L. Prince and J. M. Links, Medical Imaging Signals and Systems, and lecture notes by Prince. Figures are from the textbook except otherwise noted.

Lecture Outline

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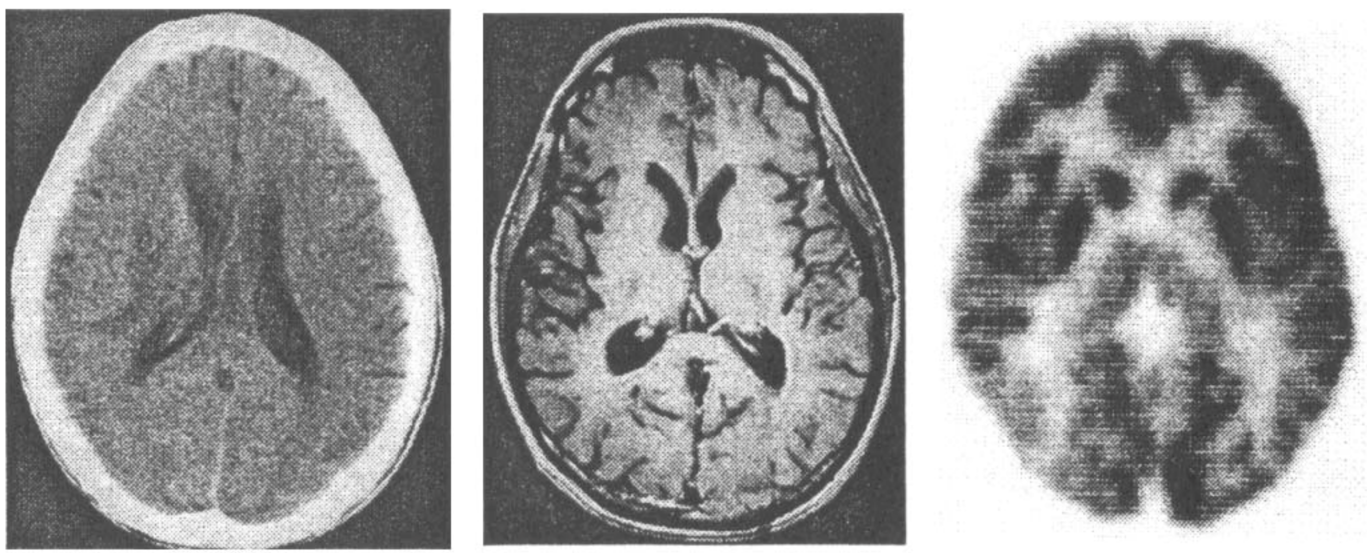
- Overview of MRI
- Nuclear spin properties
- Precession and Larmor Frequency
- RF excitation
- Relaxation
- Contrast mechanism

Magnetic Resonance Imaging

- Provide high resolution anatomic structure (as with X-ray CT)
- Provide high contrast between different soft tissues (X-ray CT cannot)
- No exposure to radiation and hence safe
- More complicated instrumentation
- Takes longer to acquire a scan than CT, more susceptible to patient motion

CT MRI PET



(a) (b) (c)

Figure I.4

X-ray projection

MRI

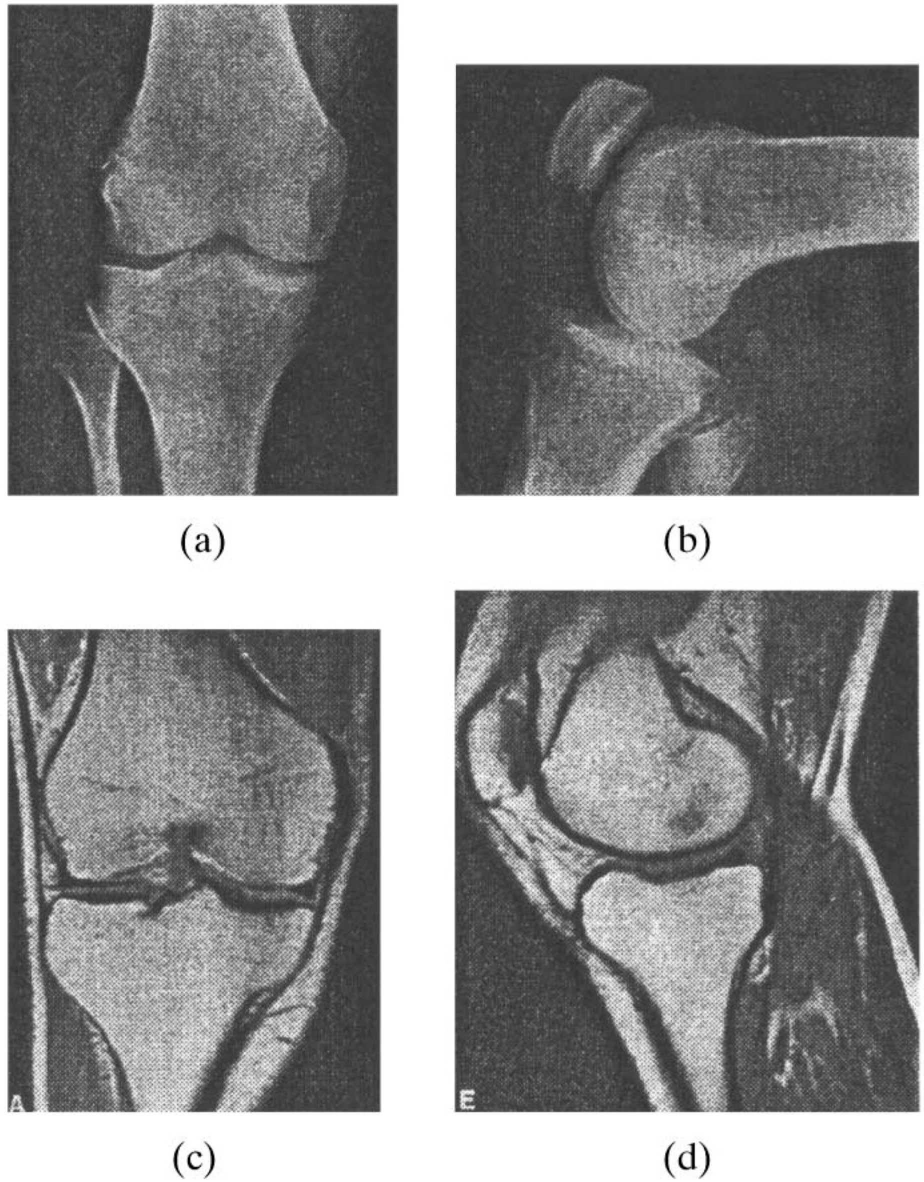


Figure V.1

Basic Principle of MRI

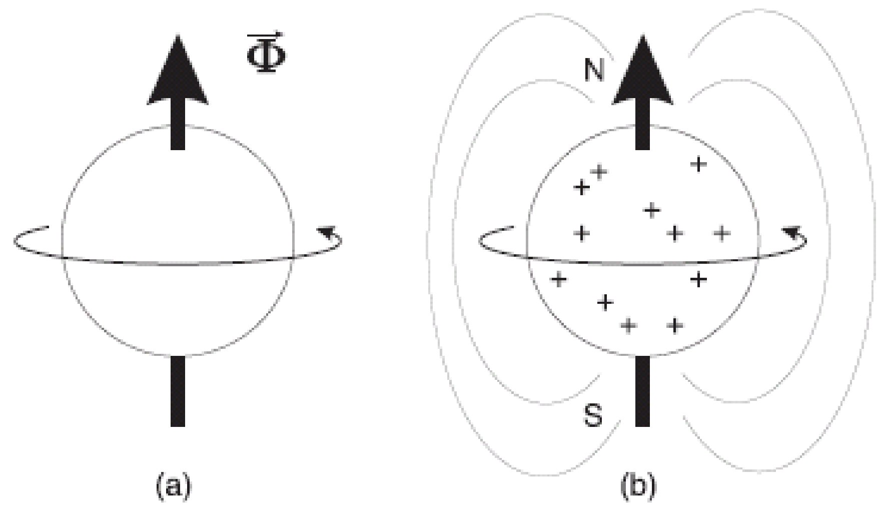
- The hydrogen (1^H) atom inside body possess “spin”
- In the absence of external magnetic field, the spin directions of all atoms are random and cancel each other.
- When placed in an external magnetic field, the spins align with the external field.
- By applying an rotating magnetic field in the direction orthogonal to the static field, the spins can be pulled away from the z-axis with an angle \alpha
- The bulk magnetization vector rotates around z at the Larmor frequency (precess)
- The precession relaxes gradually, with the xy-component reduces in time, z-component increases
- The xy component of the magnetization vector produces a voltage signal, which is the NMR signal we measure

What is Spin?

- Spin is a fundamental property of nature like electrical charge or mass. Spin comes in multiples of 1/2 and can be + or -. Protons, electrons, and neutrons possess spin. Individual unpaired electrons, protons, and neutrons each possesses a spin of ½ or - ½.
- Two or more particles with spins having opposite signs can pair up to eliminate the observable manifestations of spin.
- In nuclear magnetic resonance, it is unpaired nuclear spins that are of importance.

Nuclear Spin

- A nucleus consists of protons and neutrons
- When the total number of protons and neutrons (=mass number A) is odd or the total number of protons is odd, a nucleus has an angular momentum (\phi) and hence spin
  - Ex. Hydrogen (1^H) (1 proton), 13^C
- The spin of a nucleus generates a magnetic filed, which has a magnetic moment (\mu)
- The spin causes the nucleus behave like a tiny magnet with a north and south pole



Angular momentum vs Magnetic Moment

-]
  - Microscopic magnetic moment vector:

$$\boldsymbol{\mu} = \gamma \boldsymbol{\Phi}$$

  - γ is <u>gyromagnetic ratio</u> [radians/s-T]
  - $\bar{\gamma}$ has more convenient units [Hz/T]

$$\bar{\gamma} = \frac{\gamma}{2\pi}$$

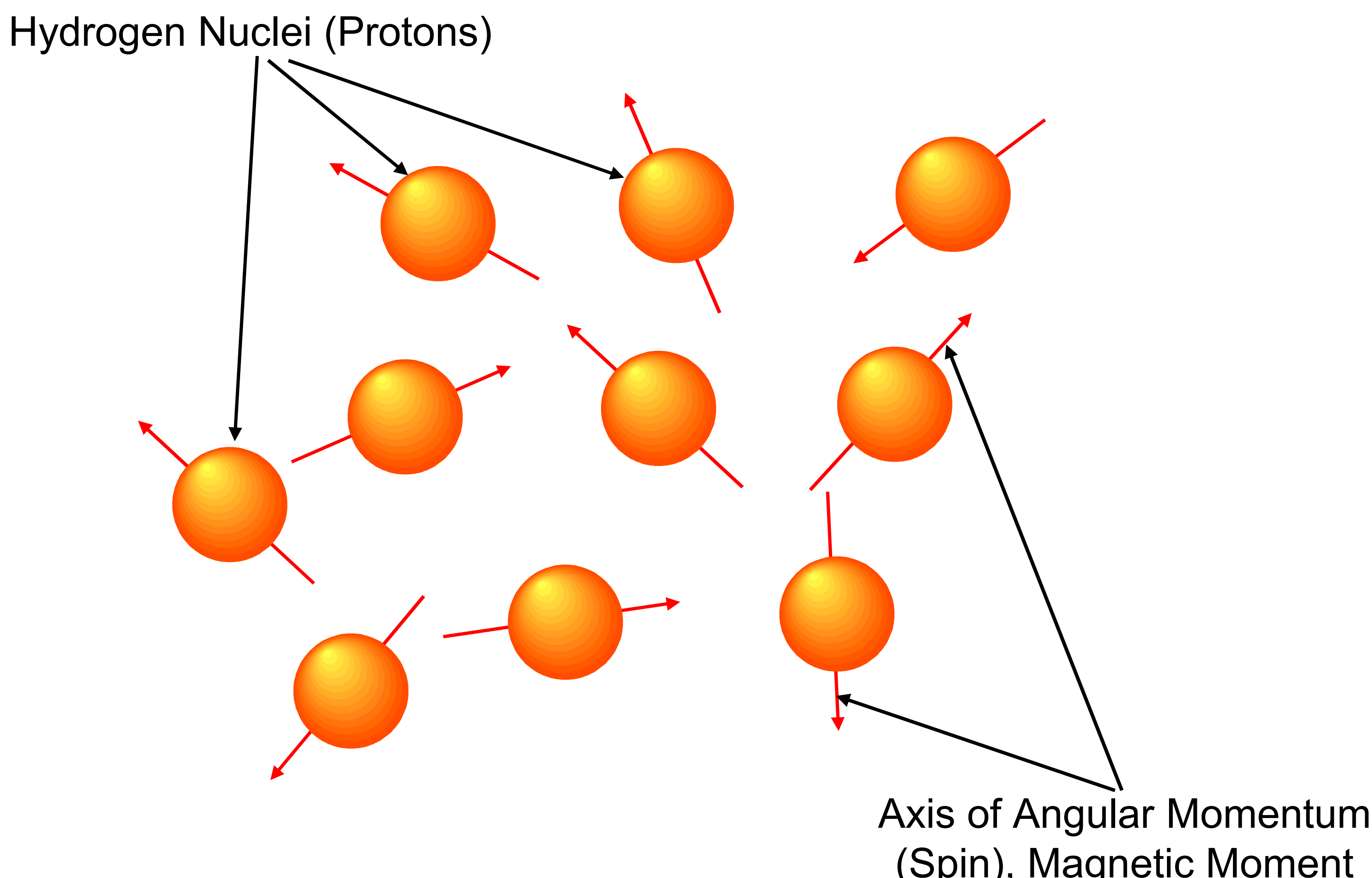
  - For ^{1}H

$$\bar{\gamma} = 42.58 \text{ MHz/T}$$

Nuclear Spin System

- Collection of identical nuclei in a given sample of material (also known as spin packet, a voxel in the imaged volume)
- In the absence of external magnetic field, the spin orientations of the nuclei are random and cancel each other
- When placed in a magnetic field, the microscopic spins tend to align with the external field, producing a net bulk magnetization aligned with the external field

In the absence of external magnetic field



From Graber, Lecture note for BMI F05

Nuclear Magnetization

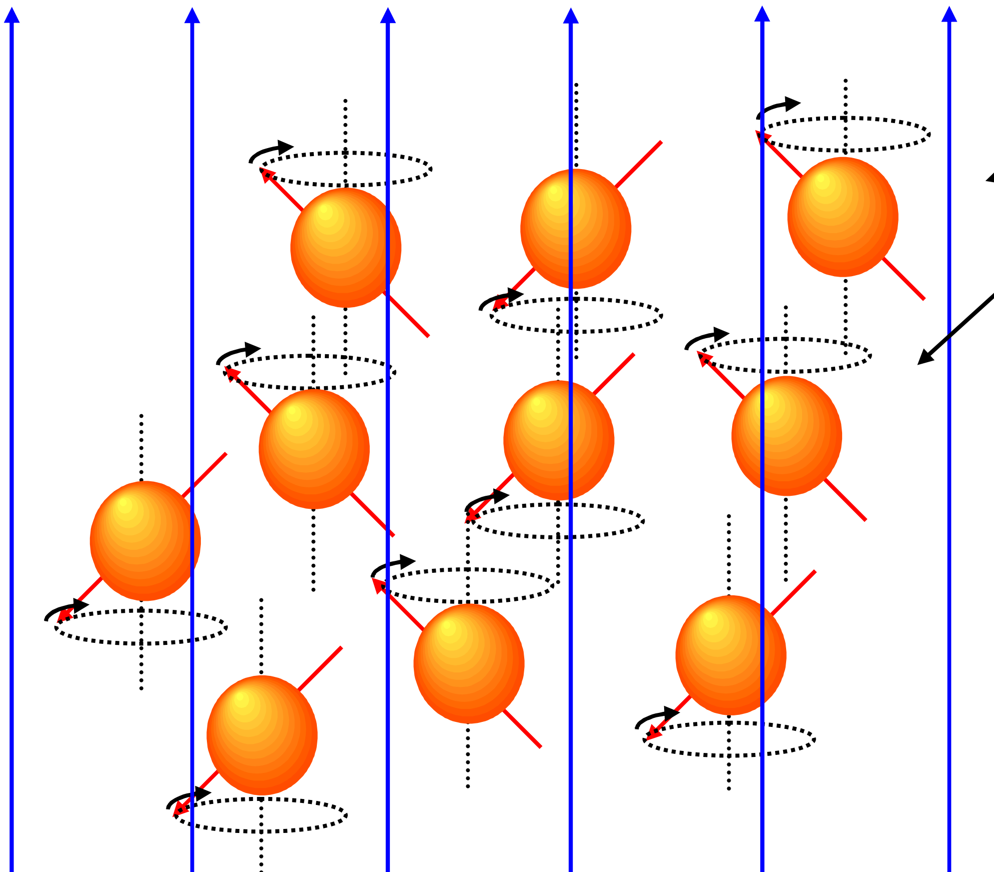
- Put sample in external magnetic field

$$\mathbf{B}_0 = B_0 \hat{z}$$

- Spins align in one of two directions
  - $54°$ off $\hat{z}$ "up" (low energy state)
  - $180 - 54°$ off $\hat{z}$ "down" (high energy state)
- Slight preference for "up" direction N-/N+ = e-E/kT
- Sample becomes magnetized
- Magnetization vector:

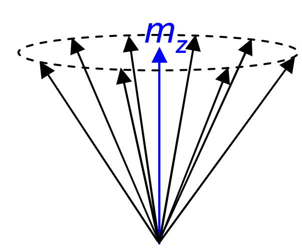
$$\mathbf{M} = \sum_{n=1}^{N_s} \boldsymbol{\mu}_n$$

Precession



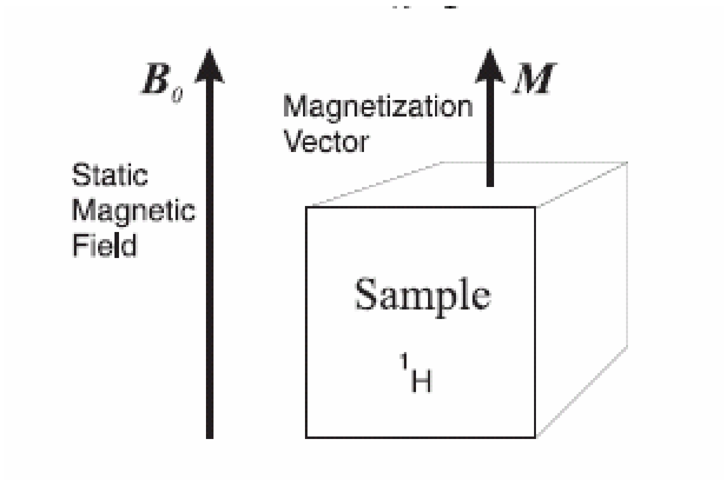
Spins **PRECESS** at a single frequency (w_0), but *incoherently* – they are not in phase, so that the sum of x-y components is 0, with net magnetization vector in z direction

W0=\gamma B_0: Larmor freq.



From Graber, Lecture note for BMI F05

Bulk Magnetization at Equilibrium



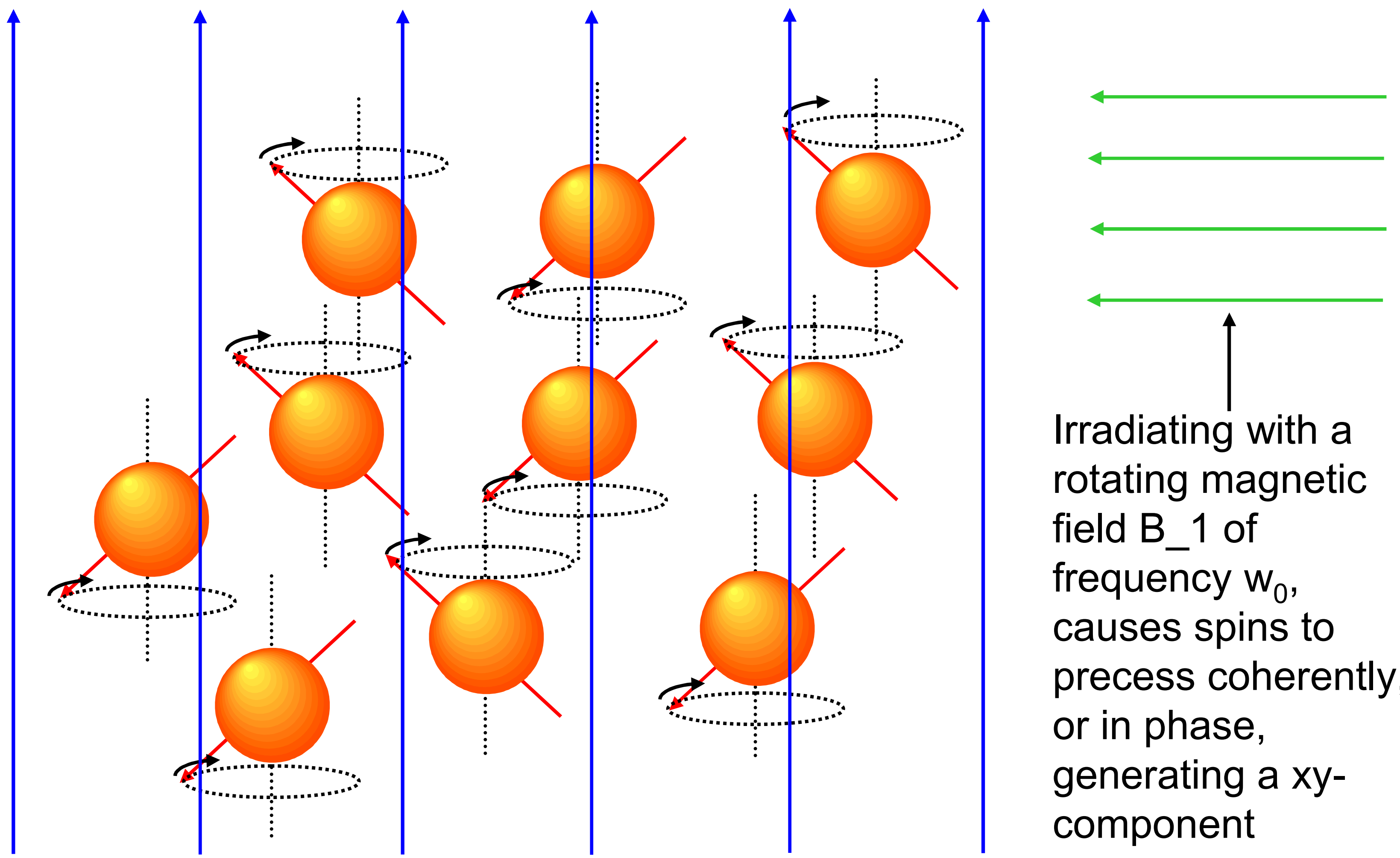
- Equilibrium value: $\mathbf{M}_0$
  - same direction as $\mathbf{B}_0$
  - depends on $\mathbf{x} = (x, y, z)$ only
- Magnitude: M_0

$$M_0 = \frac{B_0 \gamma^2 \hbar^2}{4kT} P_D$$

  - k is Boltzmann's constant
  - T is temperature
  - P_D is proton density

Which depends on tissue type

How to make the spins in phase?



Irradiating with a rotating magnetic field B_1 of frequency w_0, causes spins to precess coherently, or in phase, generating a xy-component

Process Involved in MRI

- Put patient in a static field B_0 (much stronger than the earth's field)
- (step 1) Wait until the nuclear magnitization reaches an equilibrium (align with B_0)
- Applying a rotating magnetic field B_1 (much weaker than B_0) to bring M to an initial angle \alpha with B_0 (rotating freq=Larmor freq.)
- M(t) precess around B_0 at Larmor frequency around B_0 axis (z direction) with angle \alpha
- The component in z increases in time (longitudinal relaxation) with time constant T1
- The component in x-y plane reduces in time (transverse relaxation) with time constant T2
- Measure the transverse component at a certain time after the excitation (NMR signal)
- Go back to step 1
- By using different excitation pulse sequences, the signal amplitude can reflect mainly the proton density, T1 or T2 at a given voxel

Evolution of magnetization when a Time varying magnetic field is applied

- $\mathbf{M} = \mathbf{M}(\mathbf{x}, t)$
- Relation to bulk angular momentum $\mathbf{J}$

$$\mathbf{M} = \gamma \mathbf{J}$$

- Focus on small sample $\rightarrow$ voxel
  - $\mathbf{M} = \mathbf{M}(t)$
  - Equations of motion = Bloch equations

- M(t) experiences a torque when an external magnetic field B(t) is applied

torque is $\boldsymbol{\tau} = \mathbf{M} \times \mathbf{B}$

- Torque is related to angular momentum

$$\boldsymbol{\tau} = \frac{d\mathbf{J}}{dt}$$

- Eliminate $\mathbf{J}$ to yield

$$\frac{d\mathbf{M}(t)}{dt} = \gamma \mathbf{M}(t) \times \mathbf{B}(t)$$

- Valid for "short" times

Using the right hand rule, M will rotate around z if M is not aligned with z

Cross Product: Review

$$\mathbf{M} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ M_x & M_y & M_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \left(M_y B_z - M_z B_y\right)\mathbf{i} + \left(M_z B_x - M_x B_z\right)\mathbf{j} + \left(M_x B_y - M_y B_x\right)\mathbf{k}$$

Direction of MxB follows “right hand” rule

Solution under a Static Field with an Initial Angle

- $B(t)=[0,0,B_0]$

- $M \times B = M_y B_0 \mathbf{i} - M_x B_0 j + 0 \ \mathbf{k}$

- $dM_x/dt = M_y B_0$
- $dM_y/dt = - M_x B_0$

- Solving above yields solution in the next slide

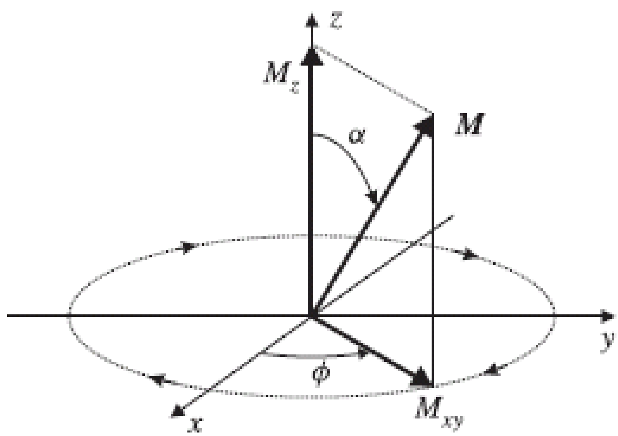
Precession Due to a Static Field with an Initial Angle

- Let $\mathbf{B}(t) = \mathbf{B}_0$; $\mathbf{M}(0)$ angle α with $\hat{z}$
- Then

$$\begin{aligned} M_x(t) &= M_0 \sin\alpha \cos\left(-\gamma B_0 t + \phi\right) \\ M_y(t) &= M_0 \sin\alpha \sin\left(-\gamma B_0 t + \phi\right) \\ M_z(t) &= M_0 \cos\alpha \end{aligned}$$

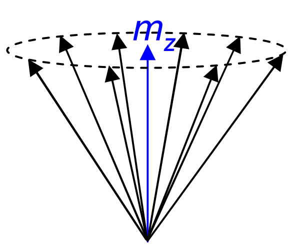
where

$$M_0 = |\mathbf{M}(0)| \quad \phi \text{ arbitrary}$$



- Precession with Larmor frequency

$$\omega_0 = \gamma B_0 \quad \text{or} \quad \nu_0 = \gamma B_0$$



This is the frequency of the photon which would cause a transition between the two energy levels of the spin.

B0=1.5T, \gamma=42.58 MHz/T, v0=63.9 MHz

Longitudinal and Transverse Components

- Magetization

$$\mathbf{M}(t) = (M_x(t), M_y(t), M_z(t))$$

- Think of $\mathbf{M}(t)$ with two components
  - Longitudinal magnetization

$$M_z(t)$$

No change

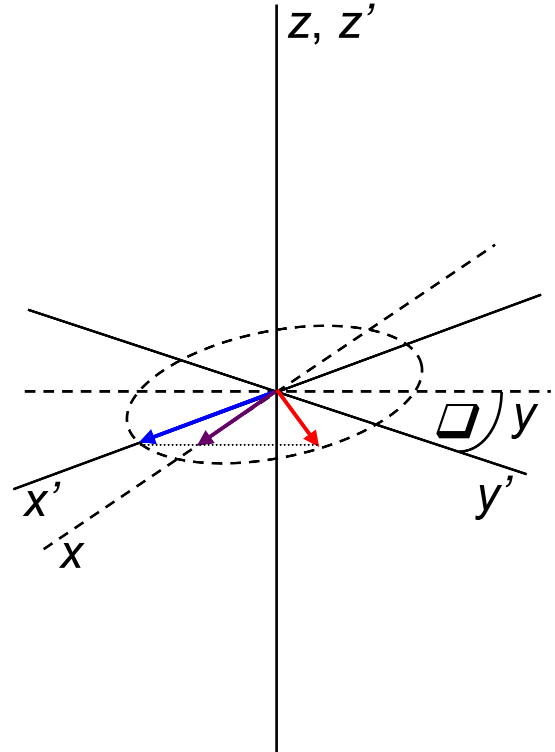
  - Transverse magnetization

$$M_{xy}(t) = M_x(t) + jM_y(t)$$

Rapidly rotating

Laboratory Frame vs. Rotating Frame

Coordinate system rotated about *z* axis at the Larmor freq.



The rotating M(t) vector appear stationary in the rotating frame

- See animation at
- http://www.cis.rit.edu/htbooks/mri/chap-3/c13-1.htm

NMR Signal

- The rapidly rotating transverse magnetization (M_xy) creates a radio frequency excitation within the sample.
- If we put a coil of wire outside the sample, the RF excitation will induce a voltage signal.
- In MRI, we measure this voltage signal.
- Voltage produced is (Faraday's Law of Induction)

$$V(t) = -\frac{\partial}{\partial t} \int_{\text{object}} \mathbf{M}(\mathbf{r}, t) \cdot \mathbf{B}^r(\mathbf{r})\, d\mathbf{r}$$

- $\mathbf{B}^r(\mathbf{r})$ is field produced at $\mathbf{r}$ by unit direct current in coil around sample.

Simplification

- B^r(r)=B^r

- Longitudinal magnetization changes too slow
- Transverse magnetization dominates

$$M_{xy}(t) = M_0 \sin\alpha e^{-j(\omega_0 t - \phi)}$$

- Final expression

$$V(t) = -\omega_0 V_s M_0 \sin\alpha\, B^r \sin(-\omega_0 t + \phi - \theta_r)$$

$$|V| = \omega_0 V_s M_0 \sin\alpha\, B^r$$

Recall $\omega_0 = \gamma B_0, M_0 = \frac{B_0 \gamma^2 h^2}{4\kappa T} P_D$

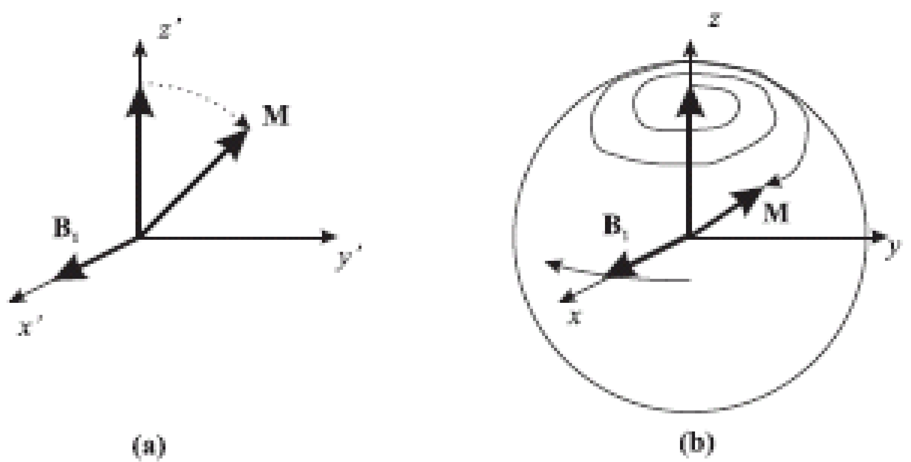
Therefore $|\mathrm{V}| \propto B_0{}^2, P_D$

How do we tilt M to an initial angle?

- Applying a circularly polarized (rotating) magnetic field B_1(t) in the x-y plane with the same Larmor frequency forces the magnetization vector to tilt down to the x-y plane

$$B_1(t) = B_1^e(t)e^{-j(\omega_0 t - \varphi)}$$

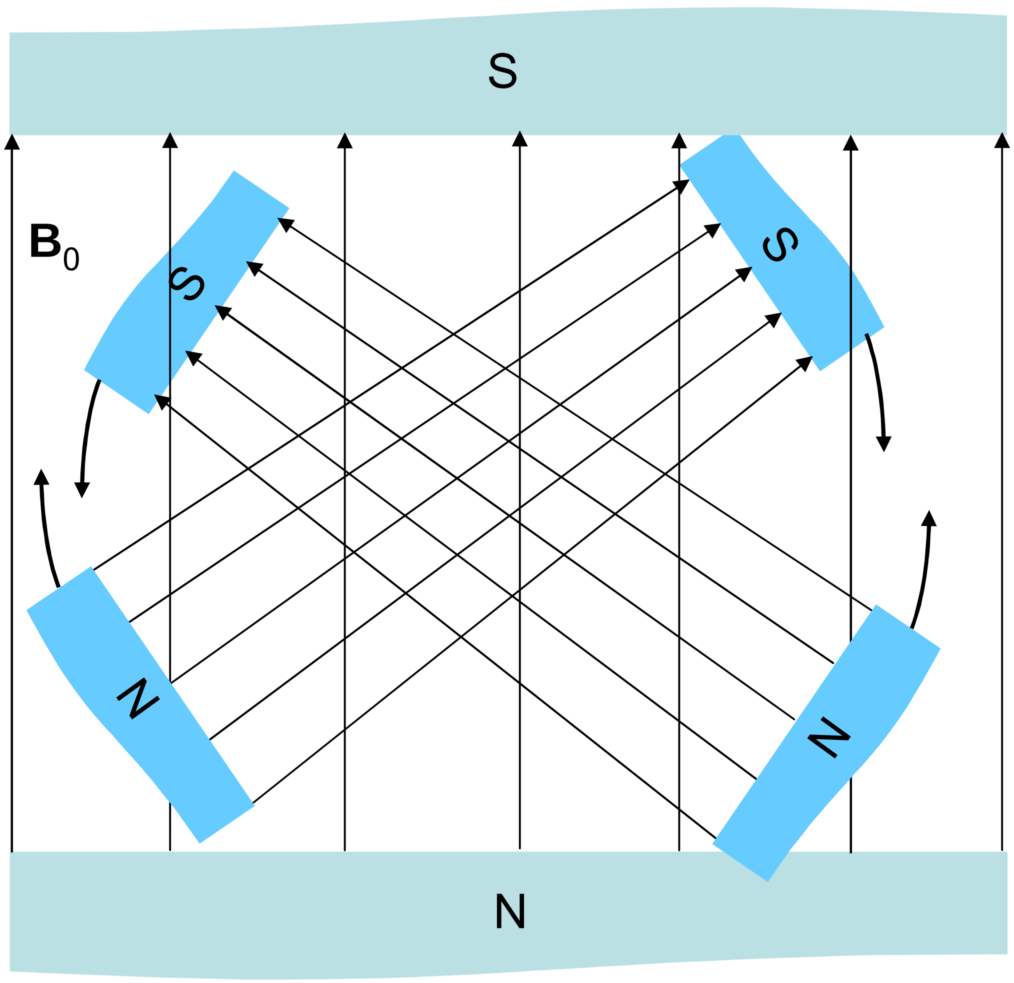
  - B_1(t) has two orthogonal components, in x and y directions respectively, and is produced by using quadrature RF coil
  - Simplest envelop B_1,e is a rectangular pulse
- Motion of M(t) is spiral



Animation of spiral motion

- Laboratory frame: http://www.cis.rit.edu/htbooks/mri/chap-3/c14-5.htm
- Rotating frame: http://www.cis.rit.edu/htbooks/mri/chap-3/c14-5.htm

Circularly Polarized Magnetic Field



two more magnets, whose fields are orthogonal to $\mathbf{B}_0$, that rotate, in opposite directions, at the Larmor frequency

Tip Angle

- If M is parallel to z-axis before the RF excitation pulse, the tip angle after the excitation (with duration \tau_p) is

$$\alpha = \gamma \int_0^{\tau_p} B_1^e(t)dt$$

- If B_1^e(t) is rectangular

$$\alpha = \gamma B_1 \tau_p$$

- Pulse that leads to \alpha=\pi/2 is called “\pi over 2 pulse”, which elicits the largest transverse component M_xy, and hence largest NMR signal
- Pulse that leads to \alpha=\pi is called “\pi pulse” or inverse pulse, which is used to induce spin echo (later)
- The excitation pulse (envelop of B_1(t)) is also called “an alpha pulse”

Relaxation

- Magnetization cannot precess forever
- Two independent relaxation processes
- Transverse relaxation
  - $\equiv$ spin-spin relaxation
- Longitudinal relaxation
  - $\equiv$ spin-lattice relaxation
- Detailed properties differ in tissues
  - Gives rise to tissue contrast

Longitudinal Relaxation

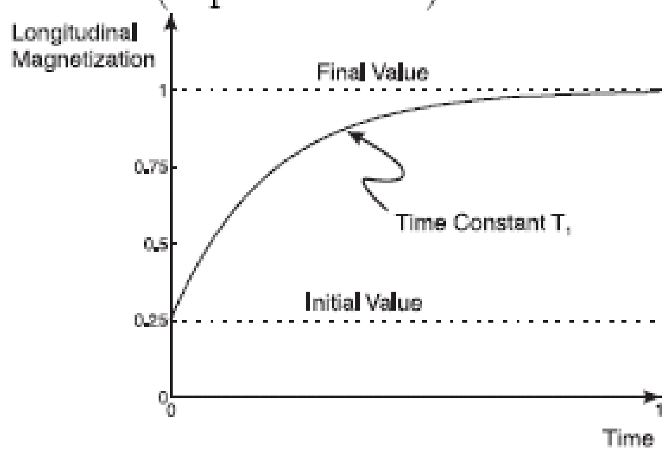
- The magnetization vectors tend to return to equilibrium state (parallel to B_0)

  - $M_z(t)$ behaves as <u>rising exponential</u>

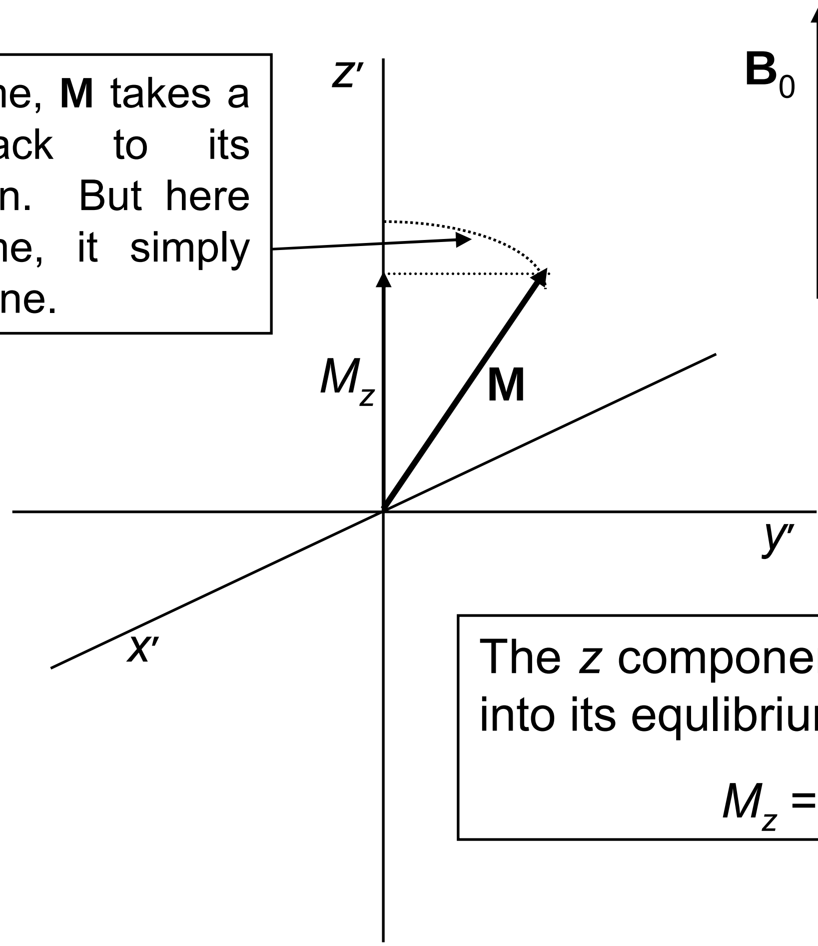
    $$M_z(t) = M_0(1 - e^{-t/T_1}) + M_z(0^+)e^{-t/T_1}$$

  - $M_z(0^+)$ is value after RF excitation pulse
  - M_0 is final (equilibrium) value

=M_0 cos\alpha
=0 for \pi/2 pulse



In the laboratory frame, **M** takes a spiralling path back to its equilibrium orientation. But here in the rotating frame, it simply rotates in the y'-z' plane.

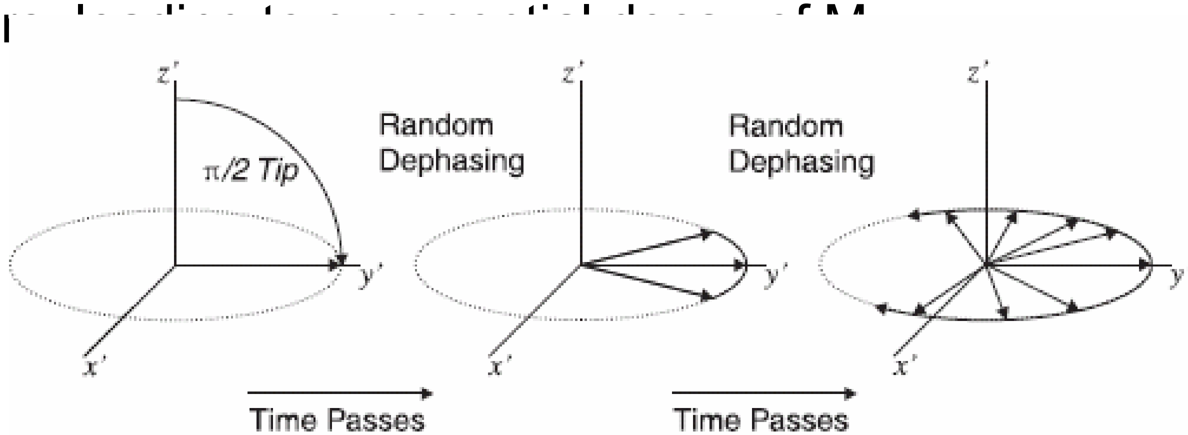


The z component of **M**, M_z, grows back into its equlibrium value, exponentially:

$$M_z = |\mathbf{M}|(1 - e^{-t/T_1})$$

Transverse Relaxation

- The strength of the magnetic field in the immediate environment of a ^1H nucleus is not homogeneous due to presence of other nucleus (and their interactions)
- Hence the Larmor frequencies of nearby nuclides are slightly different (some spins faster, some slower)
  - Spin-spin interactions
- This causes dephasing of the xy components of the magnetization vector



- Transverse relaxation decays

$$M_{xy}(t) = M_0 \sin \alpha e^{-j(\omega_0 t - \phi)} e^{-t/T_2}$$

- See animation at

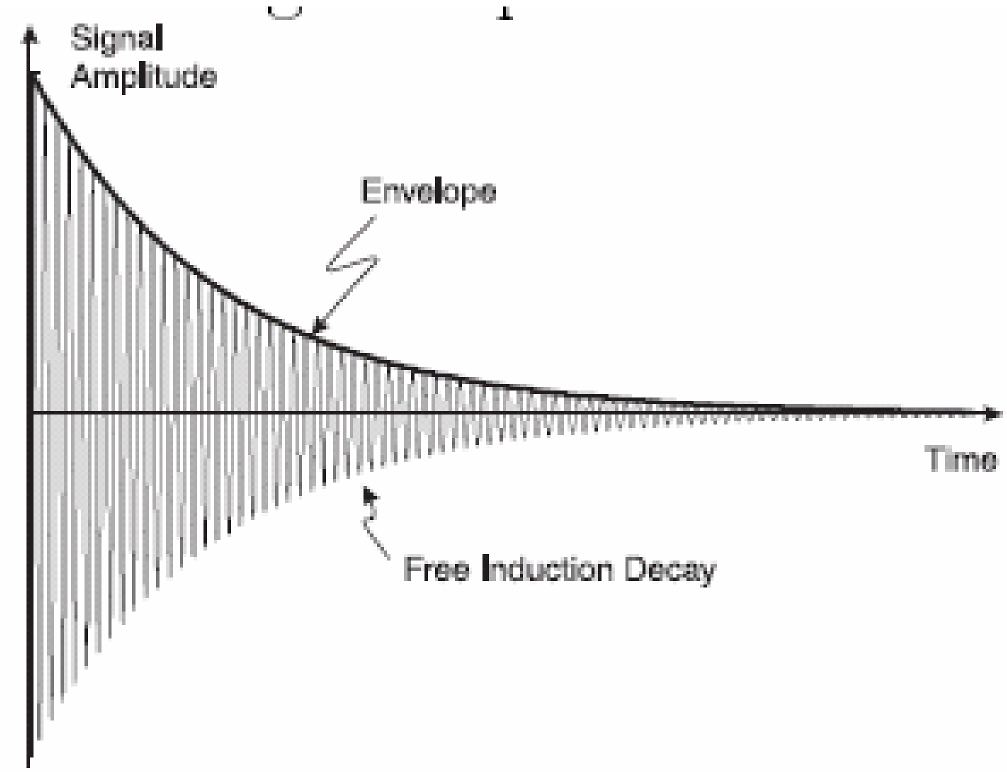
- http://www.cis.rit.edu/htbooks/mri/inside.htm
  - Under T2 processes

- Overall effect of both transverse and longitudinal relaxation:
- http://www.cis.rit.edu/htbooks/mri/chap-3/c12-2.htm

- T_2 is called transverse relaxation time, which is the time for M_{xy} to decrease by 1/e.
- Also called spin-spin relaxation time
- T2 is much smaller than T1
  - For tissue in body, T2: 25-250ms, T1: 250-2500 ms

Free Induction Decay

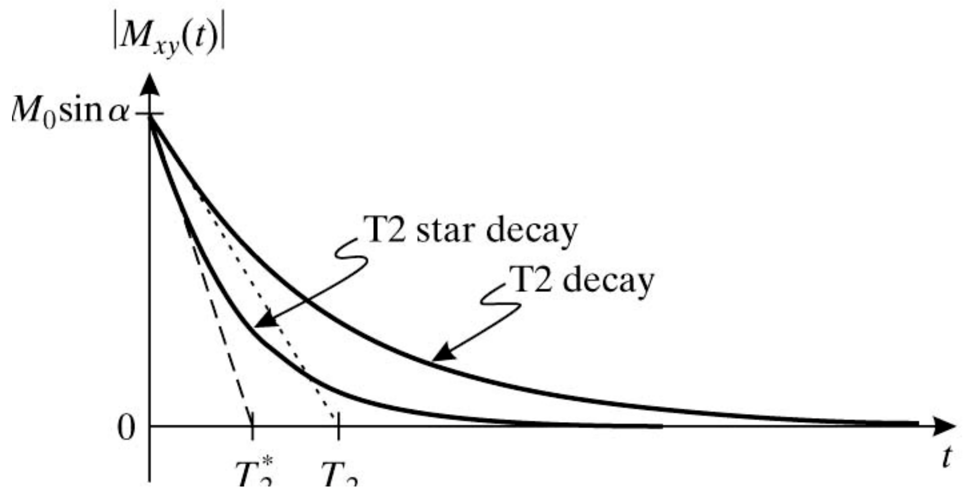
- The voltage signal (NMR signal) produced by decaying M_xy also decays



- This is called free induction decay (FID), and is the signal we measure in MRI

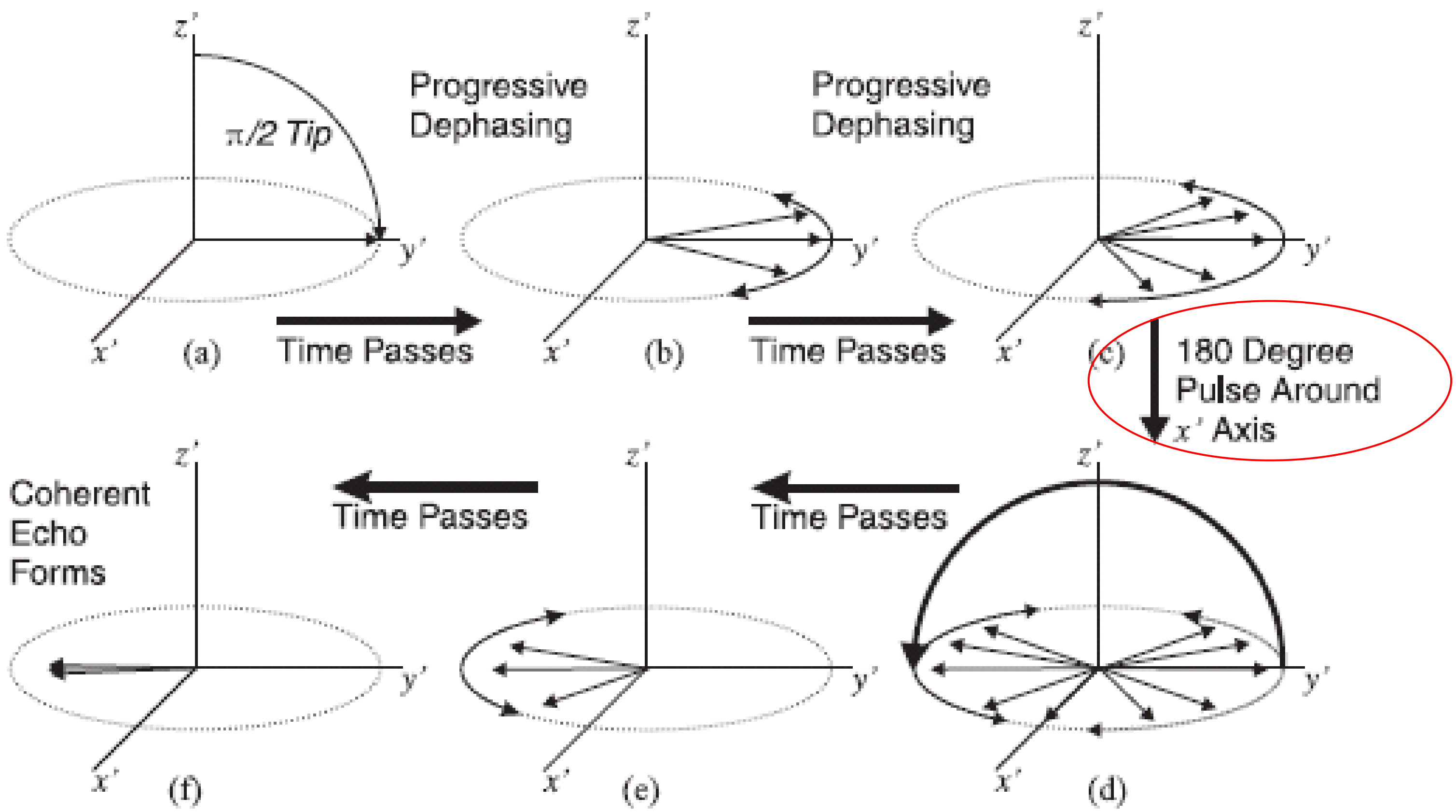
T2 Star Decay

- Received signal actually decays faster than T_2 (having a shorter relaxation time T_2^*)
- Caused by fixed spatial variation of the static field B_0 due to imperfection of the magnet
  - Accelerates the dephasing of magnetization vectors
  - Note that T2 is caused by spatial variation of the static field due to interactions of nearby spins
- The initial decay rate is governed by T_2^* , but the later decay by T_2.

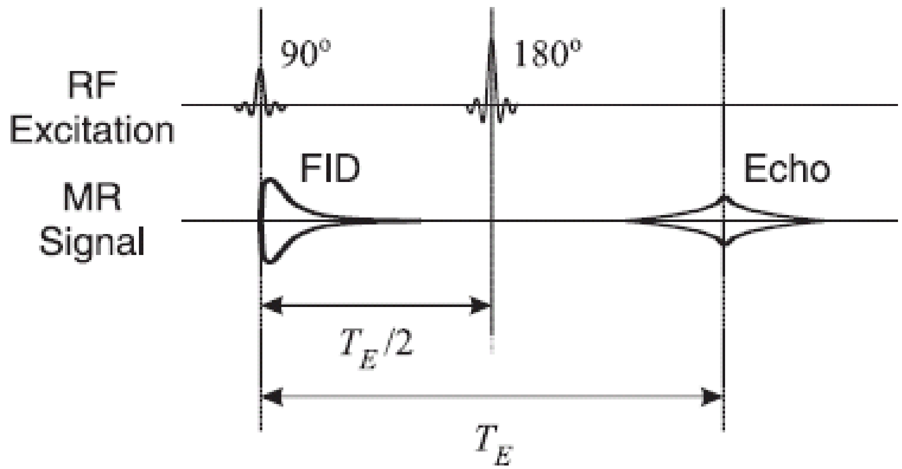


Formation of Spin Echo

- By applying a 180 degree pulse, the dephased spins can recover their coherence, and form an echo signal

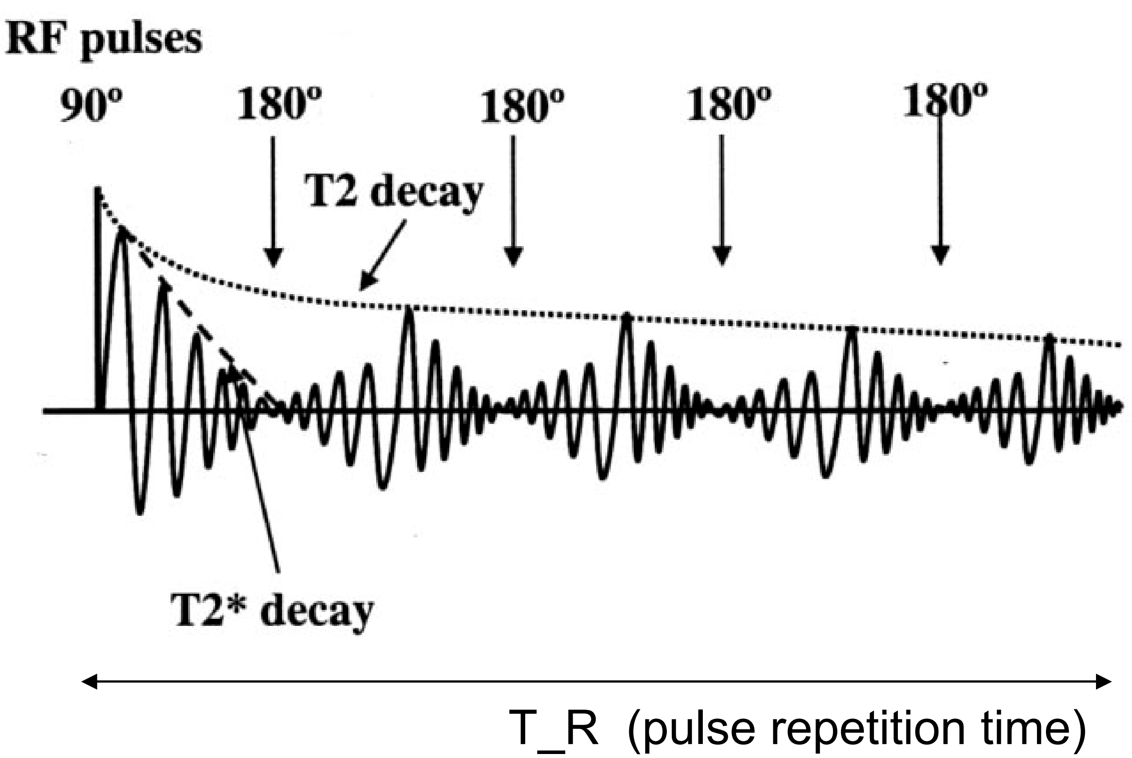


RF Pulse Sequence and Corresponding NMR Signal



Spin echo sequence

- Multiple π pulses create "Carr-Purcell-Meiboom-Gill (CPMG)" sequence
- Echo Magnitude Decays with time constant T2



Bloch Equations

Relaxation

- Equation(s) of "motion" for $\mathbf{M}(t)$

$$\frac{d\mathbf{M}(t)}{dt} = \gamma \mathbf{M}(t) \times \mathbf{B}(t) - \mathsf{R}\{\mathbf{M}(t) - \mathbf{M}_0\}$$

- Includes RF excitation

Forced precession

$$\mathbf{B}(t) = \mathbf{B}_0 + \mathbf{B}_1(t)\,,$$

- Includes relaxation

Alpha pulse
(RF excitation at
Larmor freq.)

Static field

$$\mathsf{R} = \begin{pmatrix} 1/T_2 & 0 & 0 \\ 0 & 1/T_2 & 0 \\ 0 & 0 & 1/T_1 \end{pmatrix}$$

- Solving the previous equation in x, y, z direction will yield the equations representing the transverse and longitudinal relaxations, shown previously

Source of MR Contrast

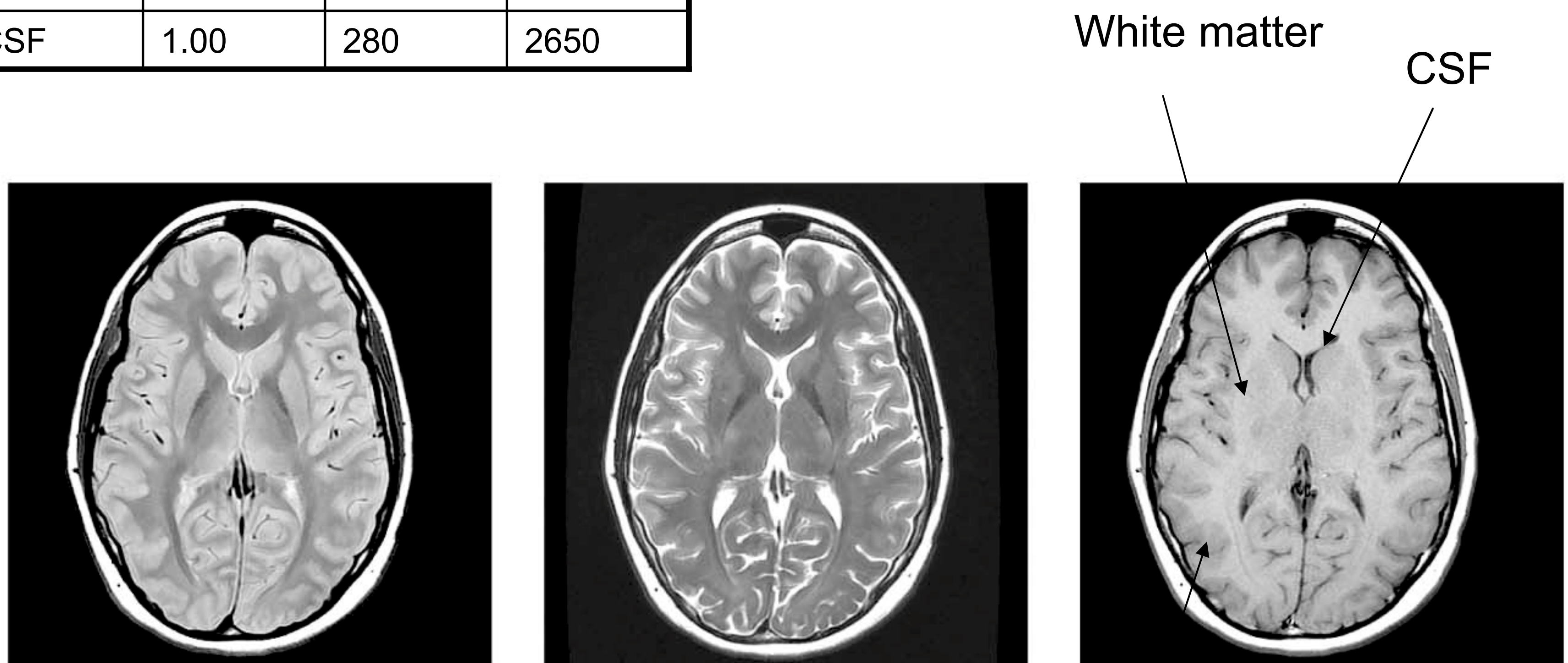
- Different tissues vary in T1, T2 and PD (proton density)
- The pulse sequence parameters can be designed so that the captured signal magnitude is mainly influenced by one of these parameters
- Pulse sequence parameters
  - Tip angle α
  - Echo time T_E
  - Pulse repetition time T_R

Typical Brain Tissue Parameters

- Table 12.2 in [Prince]

	P_D	T_2 (ms)	T_1 (ms)
White matter	0.61	67	510
Gray matter	0.69	77	760
CSF	1.00	280	2650

	P_D	T_2 (ms)	T_1 (ms)
White matter	0.61	67	510
Gray matter	0.69	77	760
CSF	1.00	280	2650



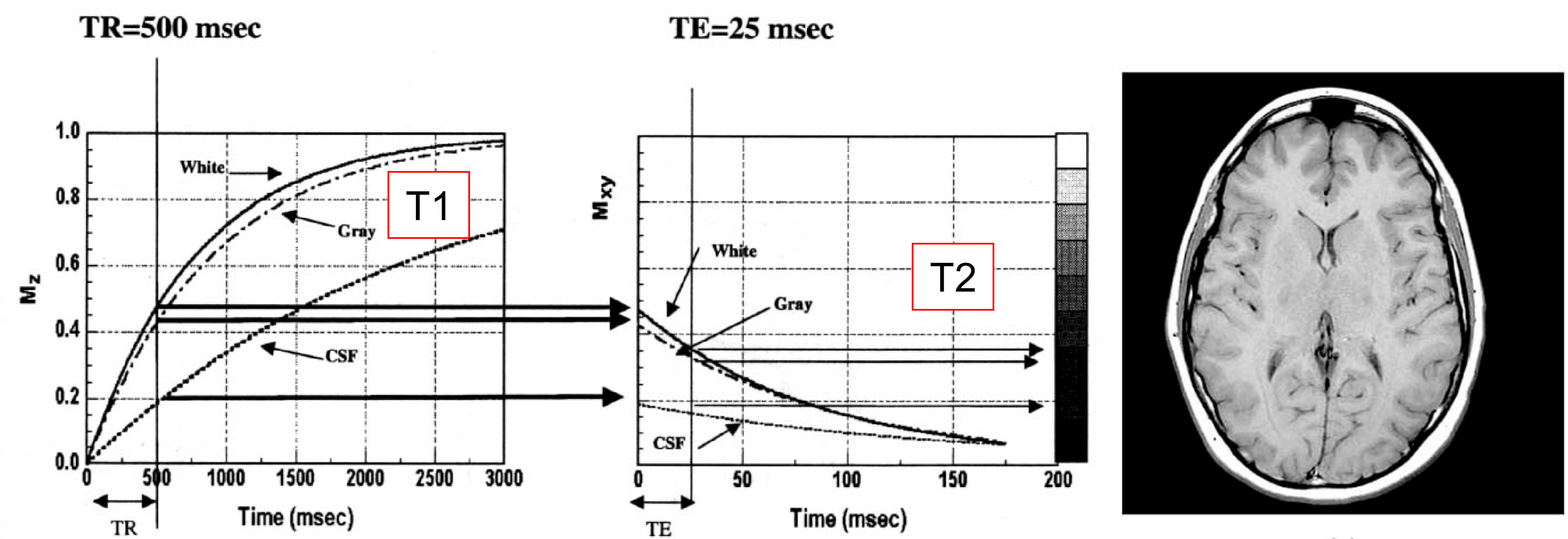
(a) PD weighted

(b) T2- weighted

(c) T1- weighted

T1-weighting

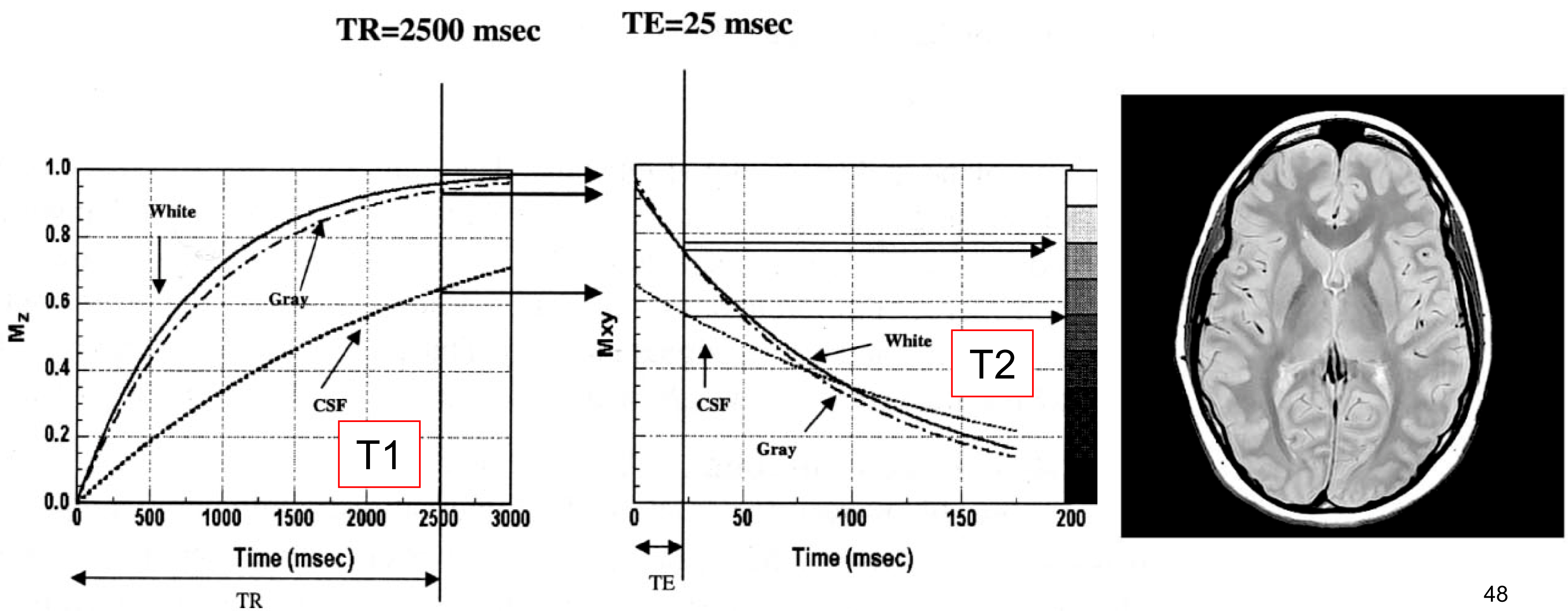
- Short TR:
  - Maximizes T1 contrast due to different degrees of saturation
  - If TR too long, tissues with different T1 all return equilibrium already
- Short TE:
  - Minimizes T2 influence, maximizes signal



Spin density weighting

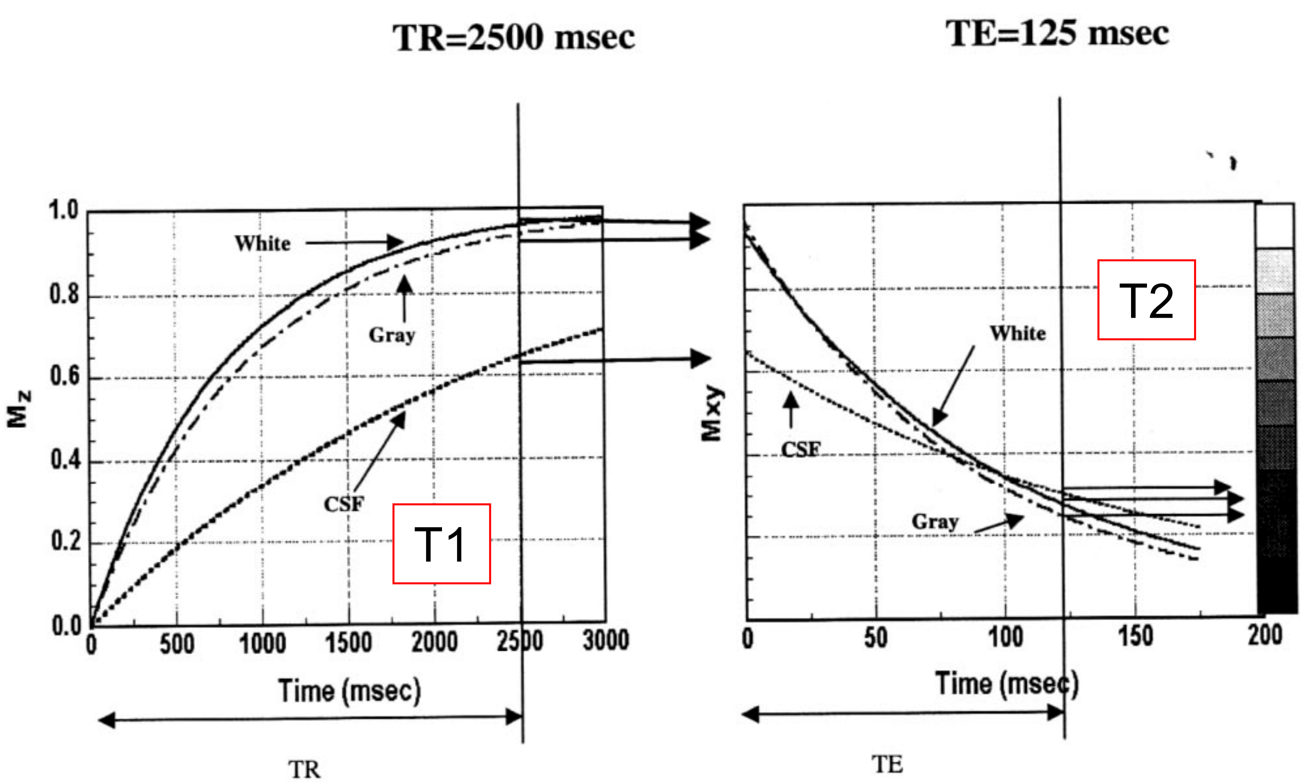
$$M_0 = \frac{B_0 \gamma^2 \hbar^2}{4kT} P_D$$

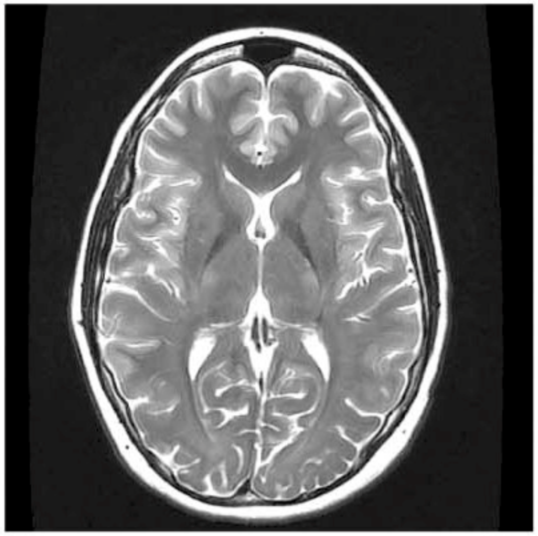
- Signal at equilibrium proportional to PD
- Long TR:
  - Minimizes effects of different degrees of saturation (T1 contrast)
  - Maximizes signal (all return to equilibrium)
- Short TE:
  - Minimizes T2 contrast
  - Maximizes signal



T2 weighting

- Long TR:
  - Minimizes influence of different T1
- Long TE:
  - Maximizes T2 contrast
  - Relatively poor SNR





Summary: Process Involved in MRI

- Put patient in a static field B_0 in z-direction
- (step 1) Wait until the bulk magnitization reaches an equilibrium (align with B_0)
- Apply a rotating field (alpha pulse) in the xy plane to bring M to an initial angle \alpha with B_0. Typically \alpha=\pi/2
- M(t) precesses around B_0 (z direction) at Larmor freq. with angle \alpha
- The component in z increases in time (longitudinal relaxation) with time constant T1
- The component in x-y plane reduces in time (transverse relaxation) with time constant T2
- Apply \pi pulse to induce echo to bring transverse components in phase to increase signal strength
- Measure the transverse component at different times (NMR signal), to deduce T1 or T2
- Go back to step 1
- By using different excitation pulse sequences (differing in TE, TR, \alpha), the signal amplitude can reflect mainly the proton density, T1 or T2 at a given voxel

Summary

- What is nuclear spin? What type of nucleus can have spin?
- What is the bulk magnetization vector in the absence of external magnetic field?
- What is the bulk magnetization vector in the presence of an external static magnetic field?
- What is precession? Under what condition will precession occur?
  - Static field, initial angle
  - Larmor frequency = \gamma B_0
- What is the function of the rotating field (\alpha pulse)
  - Tilt the magnetization vector to an angle
- What happens after?
  - Longitudinal and transversal relaxation
  - Gradually return to the equilibrium state
- Tissues differ in T1, T2 and PD
  - Using different TR, TE, so that the signal magnitude is mainly influenced by one of the parameters, T1, T2 or PD

Reference

- Prince and Links, Medical Imaging Signals and Systems, Chap. 12
- A. Webb, Introduction to Biomedical Imaging, Chap. 4
- **The Basics of MRI**, A web book by Joseph P. Horn (containing useful animation):
- http://www.cis.rit.edu/htbooks/mri/inside.htm

Homework

- Reading:
  - Prince and Links, Medical Imaging Signals and Systems, Chap. 12
  - Note down all the corrections for Ch. 10,11 on your copy of the textbook based on the provided errata (see Course website or book website for update).
- Problems (Due 12/4):
  - P12.1
  - P12.2
  - P12.4
  - P12.5
  - P12.7
  - P12.10
  - P12.11
  - P12.12