EL582/BE620 --- Medical Imaging - I

Introduction, Review of Signals & Systems, Image Quality Metrics

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Based on Prince and Links, Medical Imaging Signals and Systems and Lecture Notes by Prince. Figures are from the book.

Lecture Outline

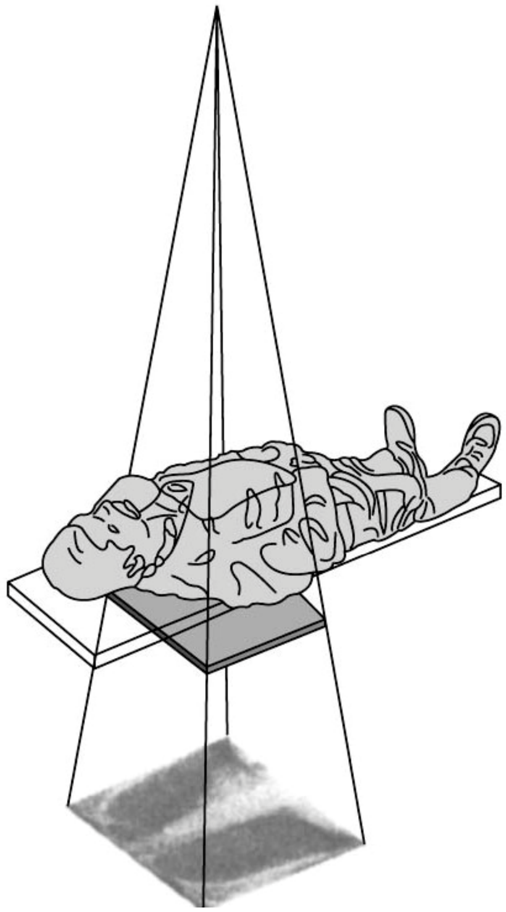
- Overview of different imaging systems
- Review of basic signals and systems
- Image quality assessment

What is Medical Imaging?

- Using an instrument to see the inside of a human body
  - Non-invasive
  - Some with exposure to small amount of radiation (X-ray, CT and nuclear medicine)
  - Some w/o (MRI and ultrasound)
- The properties imaged vary depend on the imaging modality
  - X-ray (projection or CT): attenuation coefficient to X-ray
  - Nuclear medicine (PET, SPECT): distribution of introduced radio source
  - Ultrasound: sound reflectivity
  - MRI: hydrogen proton density, spin relaxation

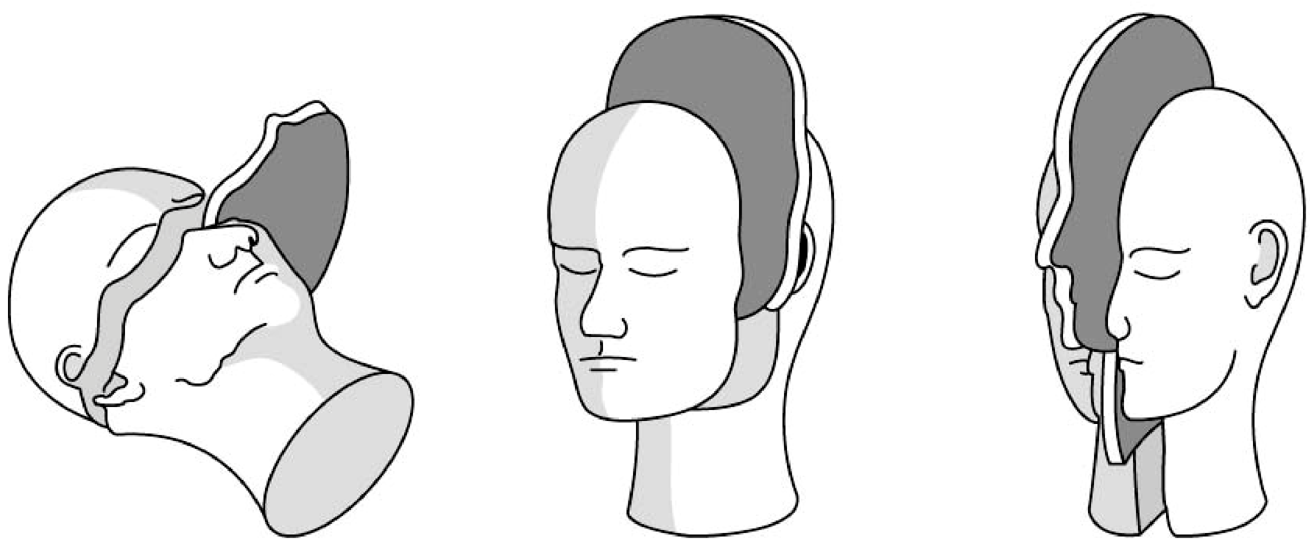
Projection vs. Tomography

- Projection:
  - A single image is created for a 3D body, which is a “shadow” of the body in a particular direction (integration through the body)



Projection vs. Tomography

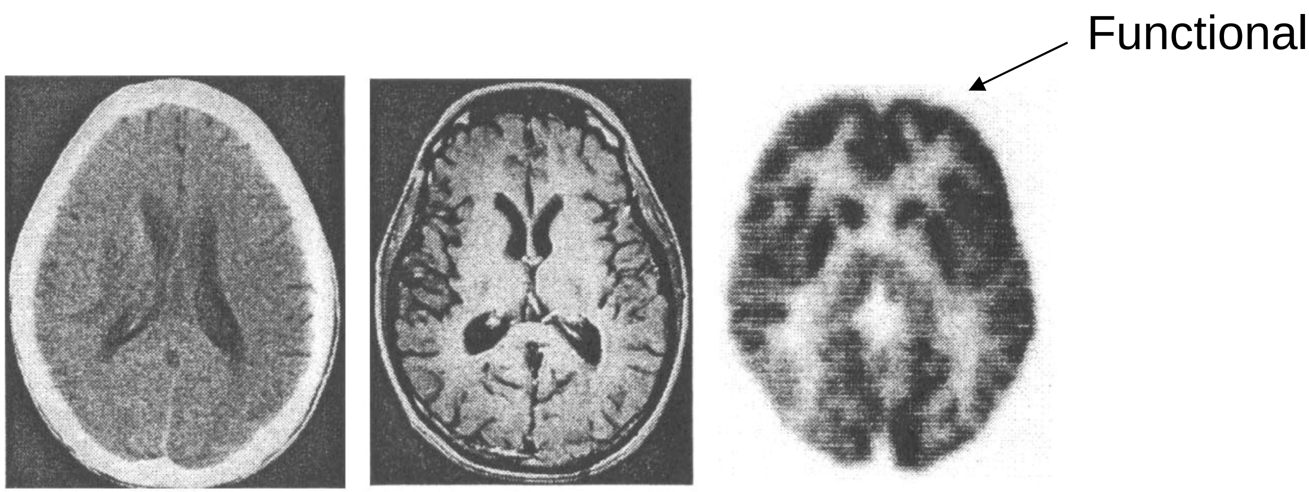
- Tomography
  - A series of images are generated, one from each slice of a 3D object in a particular direction (axial, coronal, sagital)
  - To form image of each slice, projections along different directions are first obtained, images are then reconstructed from projections (back-projection, Radon transform)



(a) (b) (c)

Anatomical vs. Functional Imaging

- Some modalities are very good at depicting anatomical structure (bone, different tissue types, boundary between different organs)
  - X-ray, X-ray CT
  - MRI
- Some modalities do not depict anatomical structures well, but reflect the functional status (blood flow, oxygenation, etc.)
  - Ultrasound
  - PET, functional MRI
- Boundaries between the two classes are blurring as the imaging resolution continues to improve



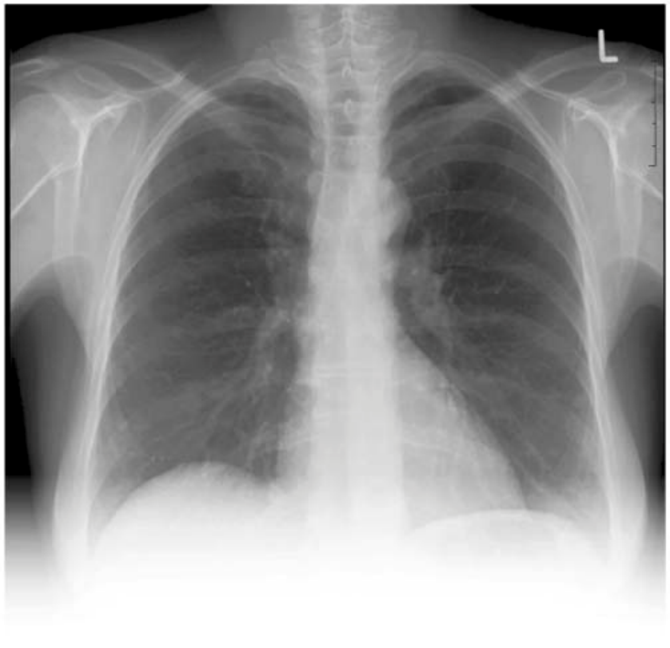
(a) CT (b) MRI (c) PET

Common Imaging Modalities

- Projection radiography (X-ray)
- Computed Tomography (CT scan or CAT Scan)
- Nuclear Medicine (SPECT, PET)
- Ultrasound imaging
- MRI
- Optical imaging

Projection Radiography

(a)



(b)

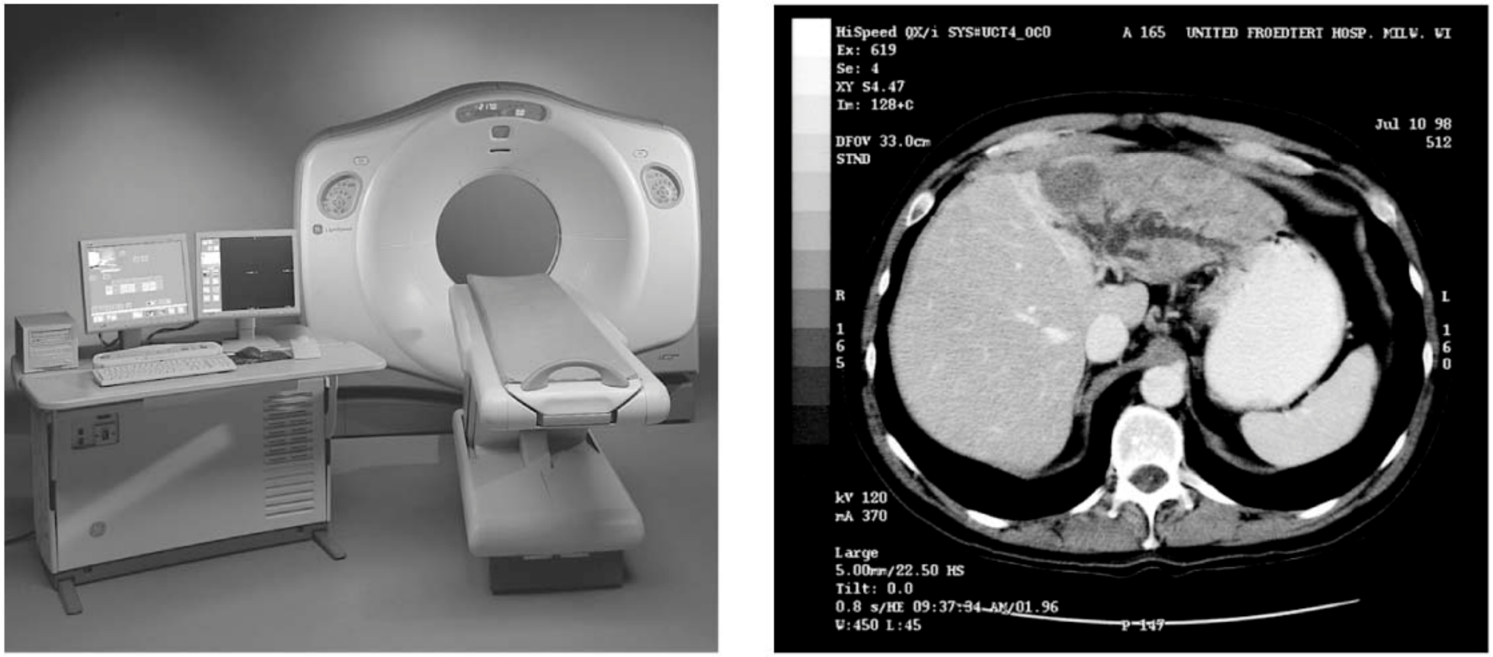
Figure 1.1

Medical Imaging Signals and Systems, by Jerry L. Prince and Jonathan Links.
ISBN 0-13-065353-5.

• Year discovered:	1895 (Röntgen, NP 1905)
• Form of radiation:	X-rays = electromagnetic radiation (photons)
• Energy / wavelength of radiation:	0.1 – 100 keV / 10 – 0.01 nm (ionizing)
• Imaging principle:	X-rays penetrate tissue and create "shadowgram" of differences in density.
• Imaging volume:	Whole body
• Resolution:	Very high (sub-mm)
• Applications:	Mammography, lung diseases, orthopedics, dentistry, cardiovascular, GI

From Graber, Lecture Note for Biomedical Imaging, SUNY

Computed Tomography

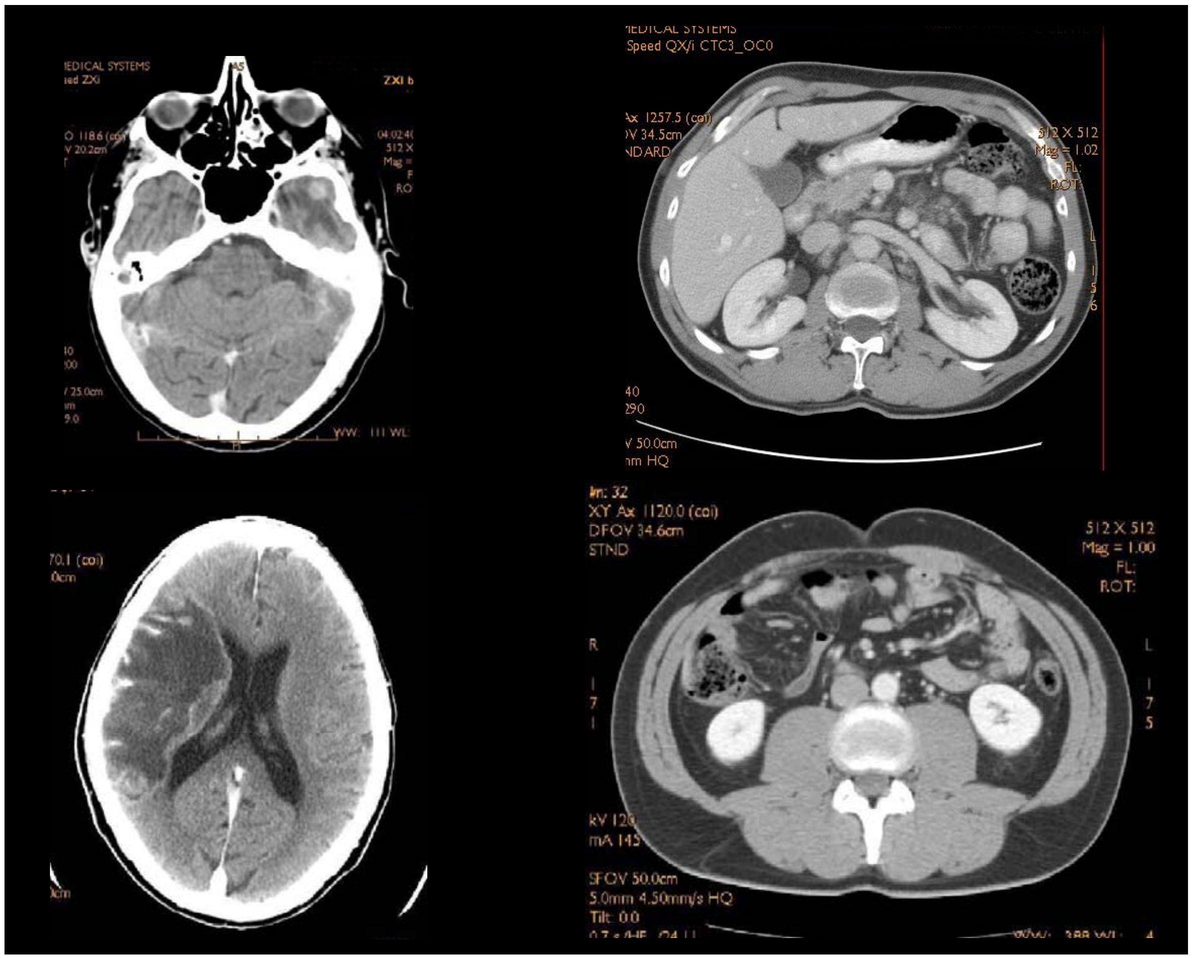


(a) (b)

Figure 1.2

Medical Imaging Signals and Systems, by Jerry L. Prince and Jonathan Links.

ISBN 0-13-065353-5.


Speed QX/i CTC3_OC0
512 X 512
Mag = 1.02
XY Ax 1120.0 (coi)
DFOV 34.6cm
STND
512 X 512
Mag = 1.00
FL:
ROT:
SFOV 50.0cm
5.0mm 4.50mm/s HQ
Tilt: 0.0

• Year discovered:	1972 (Hounsfield, NP 1979)
• Form of radiation:	X-rays
• Energy / wavelength of radiation:	10 – 100 keV / 0.1 – 0.01 nm (ionizing)
• Imaging principle:	X-ray images are taken under many angles from which tomographic ("sliced") views are computed
• Imaging volume:	Whole body
• Resolution:	High (mm)
• Applications:	Soft tissue imaging (brain, cardiovascular, GI)

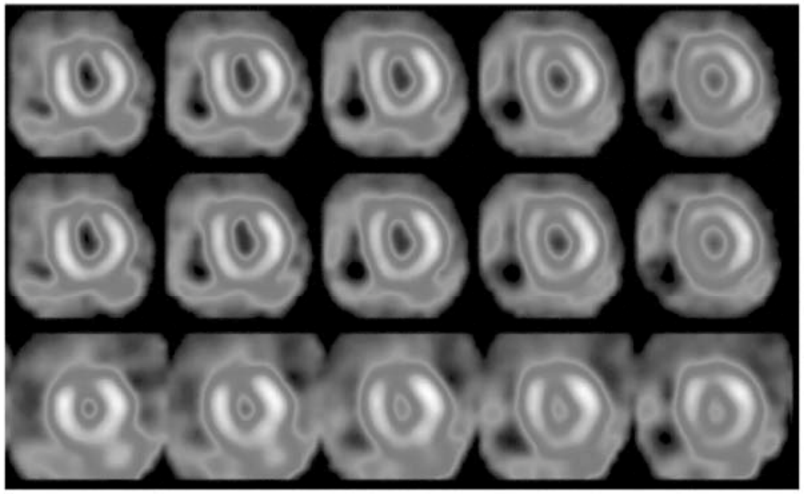
From Graber, Lecture Note for Biomedical Imaging, SUNY

Nuclear Medicine

- Images can only be made when appropriate radioactive substances (called radiotracer) are introduced into the body that emit gamma rays.
- A nuclear medicine image reflects the local concentration of a radiotracer within the body
- Three types
  – Conventional radionuclide imaging or scintigraphy
  – Single photon emission computed tomography (SPECT)
  – Positron emission tomography (PET)

SPECT

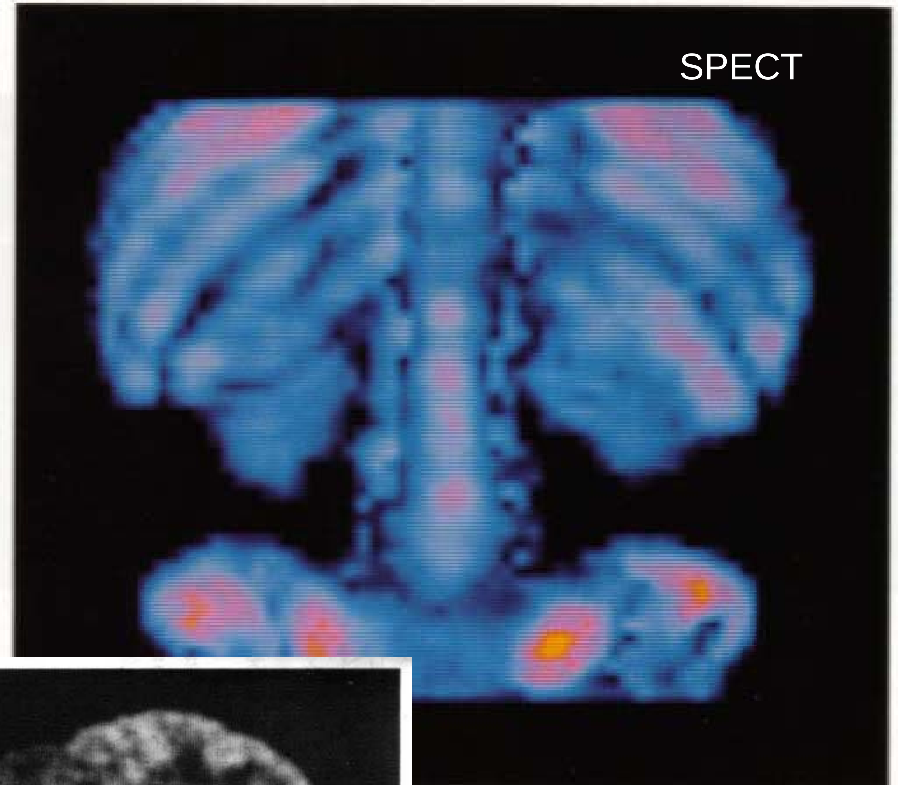
(a)

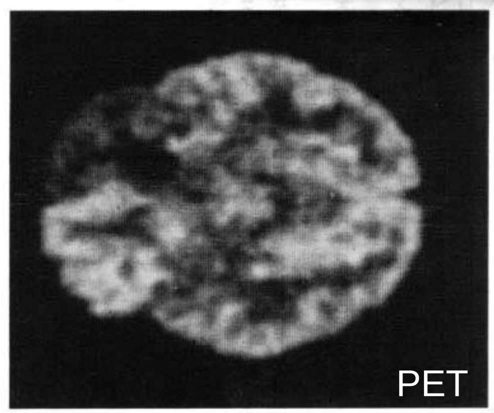


(b)

Figure 1.3

Medical Imaging Signals and Systems, by Jerry L. Prince and Jonathan Links.
ISBN 0-13-065353-5.





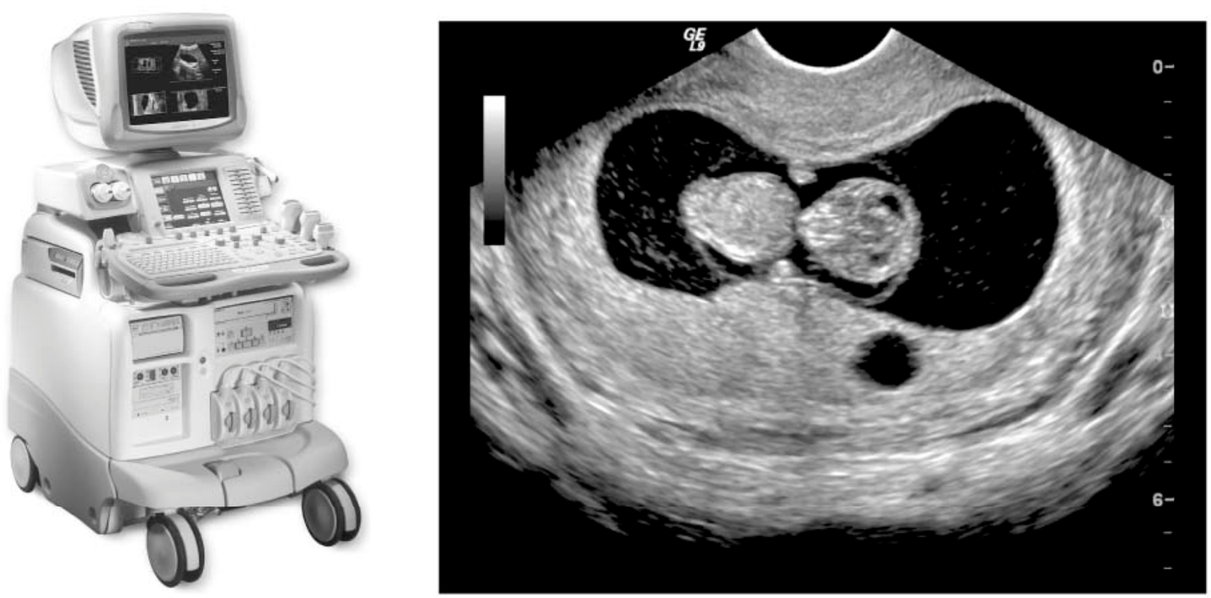
- What do you see?

• Year discovered:	1953 (PET), 1963 (SPECT)
• Form of radiation:	Gamma rays
• Energy / wavelength of radiation:	> 100 keV / < 0.01 nm (ionizing)
• Imaging principle:	Accumulation or "washout" of radioactive isotopes in the body are imaged with x-ray cameras.
• Imaging volume:	Whole body
• Resolution:	Medium – Low (mm - cm)
• Applications:	Functional imaging (cancer detection, metabolic processes, myocardial infarction)

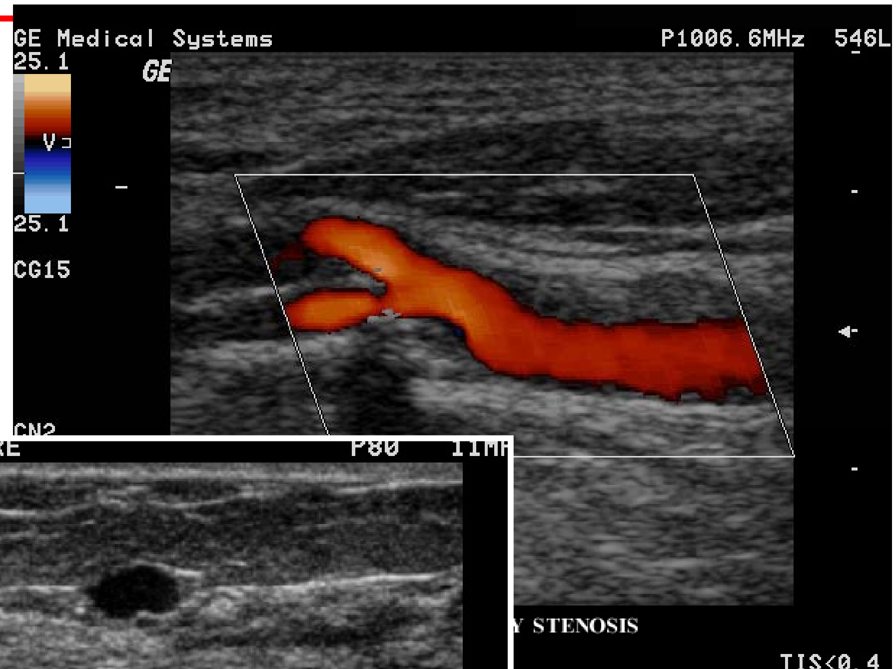
From Graber, Lecture Note for Biomedical Imaging, SUNY

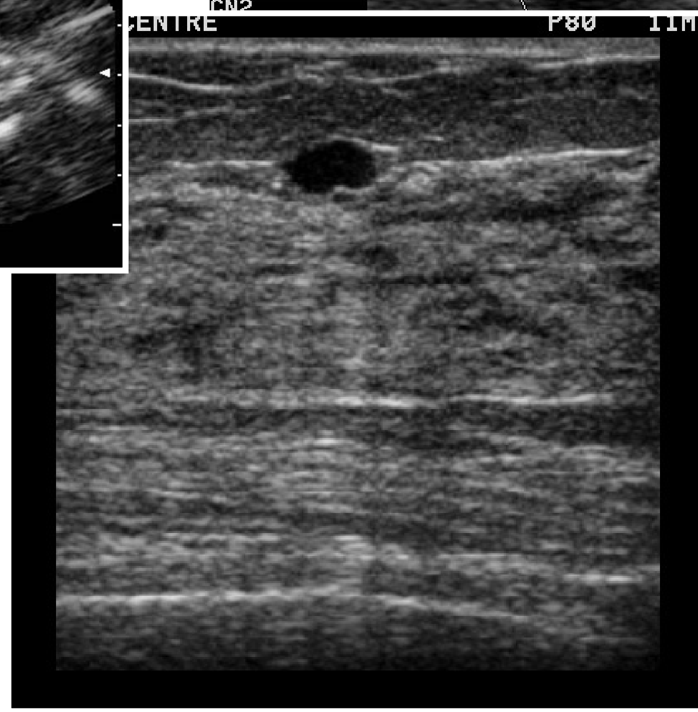
Ultrasound Imaging

- High frequency sound are emitted into the imaged body, time and strength of returned sound pulses are measured
- Comparatively inexpensive and completely non-invasive
- Image quality is relatively poor (but is improving!)



(a) (b)



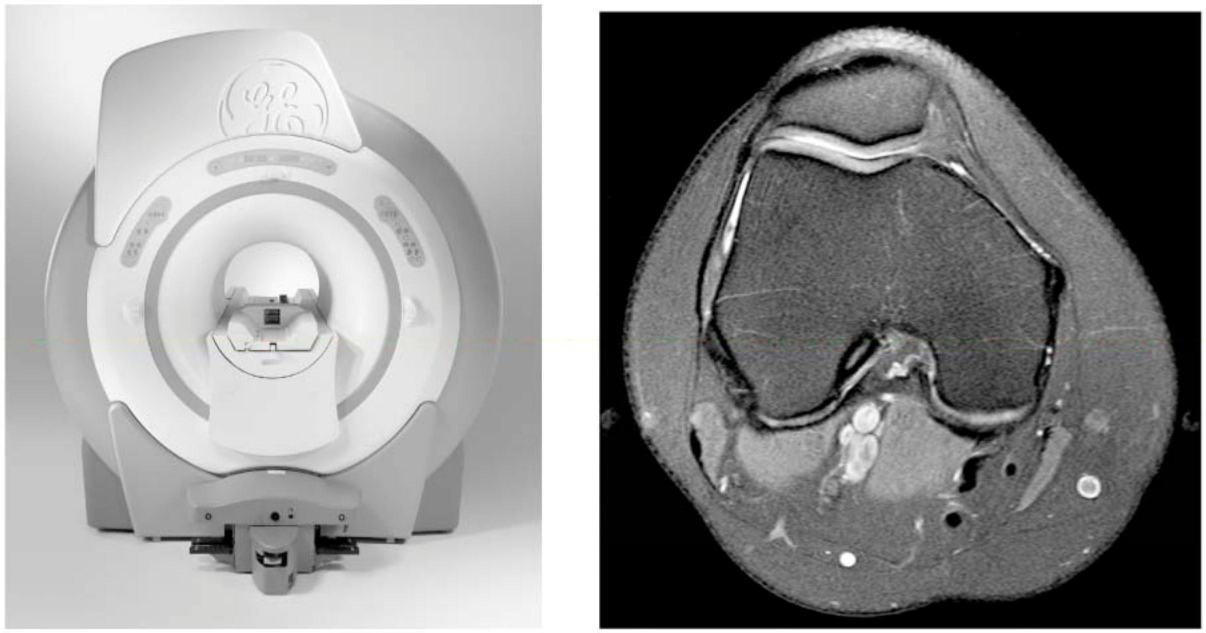


- What do you see?

- Year discovered: 1952 (clinical: 1962)
- Form of radiation: Sound waves (non-ionizing) ***NOT*** EM radiation!
- Frequency / wavelength of radiation: 1 – 10 MHz / 1 – 0.1 mm
- Imaging principle: Echoes from discontinuities in tissue density/speed of sound are registered.
- Imaging volume: < 20 cm
- Resolution: High (mm)
- Applications: Soft tissue, blood flow (Doppler)

From Graber, Lecture Note for Biomedical Imaging, SUNY

Magnetic Resonance Imaging



(a) (b)

Figure 1.5

Medical Imaging Signals and Systems, by Jerry L. Prince and Jonathan Links.
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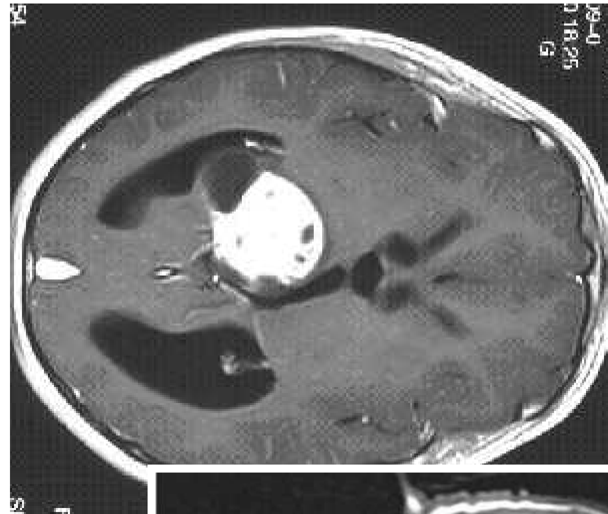
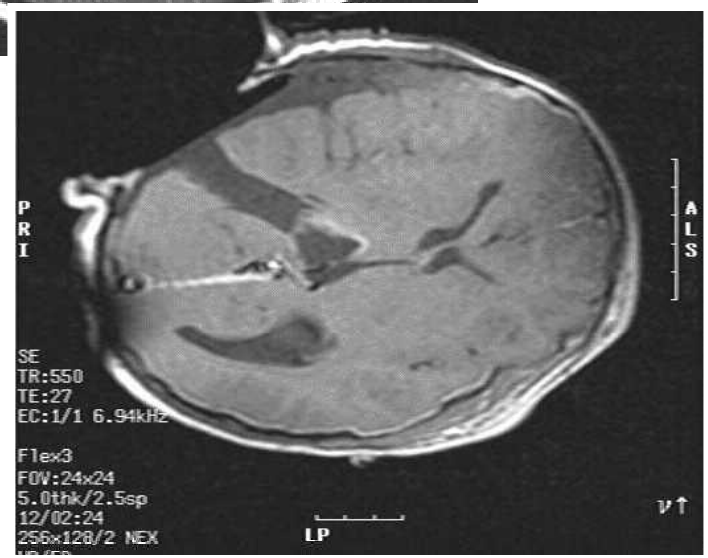


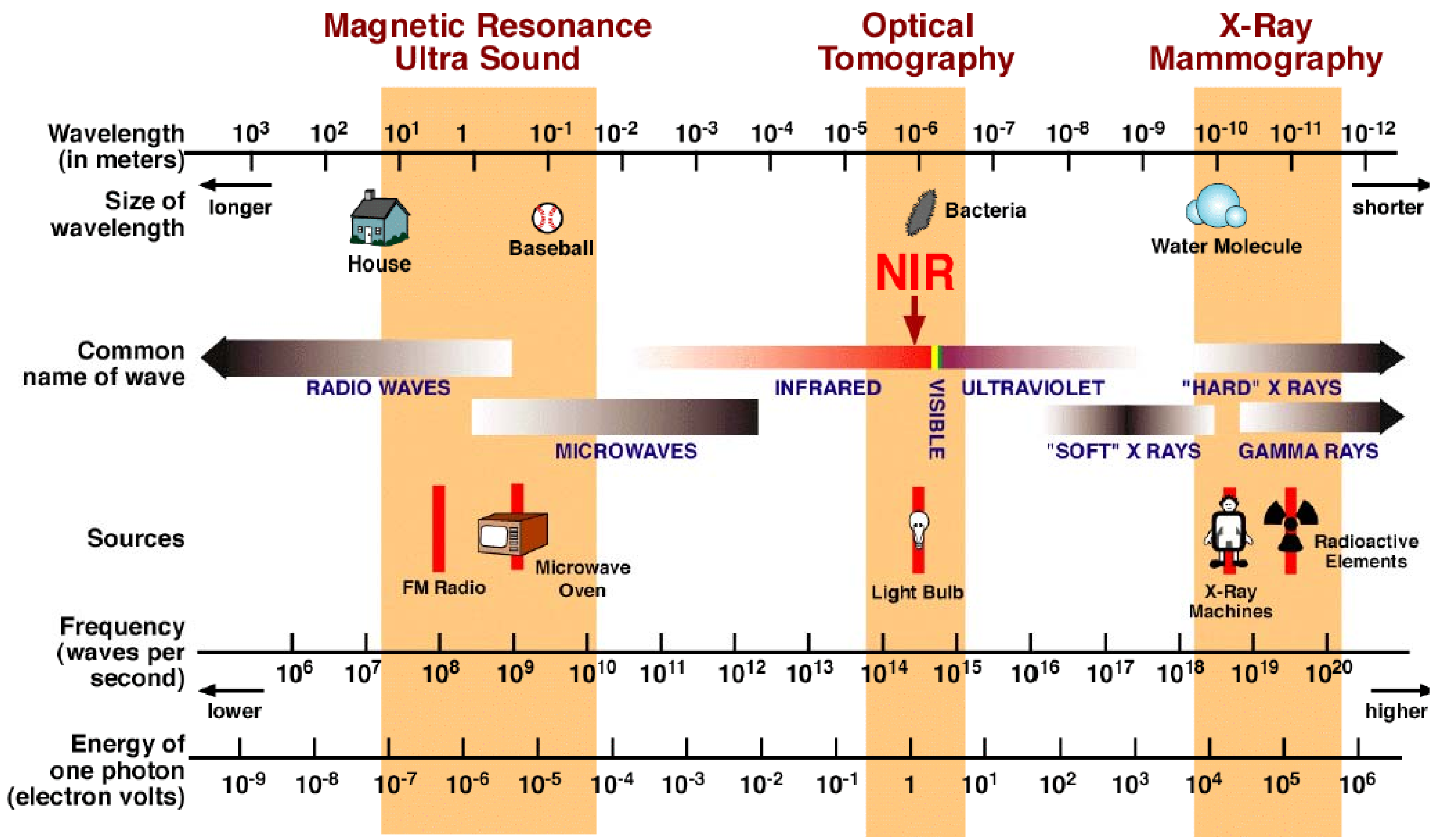


- What do you see?

- Year discovered: 1945 ([NMR] Bloch, NP 1952)
  1973 (Lauterbur, NP 2003)
  1977 (Mansfield, NP 2003)
  1971 (Damadian, SUNY DMS)
- Form of radiation: Radio frequency (RF) (non-ionizing)
- Energy / wavelength of radiation: 10 – 100 MHz / 30 – 3 m (~10-7 eV)
- Imaging principle: Proton spin flips are induced, and the RF emitted by their response (echo) is detected.
- Imaging volume: Whole body
- Resolution: High (mm)
- Applications: Soft tissue, functional imaging

From Graber, Lecture Note for Biomedical Imaging, SUNY

Waves Used by Different Modalities



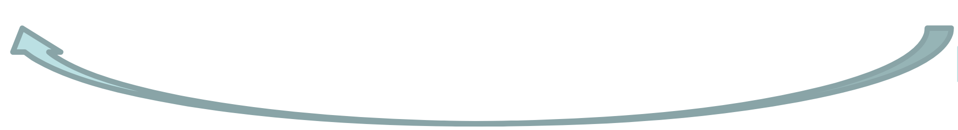
Course breakdown

- Biomedical Imaging is a multi-disciplinary field involving
  - Physics (matter, energy, radiation, etc.)
  - Math (linear algebra, calculus, statistics)
  - Biology/Physiology
  - Engineering (implementation)
  - Signal processing and Image processing (modeling imaging system as linear systems, image reconstruction and enhancement and analysis)
- Course breakdown:
  - 1/3 physics
  - 1/3 instrumentation
  - 1/3 signal processing
- Understand the imaging system from a “signals and systems” point of view

Signals and Systems View Point

- The object being imaged is an input signal
  - Typically a 3D signal
- The imaging system is a transformation of the input signal to an output signal
- The data measured is an output signal
  - A 2D signal (an image, e.g. an X-ray) or a series of 2D signals (e.g. measured projections from a CT scan), or 4D data (a series of 3D volume in time)
- Image reconstruction
  - An inverse process: from the measured output signal -> desired images of the object (a series of 2D slices)

$$\text{input signal} \rightarrow \text{system or process} \rightarrow \text{output signal}$$



Example: Projection X-Ray

- Input signal: $\mu(x; y)$ is the linear attenuation coefficient for x-rays of a body component along a line
- Imaging Process: integration over *x* variable:

$$g(y) = \int \mu(x, y)dx$$

- Output signal: $g(y)$

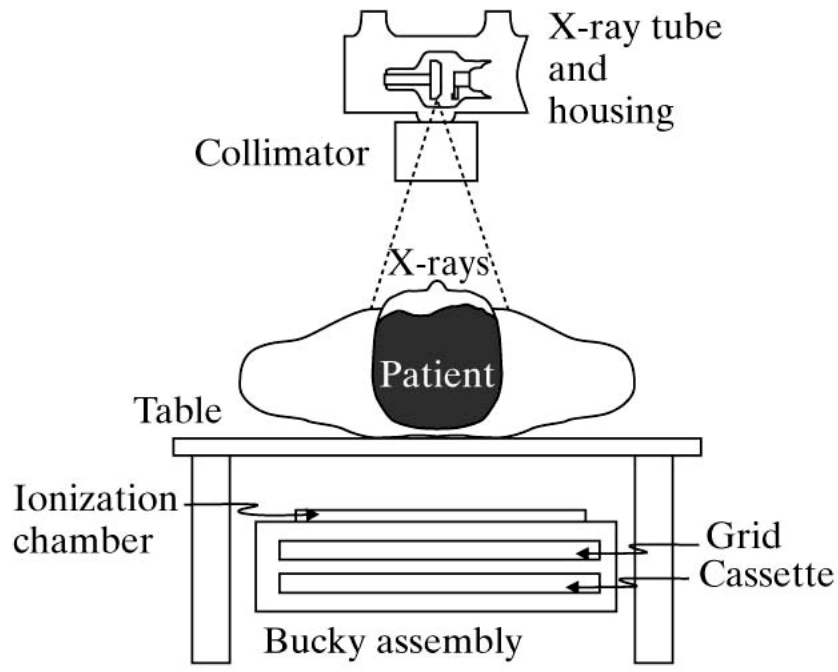
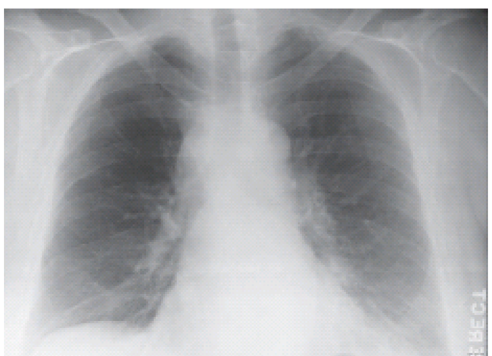


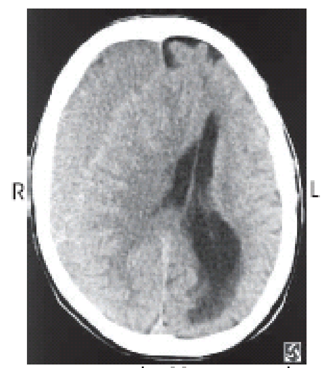
Figure 5.2

Example Signals

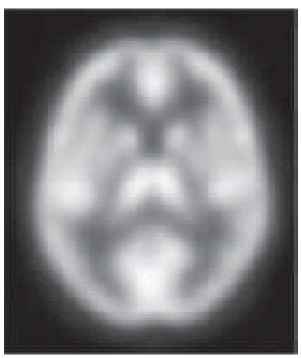
- $\mu(x, y, z)$, linear attenuation coefficient in x-rays
- $h(x, y, z)$, CT numbers in computed tomography
- $A(x, y, z)$, radioactivity in nuclear medicine



Chest X-ray: Anthrax



computed tomography
subdural hemotoma
pushing midline R to L



Positron Emission Tomography

Transformation of Signals

- Components of a transformation:
  - Input: f
  - System: $\mathcal{H}[\cdot]$
  - Output: g
- The impulse response or point spread function due to an impulse at (ξ, η) is

$$h(x, y; \xi, \eta) = \mathcal{H}[\delta(x - \xi, y - \eta)]$$

Linear Systems

- A linear system satisfies:

$$\mathcal{H}[w_1 f_1 + w_2 f_2] = w_1 \mathcal{H}[f_1] + w_2 \mathcal{H}[f_2]$$

  for all signals f_1 and f_2 and weights w_1 and w_2.

- A linear system satisfies the superposition integral

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y;\xi,\eta) f(\xi,\eta) d\xi d\eta$$

- We model most medical imaging systems as linear.

Shift-Invariant Systems

- A system is <u>shift-invariant</u> is

$$g(x - x_0, y - y_0) = \mathcal{H}[f(x - x_0, y - y_0)]$$

for every (x_0, y_0) and $f(\cdot, \cdot)$.

- A <u>linear shift-invariant (LSI) system</u> yields

$$h(x, y; \xi, \eta) \to h(x - \xi, y - \eta)$$

[Watch out for abuse of notation]

Linear and Shift-Invariant System

- An LSI system satisfies the <u>convolution</u> <u>integral</u>

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x-\xi, y-\eta) f(\xi,\eta) d\xi d\eta$$

which is abbreviated as

$$g(x,y) = h(x,y) * f(x,y)$$

- We model most medical imaging systems as LSI

h(x,y) is called the Impulse Response or Point Spread Function (PSF) of a LSI system, which indicates the output signal corresponding to a single impulse or point at origin.

Fourier Transform: 1D signals

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx$$
$$f(x) = \int_{-\infty}^{\infty} F(u) e^{+j2\pi ux} du$$

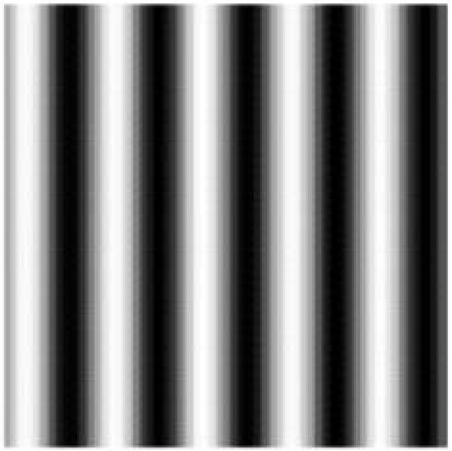
- x has units of length (mm, cm, m) or time (for 1D signal in time)
- u has units of inverse length (cycles/unit-length), which is referred to as spatial frequency, or inverse time (cycles/sec), which is referred to as temporal frequency
- $|F(u)|$ indicts the amount of signal component in f(x) with frequency u

Fourier Transform: 2D signals

$$\begin{aligned} F(u,v) &= \mathcal{F}\{f\} \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(ux+vy)}dxdy \\ f(x,y) &= \mathcal{F}^{-1}\{F\} \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)e^{+j2\pi(ux+vy)}dudv \end{aligned}$$

- 2D signal's frequency can be measured in different directions (horizontal, vertical, 45^, etc.), but only two orthogonal directions are necessary
- u and v indicate cycles/horizontal-unit and cycles/vertical-unit
- $|F(u,v)|$ indicates the amount of signal component with frequency u,v.

Spatial Frequency



(a)

(b)

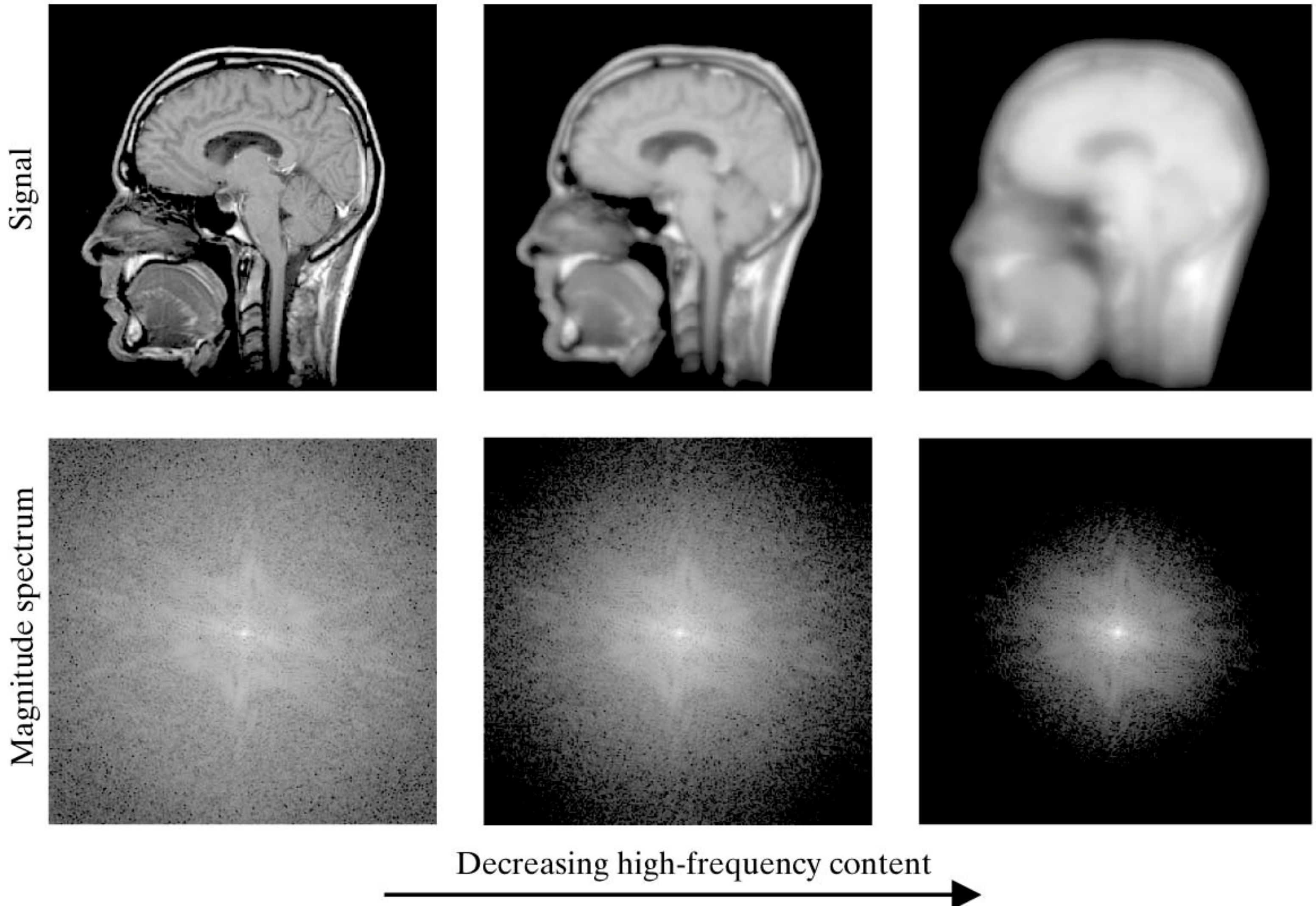
Figure 2.1 Two-dimensional sinusoidal signals: (a) $(f_x, f_y) = (5, 0)$; (b) $(f_x, f_y) = (5, 10)$. The horizontal and vertical units are the width and height of the image, respectively. Therefore, $f_x = 5$ means that there are five cycles along each row.

Spatial Frequency

- Spatial frequency measures how fast the image intensity changes in the image plane
- Spatial frequency can be completely characterized by the variation frequencies in two orthogonal directions (e.g horizontal and vertical)
  - f_x: cycles/horizontal unit distance
  - f_y : cycles/vertical unit distance
- It can also be specified by magnitude and angle of change

$$f_m = \sqrt{f_x^2 + f_y^2}, \theta = \arctan(f_y / f_x)$$

FT of Typical Images



Convolution Property and Frequency Response

- Convolution in space domain = Product in frequency domain

$$\mathcal{F}\{f_1 * f_2\} = F_1 F_2$$

- For LSI system

Impulse response

$$g(x,y) = h(x,y) * f(x,y)$$
$$G(u,v) = H(u,v)\,F(u,v)$$

Frequency response

H(u,v) indicates how a complex exponential signal with frequency *u,v* will be modified by the system in its magnitude and phase

$$e^{-j2\pi(ux+vy)} \rightarrow H(u,v)e^{-j2\pi(ux+vy)} = |H(u,v)|e^{-j\left(2\pi(ux+vy)+\angle H(u,v)\right)}$$

Extra Readings

- See Chap 2 of textbook for more extensive reviews of signals and systems
- For more exposition, see
  - Oppenheim and Wilsky, Signals and Systems
- We will review a particular subject more when needed

Image Quality

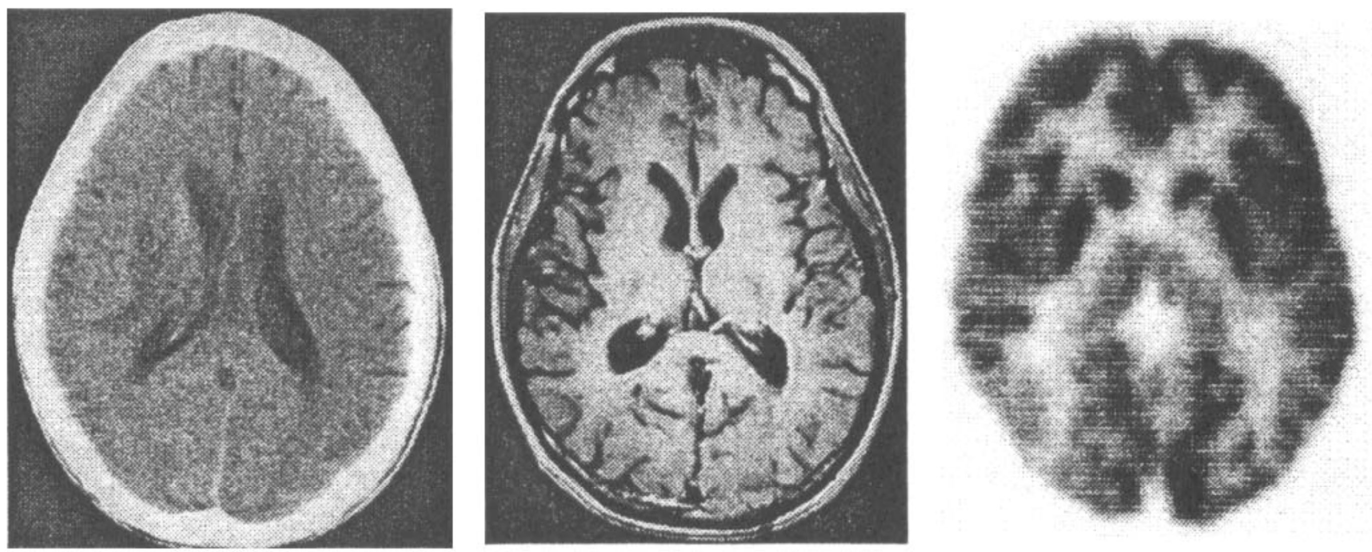
- Introduction
- Contrast
- Resolution
- Noise
- Artifacts
- Distortions

Measures of Quality

- Physics-oriented issues:
  - contrast, resolution
  - noise, artifacts, distortion
  - Quantitative accuracy
- Task-oriented issues:
  - sensitivity, specificity
  - diagnostic accuracy

What is Contrast?

- Difference between image characteristics (e.g. gray scale intensity) of an object of interest and surrounding objects or background
- Which image below has higher contrast?



(a) (b) (c)

Figure I.4

Medical Imaging Signals and Systems, by Jerry L. Prince and Jonathan Links.

Contrast

- General definition
  - f_{max}, f_{min}: maximum and minimum values of the signal in an image

$$\underline{\text{Contrast}} = \underline{\text{modulation}} =$$

$$m_f = \frac{\text{amplitude}}{\text{average}} = \frac{f_{\max} - f_{\min}}{f_{\max} + f_{\min}}$$

- For a sinusoidal signal

$$f(x, y) = A + B \sin(2\pi u_0 x) \qquad m_f = \frac{B}{A}$$

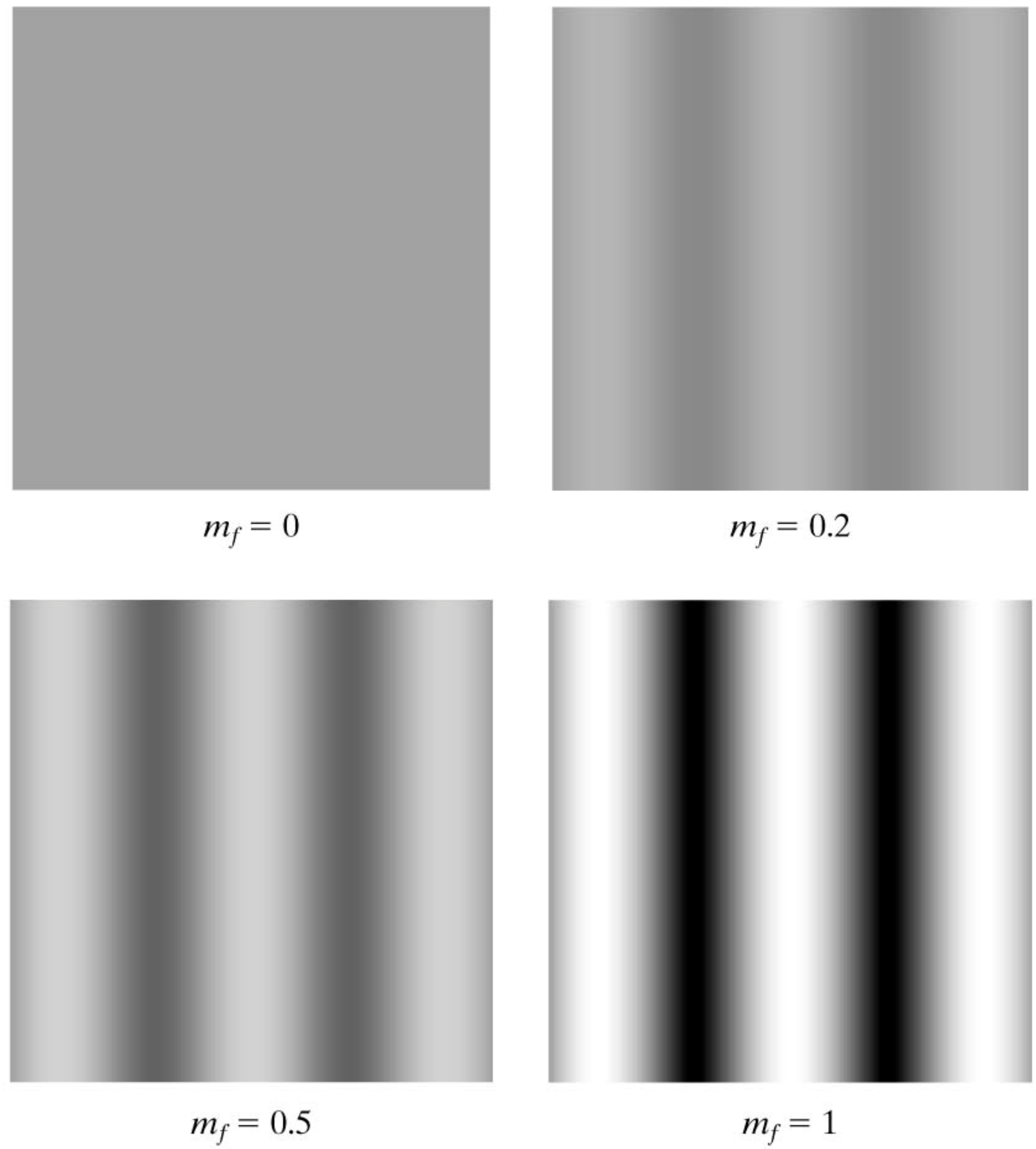


Figure 3.1

Medical Imaging Signals and Systems, by Jerry L. Prince and Jonathan Links.
ISBN 0-13-065353-5.

Modulation Transfer Function

- The actual signal being imaged can be decomposed into many sinusoidal signals with different frequencies

$$f(x,y) = A + \sum_k B_k \sin(2\pi u_k x + 2\pi v_k y); \quad m_{f,k} = \frac{B_k}{A}$$

- Suppose the imaging system can be considered as a LSI system with frequency response $H(u,v)$
- Imaged signal is

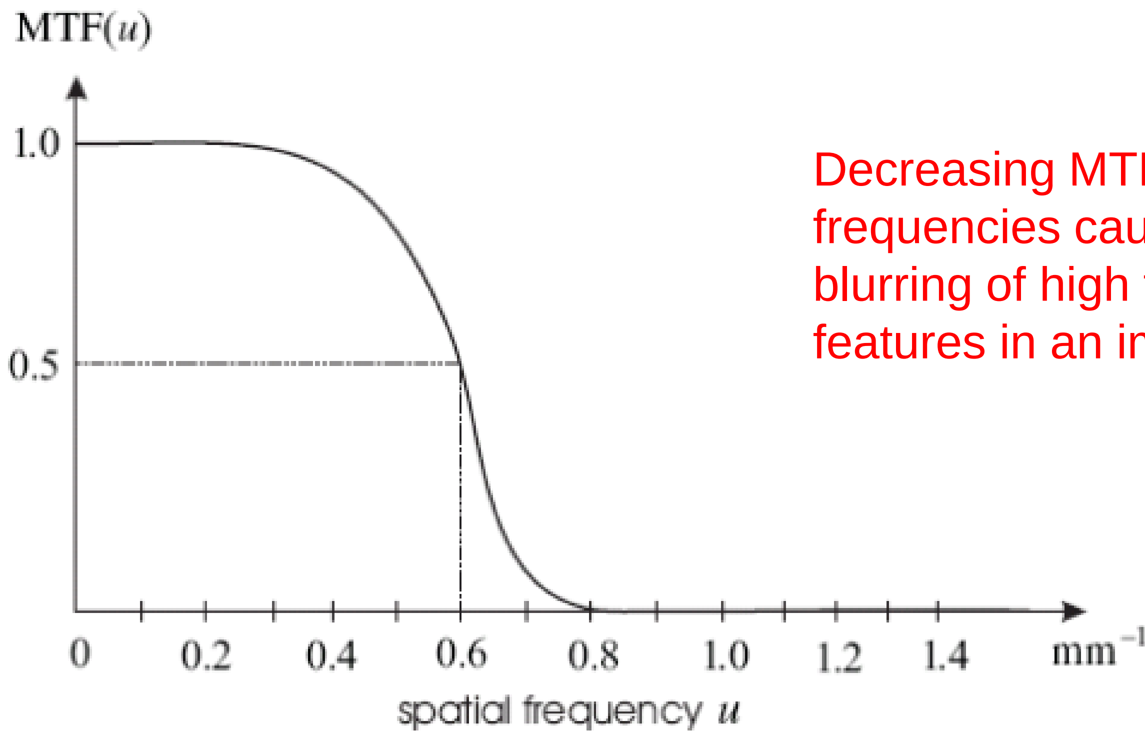
$$g(x,y) = H(0,0)A + \sum_k H(u_k, v_k) B_k \sin(2\pi u_k x + 2\pi v_k y); \quad m_{g,k} = \frac{|H(u_k, v_k)| B_k}{H(0,0)A}$$

- The MTF refers to the ratio of the contrast (or modulation) of the imaged signal to the contrast of the original signal at different frequencies

$$MTF(u,v) = \frac{m_{g,u,v}}{m_{f,u,v}} = \frac{|H(u,v)|}{H(0,0)}$$

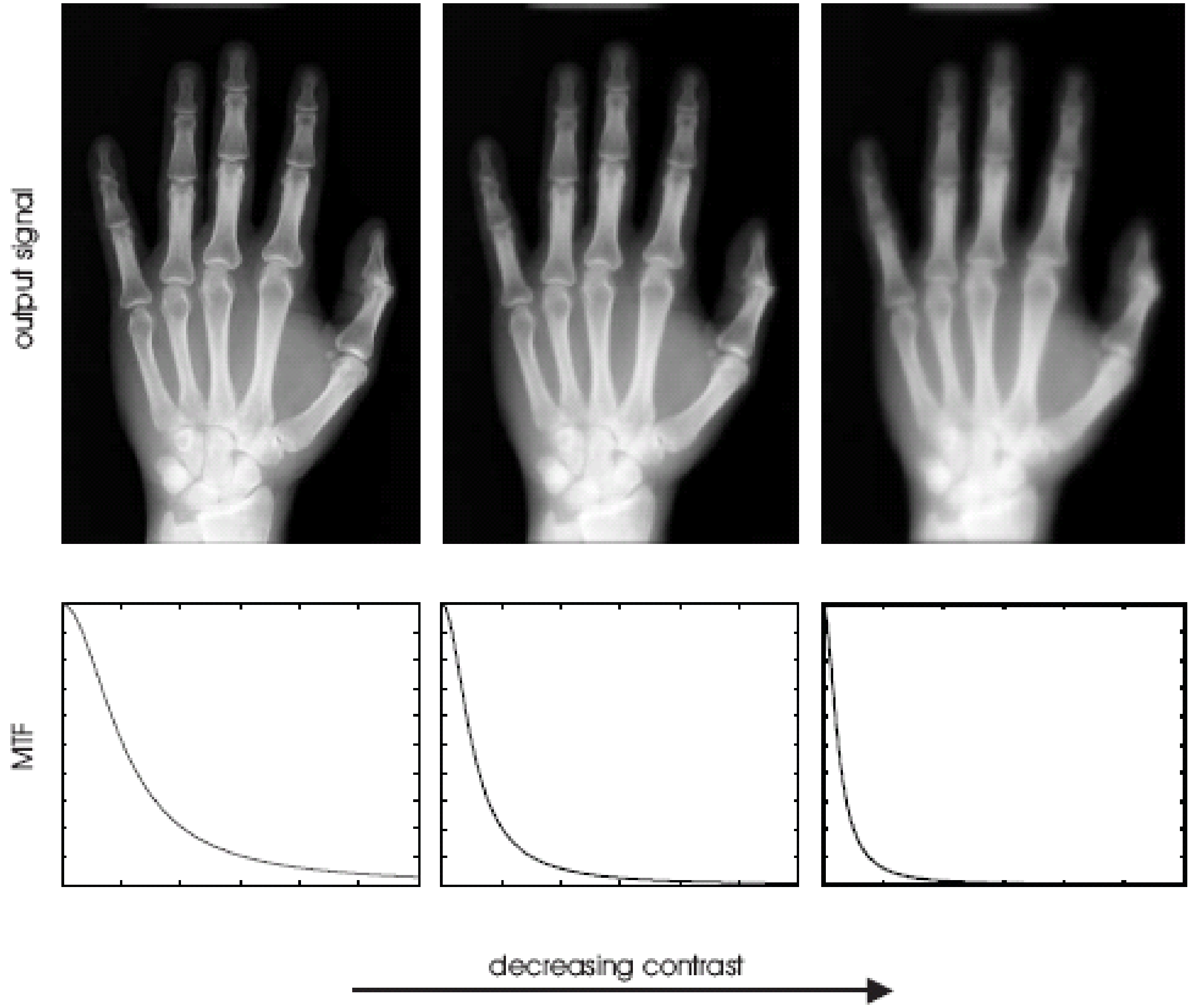
More on MTF

- MTF characterizes how the contrast (or modulation) of a signal component at a particular frequency changes after imaging
- MTF = magnitude of the frequency response of the imaging system (normalized by H(0,0))
- Typically $0 \le MTF(u,v) \le MTF(0,0) = 1$



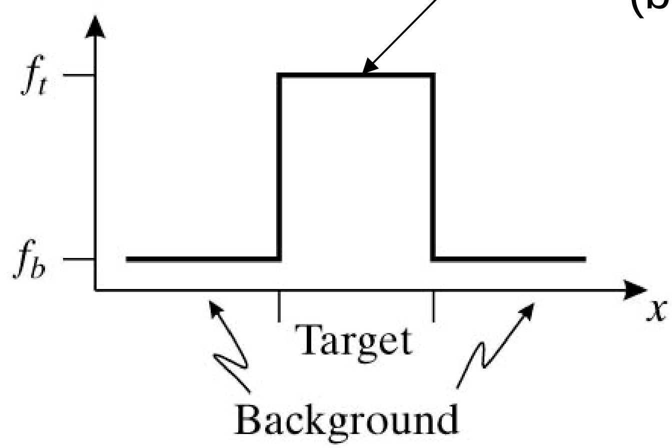
Decreasing MTF at higher frequencies causes the blurring of high frequency features in an image

Impact of the MTF on the Image Contrast



Local Contrast

A target is an object of interest in an image
Eg. a tumor (target) in a liver (background)



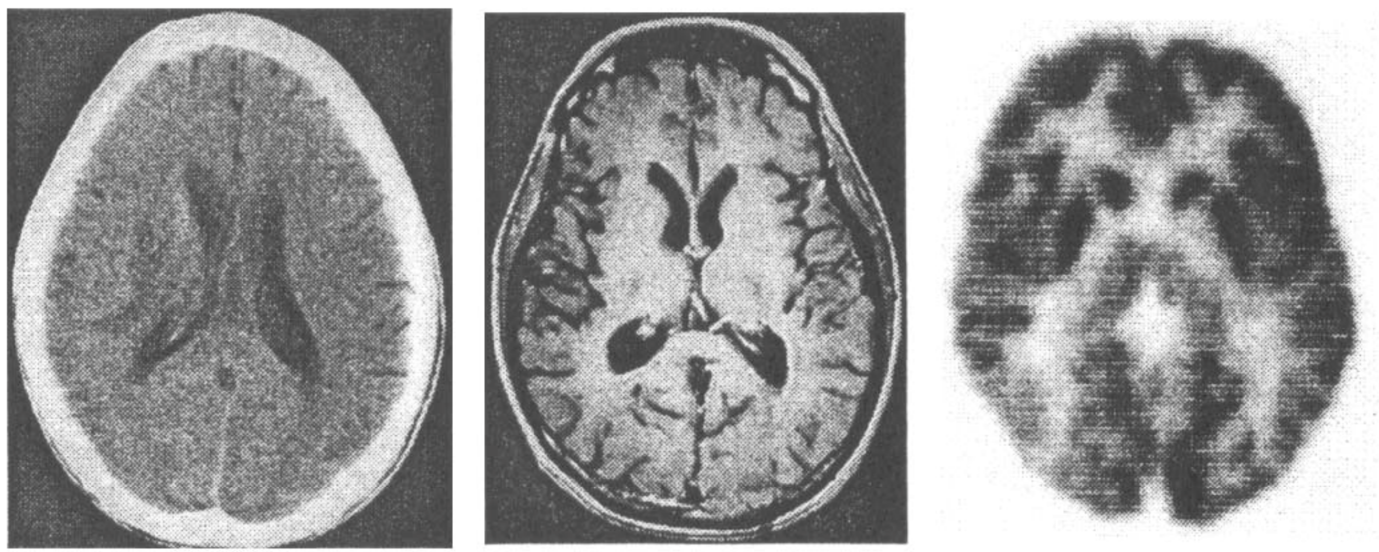
$$C = \frac{f_t - f_b}{f_b}$$

Figure 3.5

Medical Imaging Signals and Systems, by Jerry L. Prince and Jonathan Links.
ISBN 0-13-065353-5.

What is Resolution?

- The ability of a system to depict spatial details.
- Which image below has higher resolution?



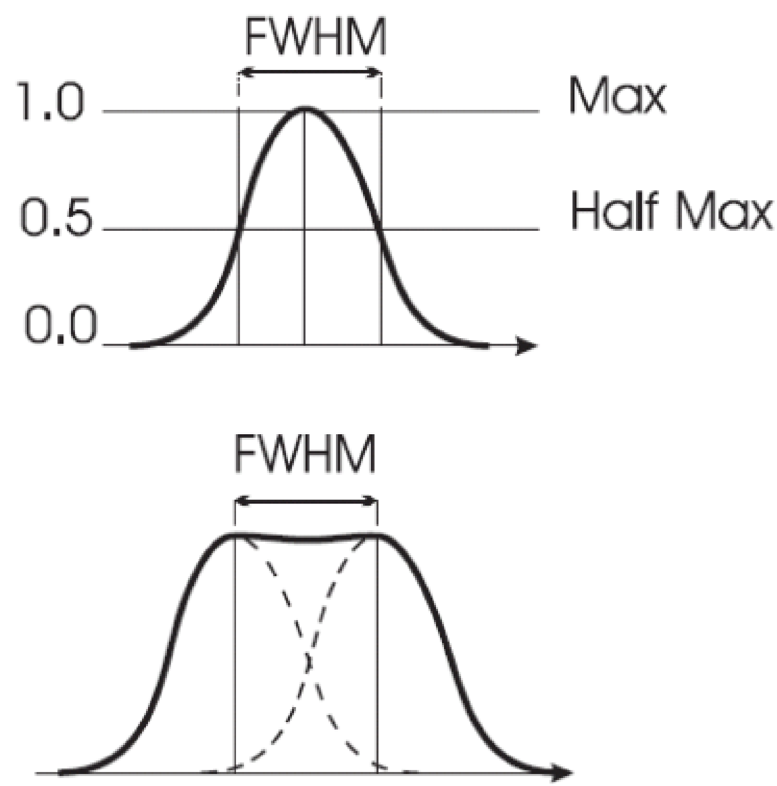
(a) (b) (c)

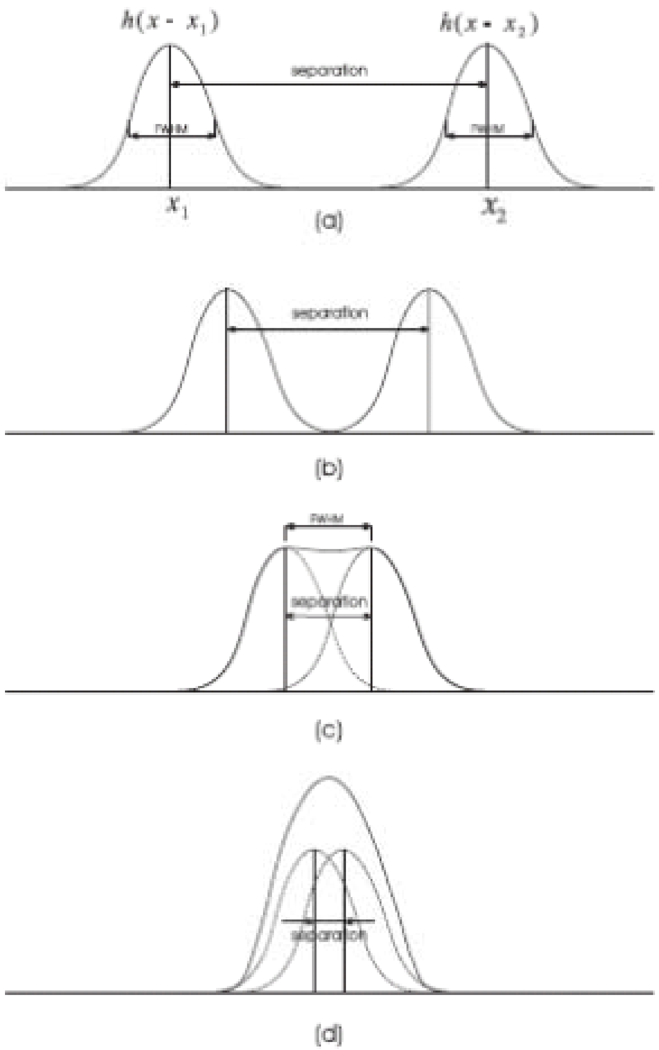
Figure I.4

Medical Imaging Signals and Systems, by Jerry L. Prince and Jonathan Links.

Resolution

- Resolution refers to the ability of a system to depict spatial details.
- Resolution of a system can be characterized by its line spread function
  - How wide a very thin line becomes after imaging
  - Full width at half maximum (FWHM) determines the distance between two lines which can be separated after imaging
  - The smaller is FWHM, the higher is the resolution





Distance > FWHM

Distance > FWHM

Distance = FWHM
(barely separate)

Distance < FWHM
(cannot separate)

Resolution and MTF

- A pure vertical sinusoidal pattern can be thought of as the blurred image of uniformly spaced vertical lines
- The distance between lines is equal to distance between maxima
- If the frequency = u_0, the distance = 1/ u_0

$$f(x,y) = A + B\sin(2\pi u_0 x)$$

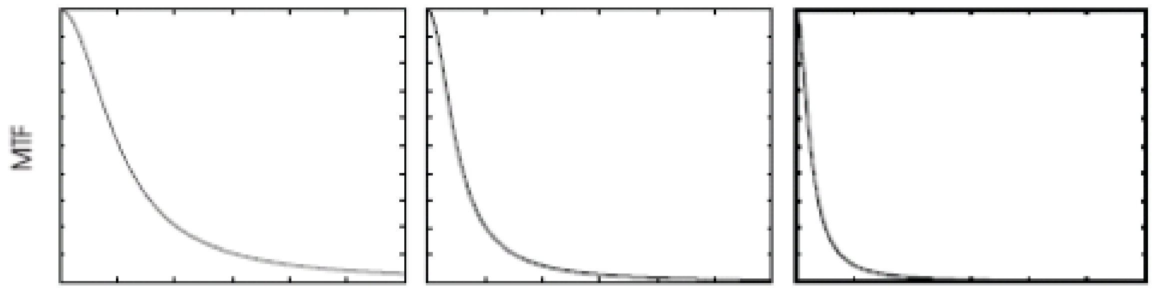
$$g(x,y) = H(0,0)A + H(u_0,0)\sin(2\pi u_0 x)$$

$$= H(0,0)A + MTF(u_0,0)H(0,0)\sin(2\pi u_0 x)$$

- If MTF(u_0)=0, the sinusoidal patterns become all constant and one cannot see different lines
- If MTF(u) first becomes 0 at frequency $u_{c\,,}$ the minimum distance between distinguishable lines = 1/ u_c
- Resolution is directly proportional to the stopband edge in MTF

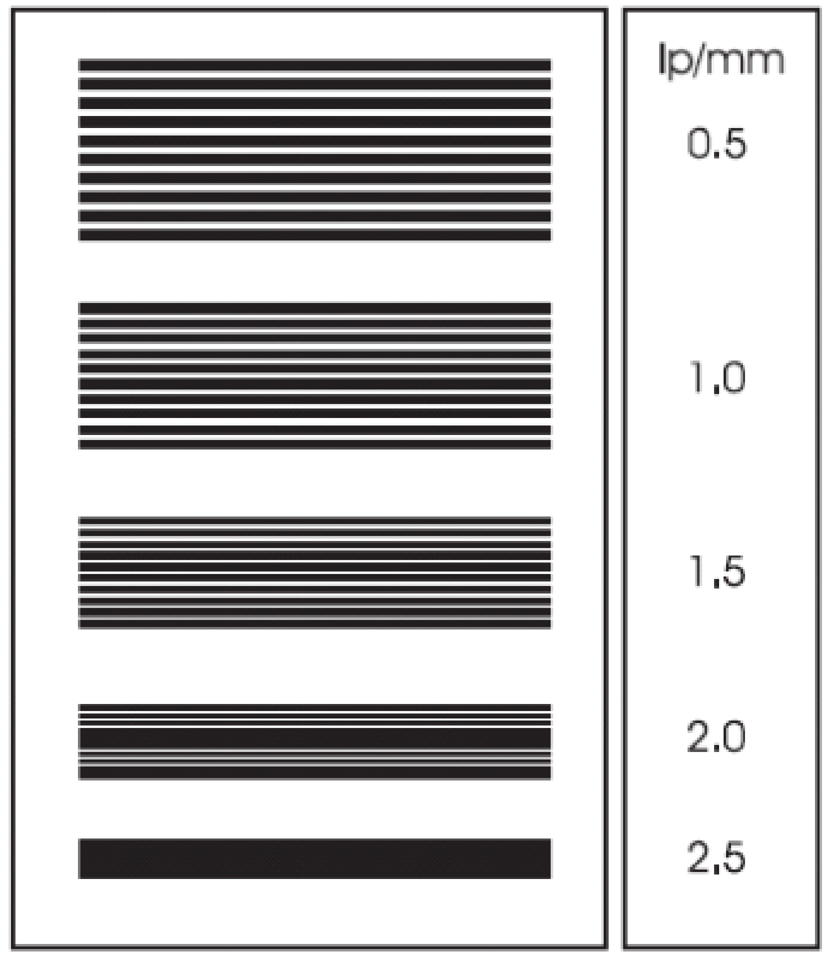
Example

- Which system below has better contrast and resolution?



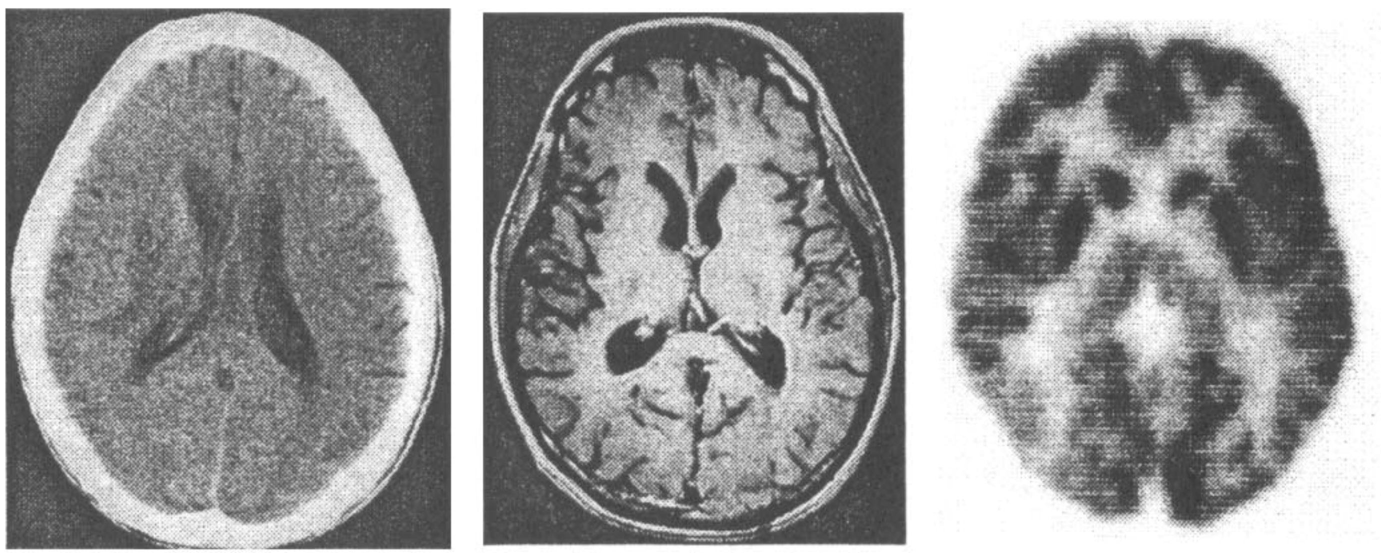
Bar Phantom

- The resolution of an imaging system can be evaluated by imaging a bar phantom.
- The resolution is the frequency (in lp/mm) of the finest line group that can be resolved after imaging.
  - Gamma camera: 2-3 lp/cm
  - CT: 2 lp/mm
  - chest x-ray: 6-8 lp/mm
-



What is noise?

- Random fluctuations in image intensity that are not due to actual signal
- The source of noise in an imaging system depends on the physics and instrumentation of the imaging modality
- Which image below is most noisy?



(a) (b) (c)

Figure I.4

Noise

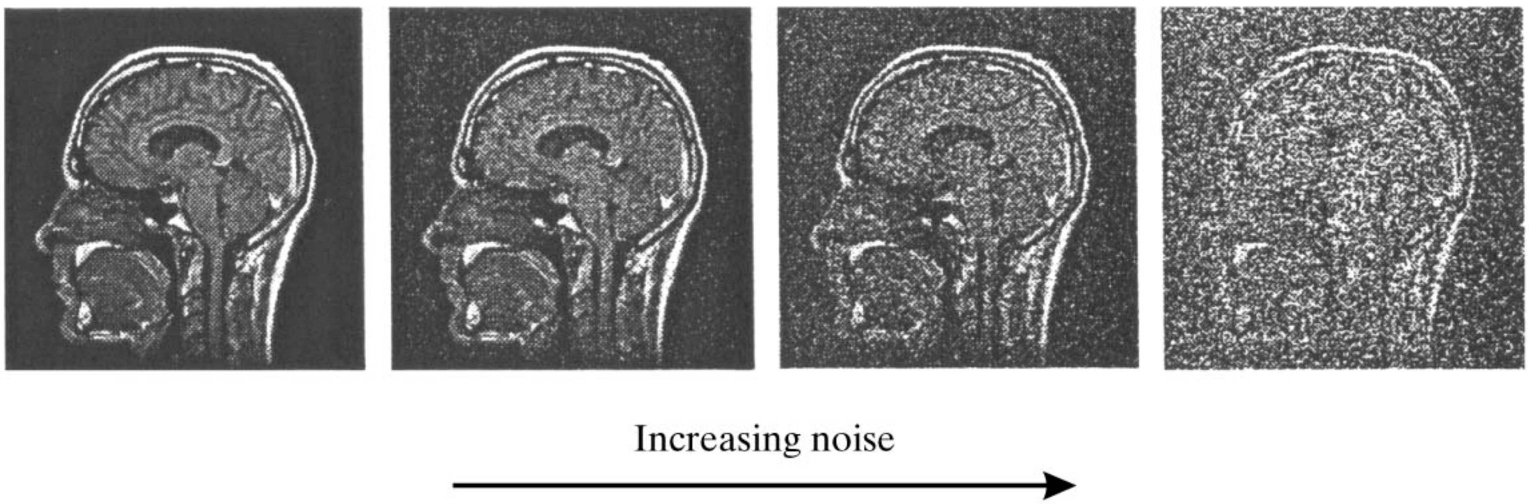


Figure 3.10

Medical Imaging Signals and Systems, by Jerry L. Prince and Jonathan Links.
ISBN 0-13-065353-5.

White vs. Correlated Noise

- Model of a typical imaging system

$$g(x,y) = f(x,y) * h(x,y) + N(x,y)$$

$N(x,y)$ is noise
$N(x,y)$ is a <u>random variable</u> at each (x,y)
$N(x,y)$ could be <u>continuous</u> or <u>discrete</u>

- White Noise: Noise values at different positions are independent of each other, and position independent
  - Mean and variance at different (x,y) are same
- Correlated noise: noise at adjacent positions are correlated
  - Described by the correlation function R(x,y), whose Fourier transform is the noise power spectrum density NPSD(u,v) or simply NPS(u,v)
  - White noise has a PSD = constant = variance

Random Variables

- The most complete description of a random variable is its probability density function (pdf) for continuous-valued RV, or probability mass function (pmf) for discrete-valued RV.
- The two most important statistics of a random variable is mean (μ) and standard deviation (σ). The power of a random signal = variance = σ^{2}. Both η and σ can be derived from the pdf or pmf of a RV.
- Noise typically has zero mean ($\eta=0$).

Amplitude Signal to Noise Ratio

- Amplitude SNR

$$\mathrm{SNR}_a = \frac{\mathrm{amplitude}(f)}{\mathrm{amplitude}(N)}$$

  - Meaning of “signal amplitude” and “noise amplitude” are case-dependent.
  - For projection radiography, the number of photons G counted per unit area follows a Poisson distribution. The signal amplitude is the average photon number per unit area (μ) and the noise amplitude is the standard deviation of G

$$SNR_a = \frac{\mu_G}{\sigma_G} = \frac{\mu}{\sqrt{\mu}} = \sqrt{\mu}$$

  - A higher exposure can lead to higher SNR_a

Power SNR

- Power SNR

$$\mathrm{SNR}_p = \frac{\mathrm{power}(f)}{\mathrm{power}(N)}$$

- Signal power:

$$power(f) = \iint_{x,y} |h(x,y) * f(x,y)|^2 dxdy = \iint_{u,v} |H(u,v)F(u,v)|^2 dudv$$

$$\text{Approximation: } \quad power(f) = A^2, \text{ A is the average value of the signal}$$

$$\text{Approximation: } \quad power(f) = \sigma_f^2, \text{ variance of the signal}$$

- Noise power: $power(N) = \iint_{u,v} NPS(u,v)dudv$

- For white noise: $power(N) = \sigma_N^2$

SNR in dB

- SNR is more often specified in decibels (dB)

- SNR in dB
  - SNR (dB) = 20 $\log_{10} SNR_a$
  - = 10 $\log_{10} SNR_p$

- Example:
  - SNR_p =2, SNR (dB)=3 dB
  - SNR_p =10, SNR (dB)=10 dB
  - SNR_p =100, SNR (dB)=20 dB

Artifacts, distortion & accuracy

- Artifacts:
  - Some imaging systems can create image features that do not represent a valid object in the imaged patient, or false shapes/textures.
- Distortion
  - Some imaging system may distort the actual shape/position and other geometrics of imaged object.
- Accuracy
  - Conformity to truth and clinical utility

Non-Random Artifacts

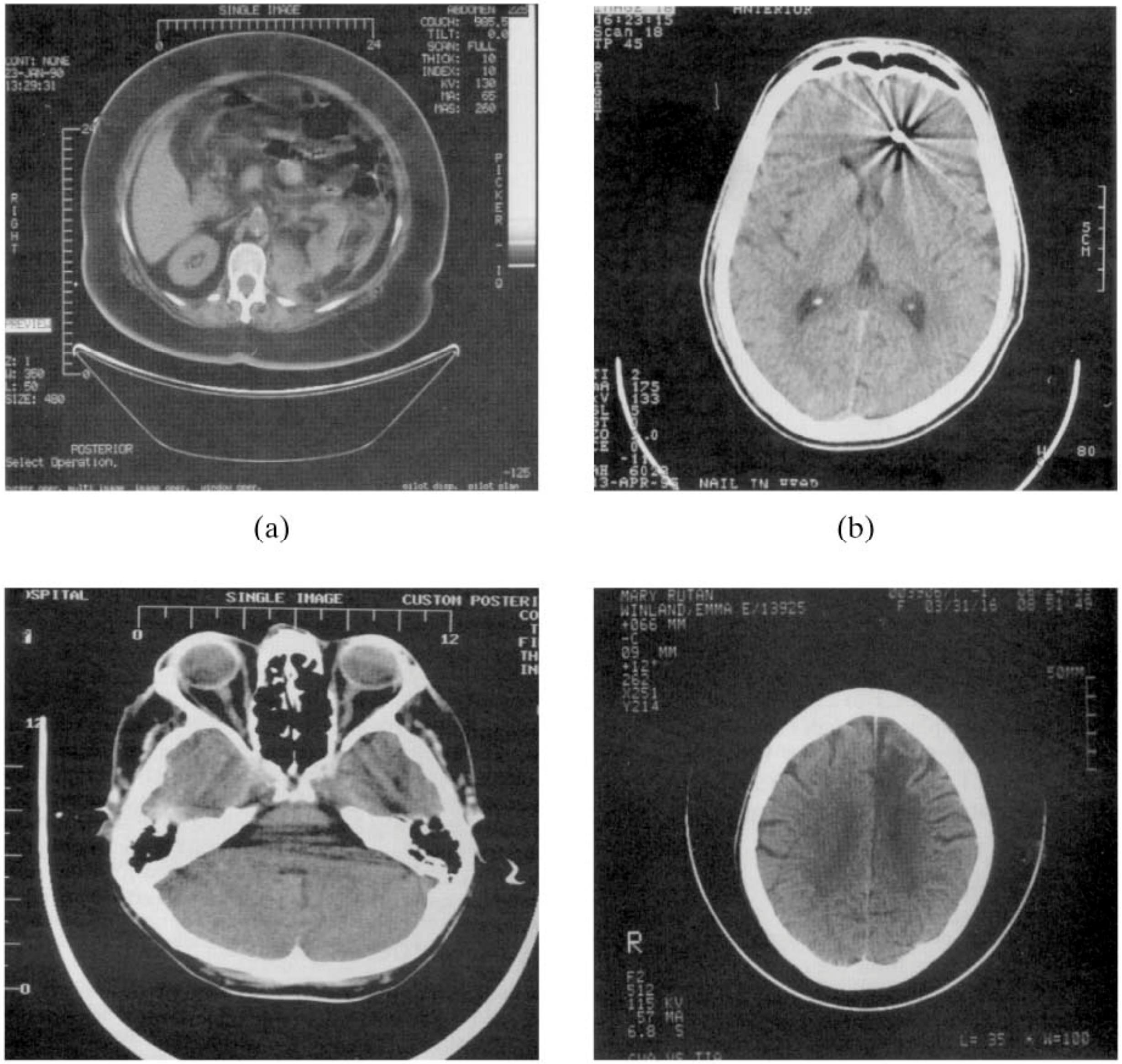
- Artifacts: image features that do not correspond to a real object, and are not due to noise
  - Motion artifacts: blurring or streaks due to patient motion
  - star artifact: in CT, due to presence of metallic material in a patient
  - beam hardening artifact: broad dark bands or streaks, due to significant beam attenuation caused by certain materials
  - ring artifact: because detectors are out of calibration

Motion artifact

Star artifact

Beam hardening

Ring artifact



(a) (b) (c) (d)

Figure 3.12

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Geometric Distortion

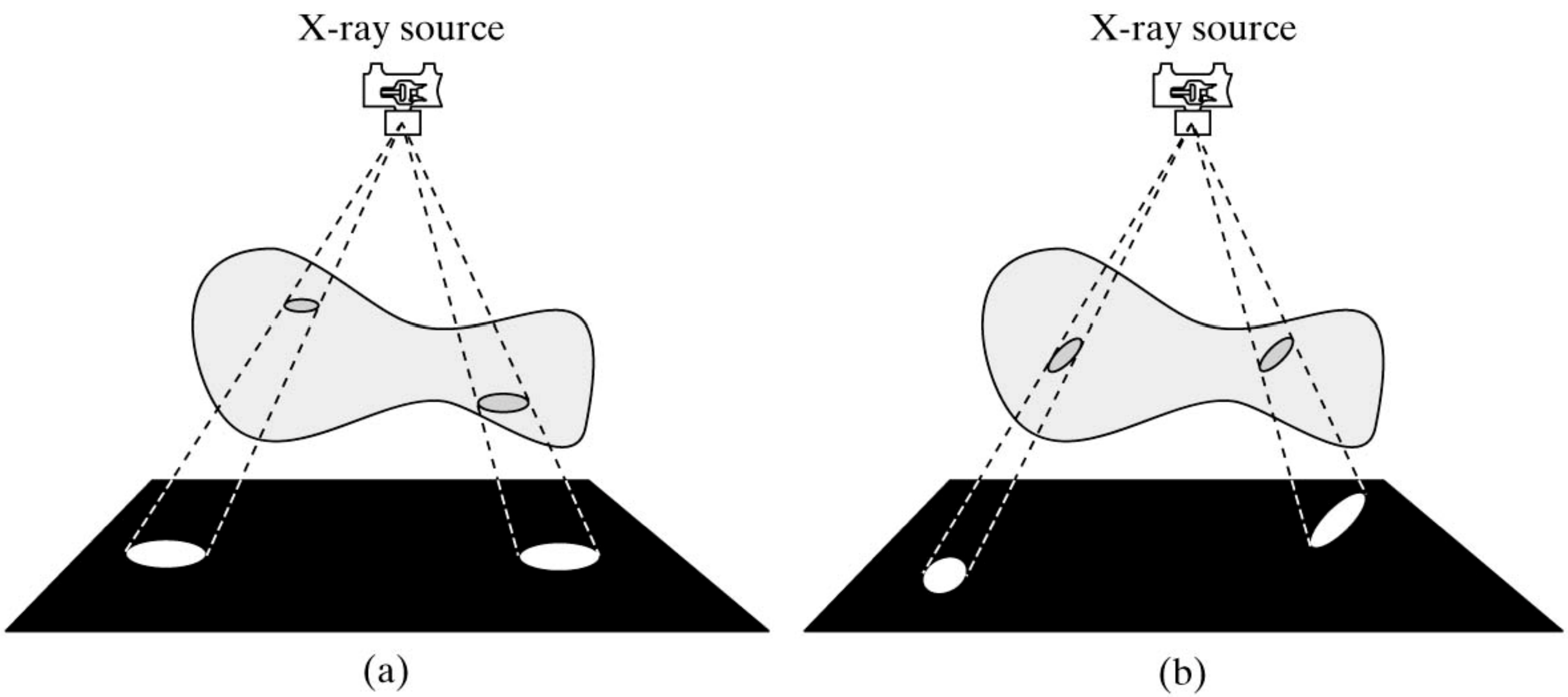


Figure 3.13

Medical Imaging Signals and Systems, by Jerry L. Prince and Jonathan Links.

- In (a): two objects with different sizes appear to have the same size
- In (b): two objects with same shape appear to have different shapes

Accuracy

- Accuracy:
  - conformity to truth
    - quantitative accuracy
  - clinical utility
    - diagnostic accuracy
- Quantitative accuracy:
  - numerical accuracy: accuracy in terms of signal value
    - bias (systematic, e.g. due to miscalibration), imprecision (random)
  - geometric accuracy: accuracy in terms of object size/shape

Diagnostic Accuracy

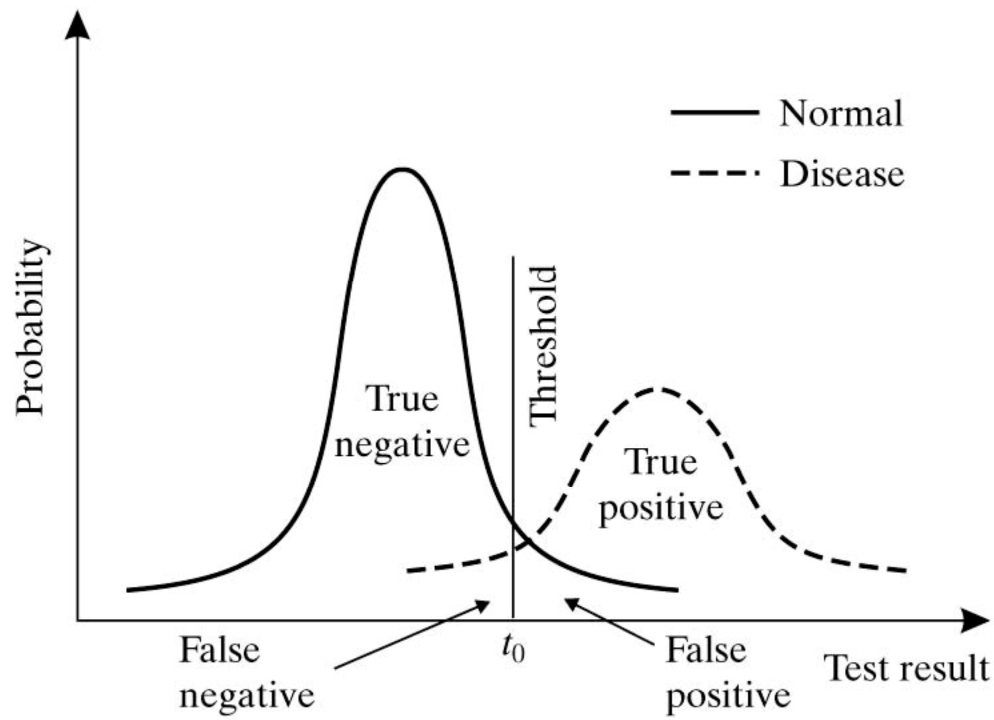
- Contingency Table

		Disease	
		+	−
Test	+	*a*	*b*
	−	*c*	*d*

$$
\begin{aligned}
a &= \#\ \text{w/ disease \& test says disease} \\
b &= \#\ \text{w/o disease \& test says disease} \\
c &= \#\ \text{w/ disease \& test says normal} \\
d &= \#\ \text{w/o disease \& test says normal}
\end{aligned}
$$

$$
\begin{aligned}
\text{sensitivity} &= \frac{a}{a+c} \\
\text{specificity} &= \frac{d}{b+d} \\
\text{diagnostic accuracy} &= \frac{a+d}{a+b+c+d}
\end{aligned}
$$

- If the diagnosis is based on a single value of a test result and the decision is based on a chosen threshold, the sensitivity and specificity can be visualized as follows



Reference

- Prince and Links, Medical Imaging Signals and Systems, Chap 1-3.

Homework

- Reading:
  - Prince and Links, Medical Imaging Signals and Systems, Chap 1-3.
- Note down all the corrections on your copy of the textbook based on the provided errata on the course webpage.
- Problems for Chap 3 of the text book (due at the beginning of next lecture):
  - P3.2
  - P3.5
  - P3.7
  - P3.9
  - P3.11
  - P3.16
  - P3.22 (note correction in the Errata)