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Medical Imaging

MRI Instrumentation, Data Acquisition, Image Reconstruction

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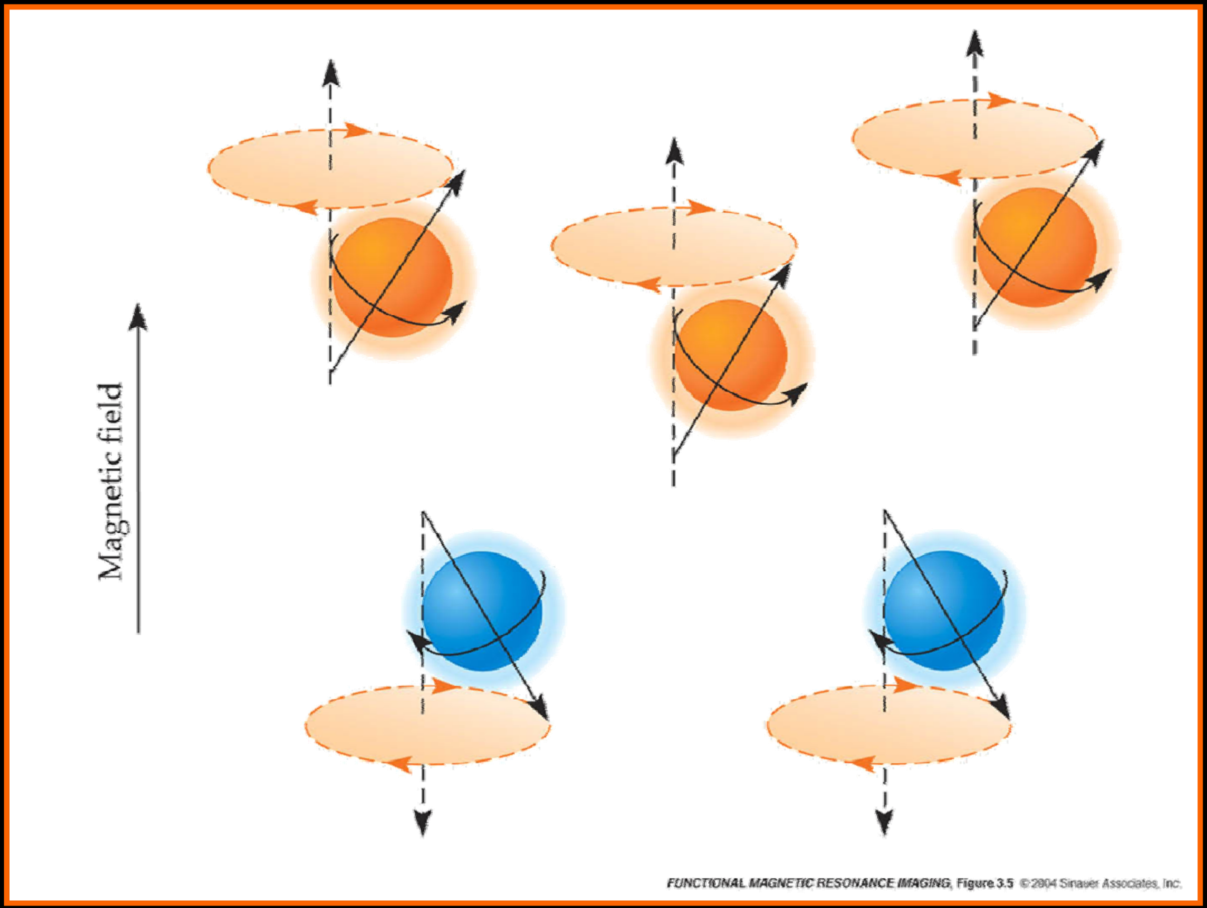
Outline

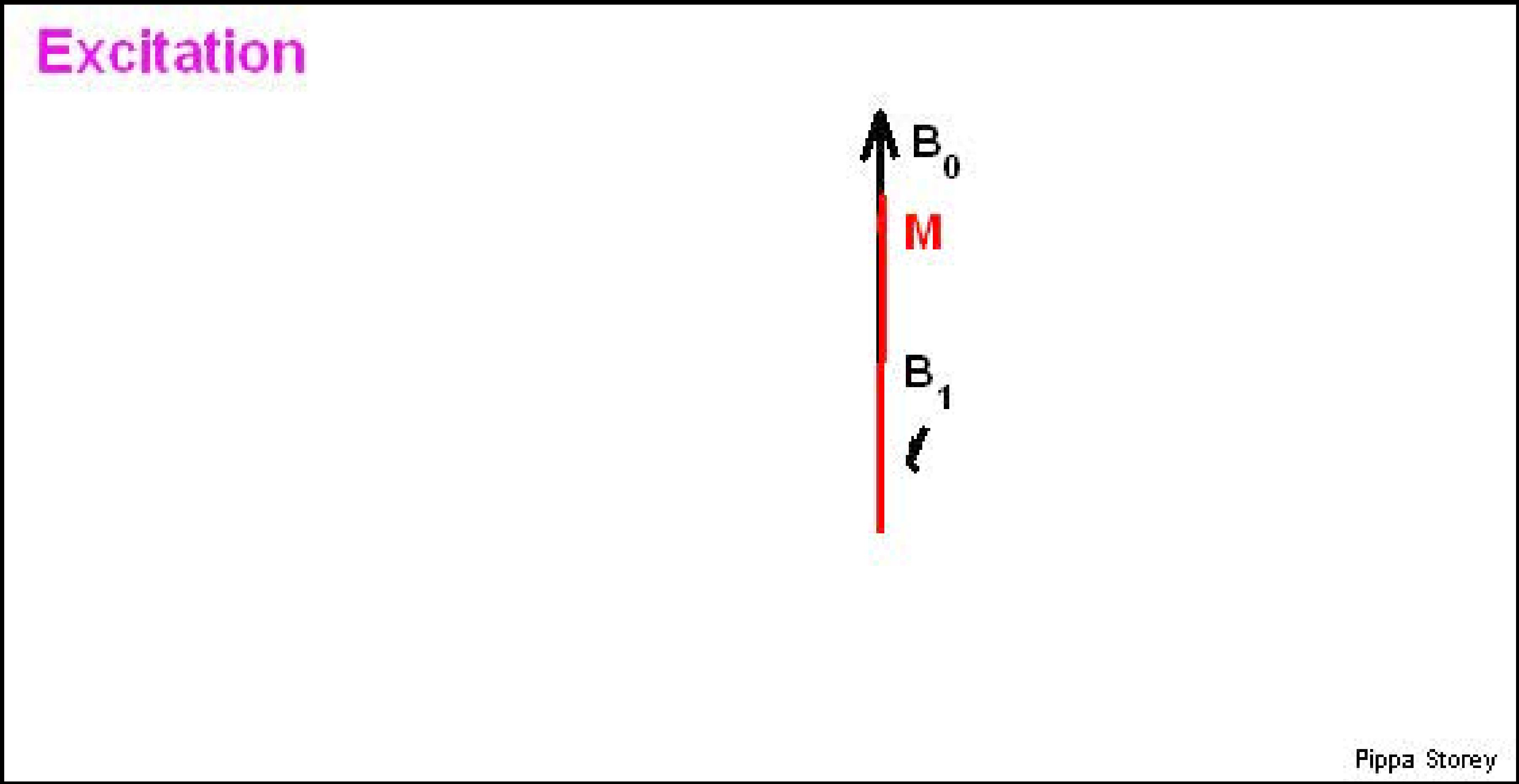
- Review of MR Physics
- MRI hardware
  – Magnet
  – Gradient coils
  – RF coils
- Data acquisition and image reconstruction

Nuclei in an External Magnetic Field



In the presence of an external magnetic field, the spins align with its direction and precess about it

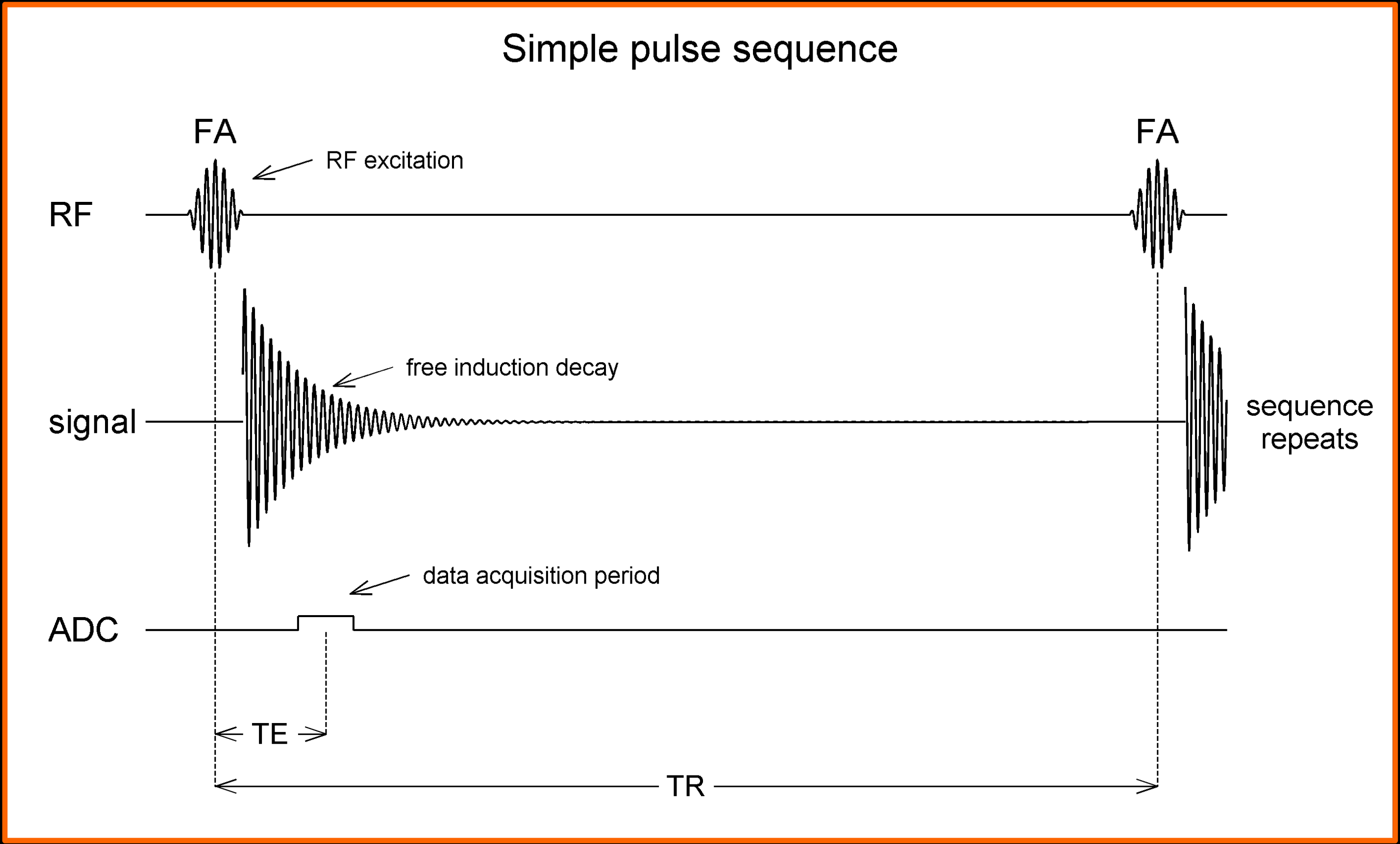
Excitation and Signal Detection



Recap of Excitation and Signal Detection

- Apply a transverse RF magnetic field B_1 at the Larmor frequency
- Net magnetization is tipped away from direction of B_0
- Spins precess in phase
- When switch off field, magnetization continues to rotate
- Signal comes from transverse component of magnetization
- Spins eventually dephase and the signal dies away (transverse relaxation)
- Spins lose energy and return to equilibrium (longitudinal relaxation)
- Relaxation processes are important for producing image contrast

Free-Induction Decay



- TE is how long we wait before measuring the signal
- TR is how long we wait before the next excitation
- Choice of TE and TR determines the signal contrast

Outline

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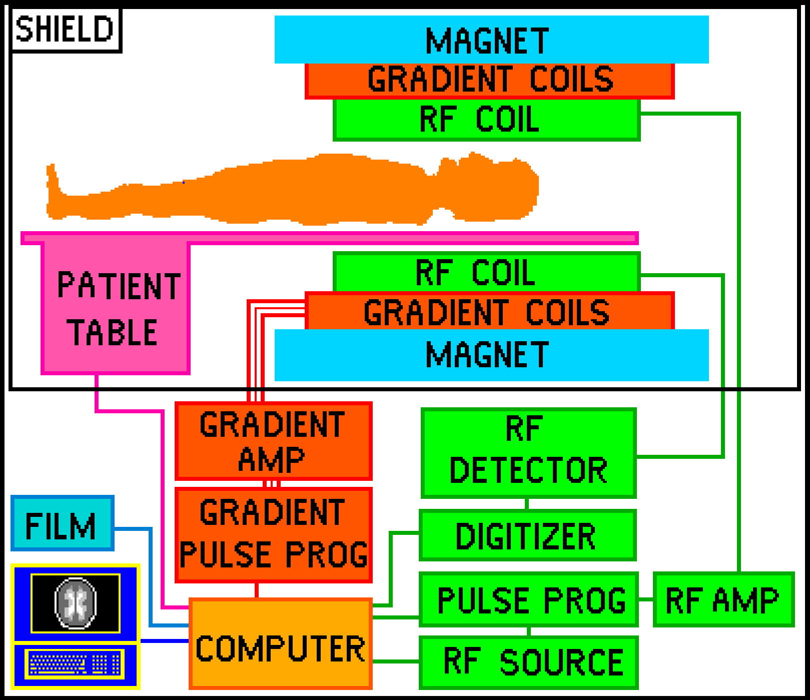
  - Gradient coils
  - RF coils
- Data acquisition and image reconstruction
- Image quality

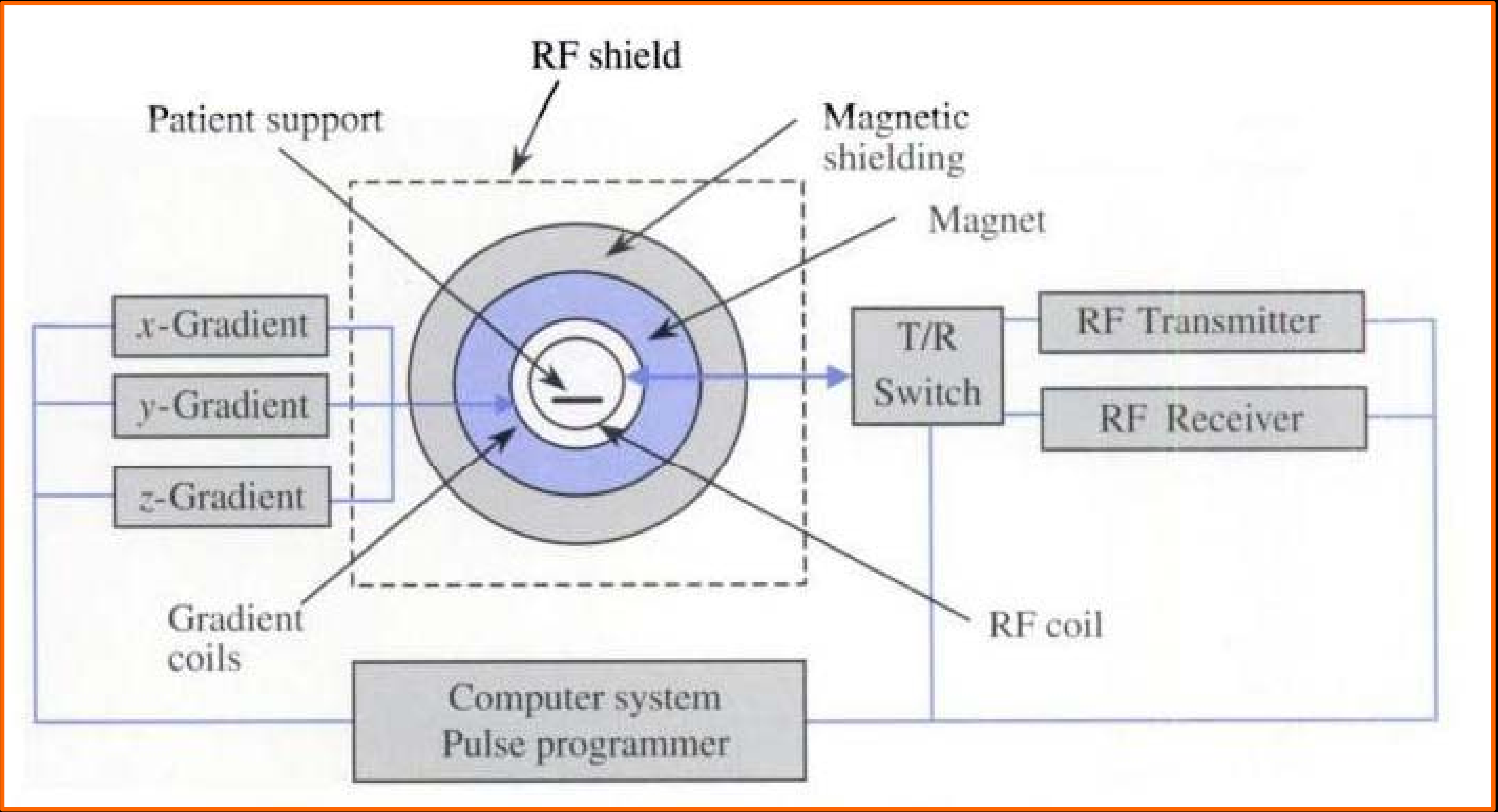
A Look Under the Hood

A Look Under the Hood



http://www.cis.rit.edu/htbooks/mri/

Schematic Diagram of an MR Scanner



Essential components of an MRI system

Magnet

Required to polarize the spins

Must produce a magnetic field that is ***strong***, ***uniform*** and ***stable*** (does not drift over time). Generally a superconducting electromagnet

Gradient and shim coils

Required to:

- 'shim' the primary magnet (compensate for field inhomogeneities)
- provide imaging capability

Gradient coils are used to produce linear field variations, and shim coils to compensate for quadratic and higher-order field variations. Three gradient coils are required to provide linear field variations in three orthogonal directions

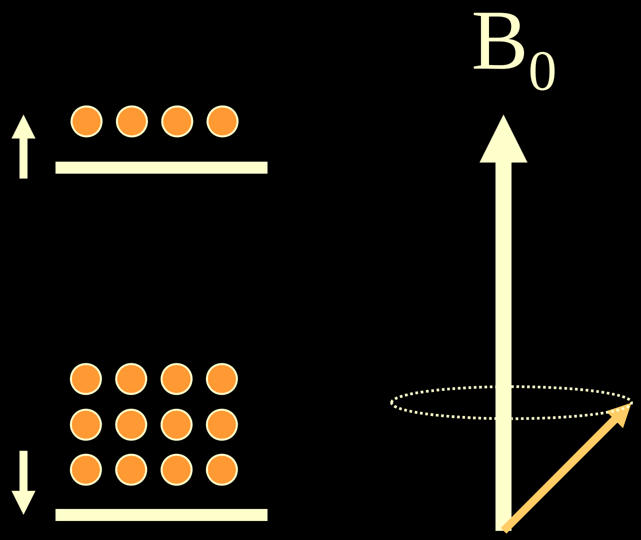
RF coils

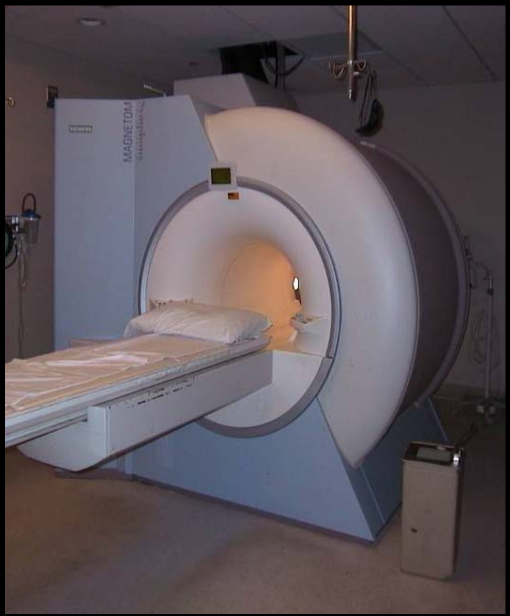
Required to:

- excite the spins
- detect the emitted signal

Separate coils may be used for transmission and reception, or a single coil may serve both purposes. Clinical scanners generally incorporate an integrated 'body coil' inside the magnet housing, and also interface with a variety of anatomy-specific volume and surface coils

The Magnet





- Polarize nuclear spins to create reservoir of MR signal
- Establish operating (Larmor) frequency

Type of Magnets Clinically Available

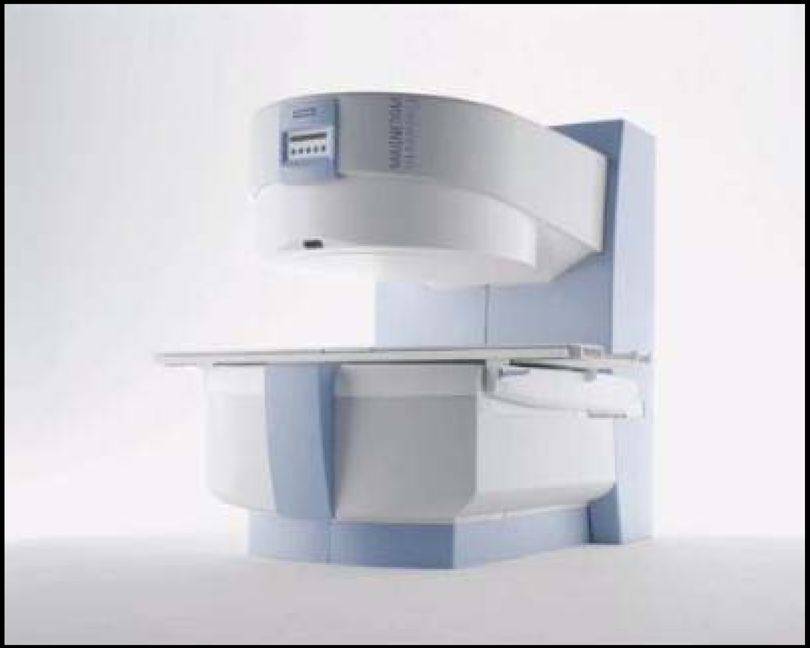
- Permanent
  - 0.2 to 0.35 T
- Resistive
  - 0.2 to 0.6 T
- Superconducting
  - 0.35 to 3 T (up to 9.4 T for research)

Permanent Magnets

- Made of special alloys (ferromagnetic materials)
- Loose a fraction of their magnetism over time
- Advantages
  - Low cost
  - Do not need electrical power and cooling system
  - Low maintenance cost
- Disadvantages
  - Always on
  - Weight average 15 tonnes

Permanent Magnets

Philips and Neusoft
0.35 T



Siemens Magnetom Concerto
0.2 T

Resistive Magnets

- Made with copper-based winding
- Advantages
  - Easily turn on/off
  - Do not need cryogenic cooling system
- Disadvantages
  - Stability of power supply (if current varies so will the magnetic field)
  - Cost of electricity

Resistive Magnets

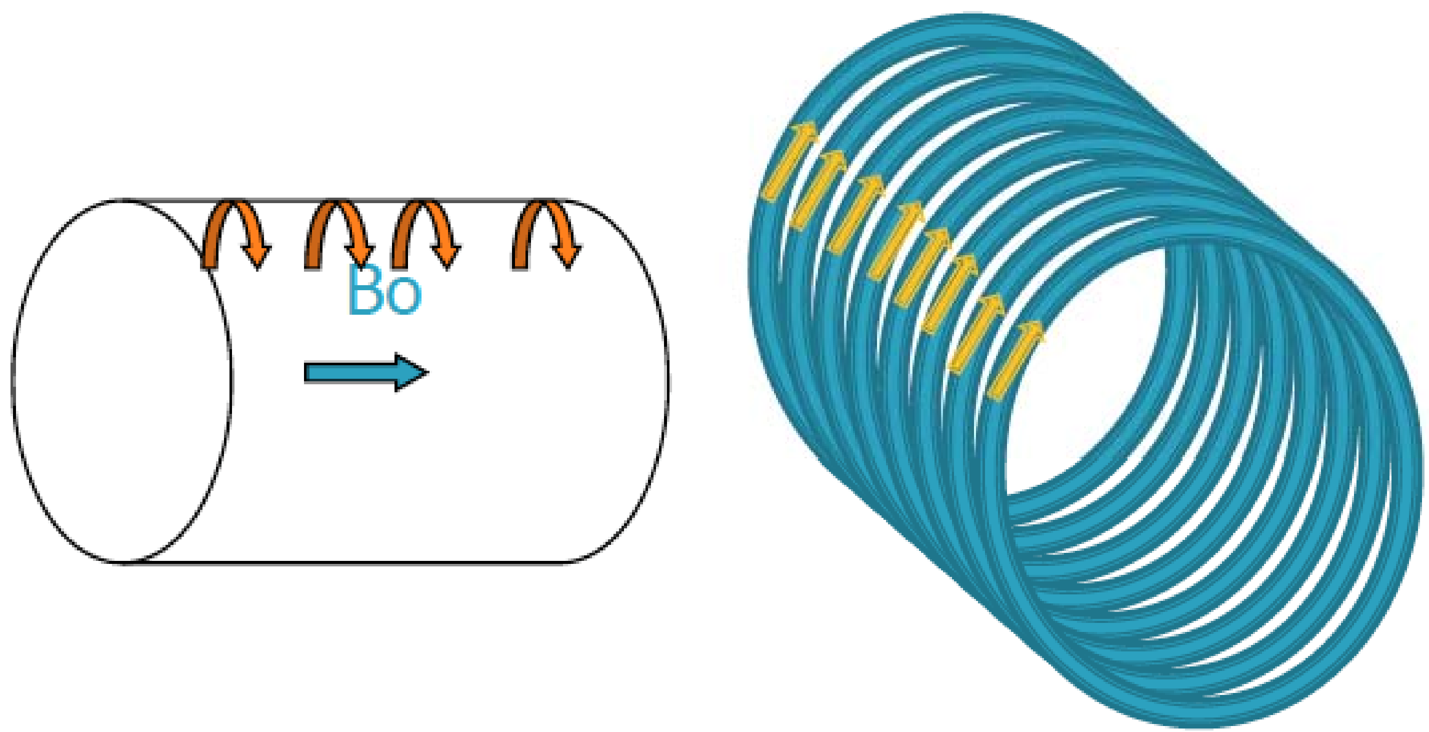
Fonar Standing Ovation
0.6 T

Siemens Magnetom Open Viva
0.2 T

Superconducting Magnets

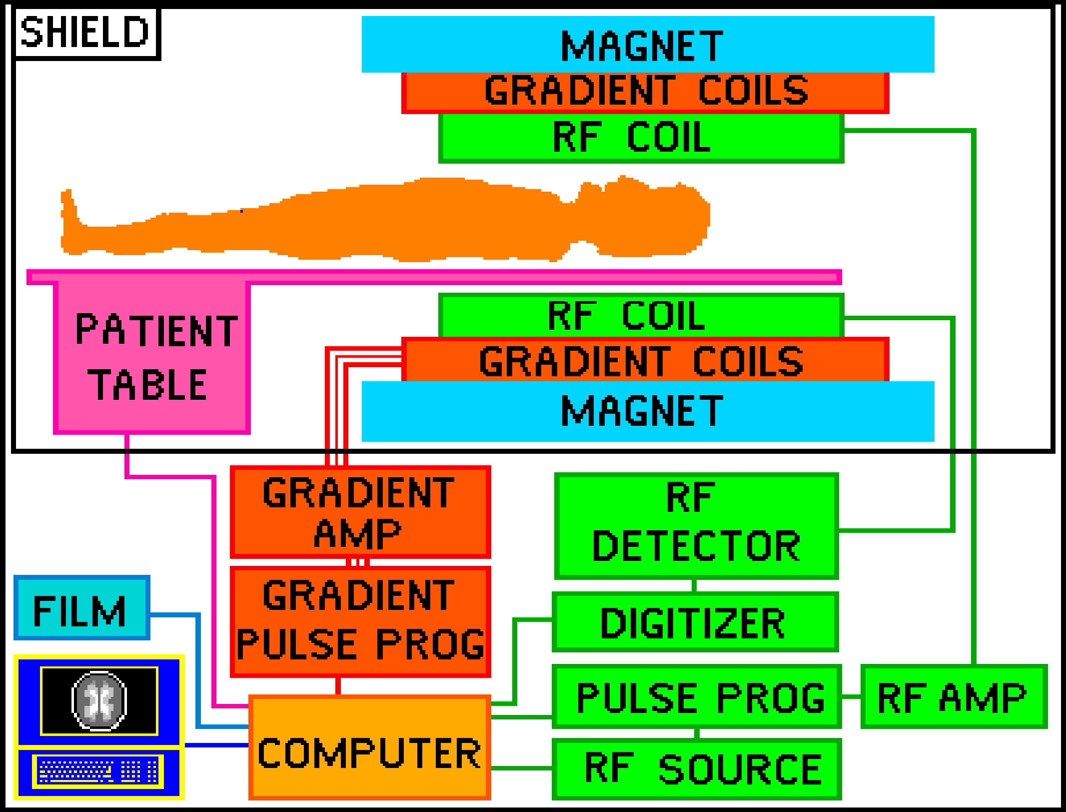
- Different version of the resistive magnet
- Cooled at 77 K (-321 F)
- Advantages
  - No electric resistance at low temperatures (less current)
  - Field can be contained with active shielding
  - Weight about 5 tonnes
- Disadvantages
  - Expensive to make
  - Price goes up per tesla
  - Supply of helium limited and expensive
  - Large installations

Superconducting Solenoidal Magnet



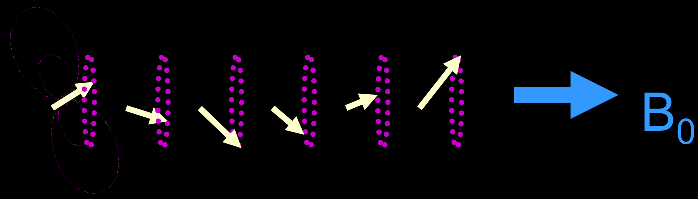
Coil is immerged in liquid helium

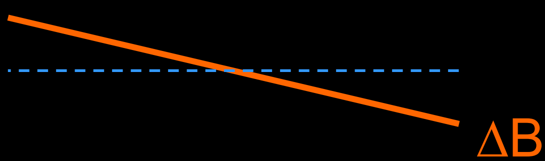
The Gradient Coils



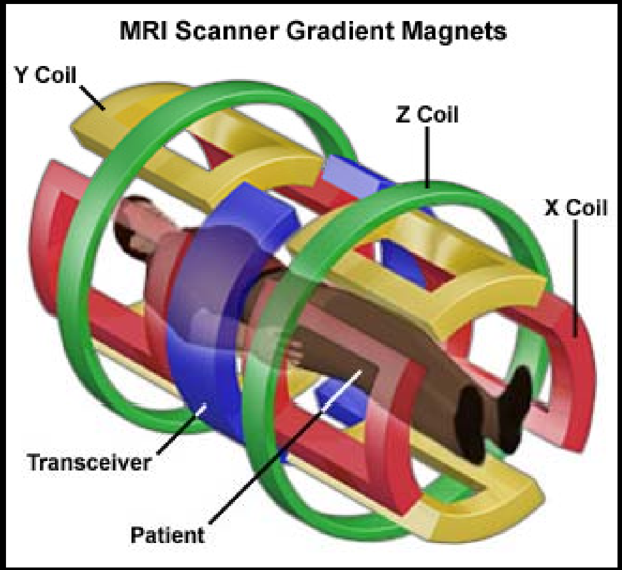
http://www.cis.rit.edu/htbooks/mri/

The Gradient Coils





"Fast" spins "Slow" spins

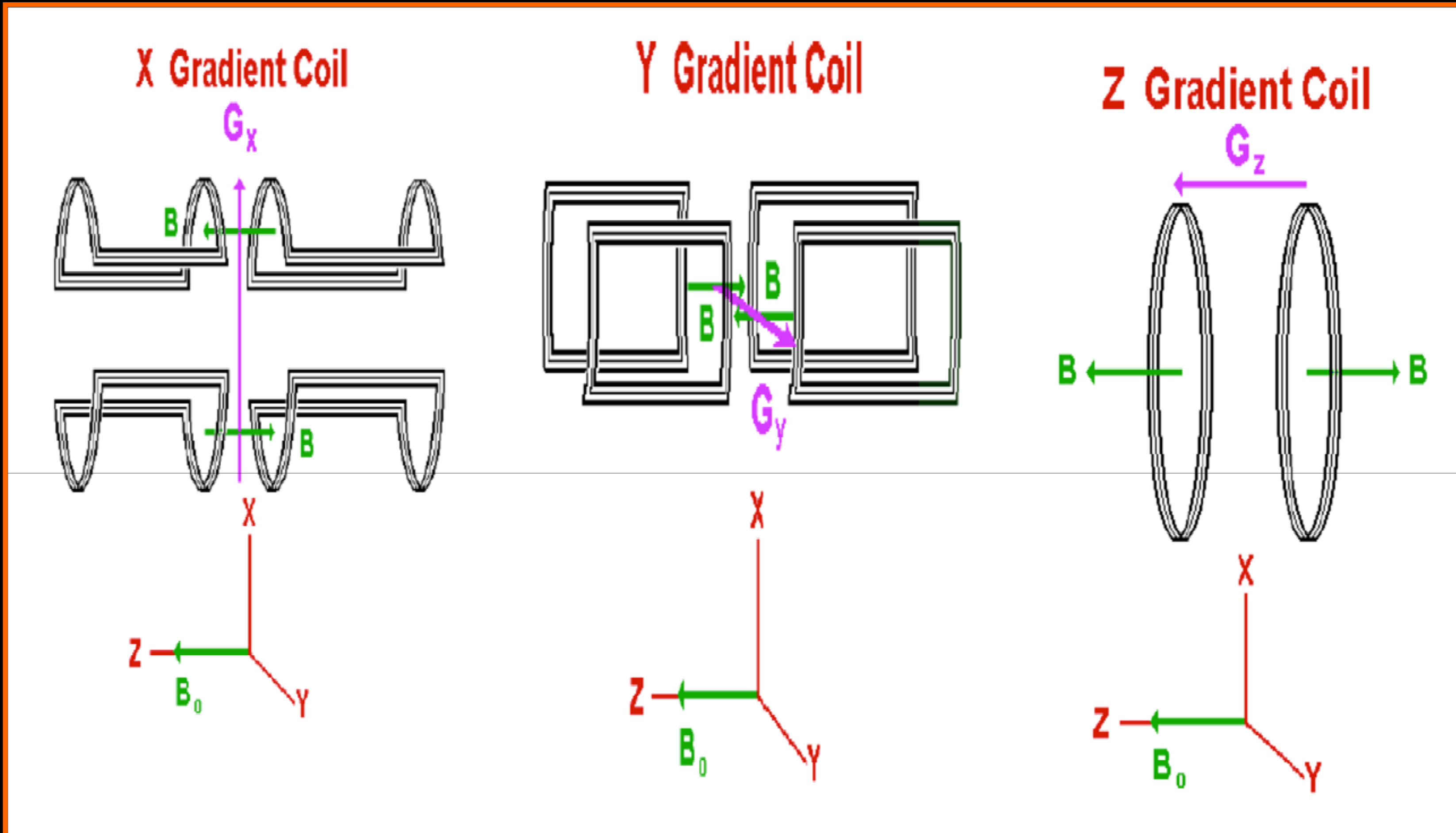


Lauterbur & Mansfield
Nobel Prize in Physiology or Medicine, 2003

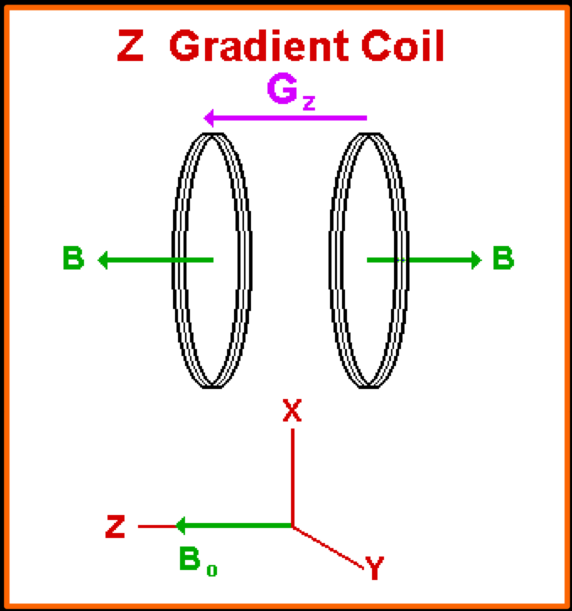
The Gradients

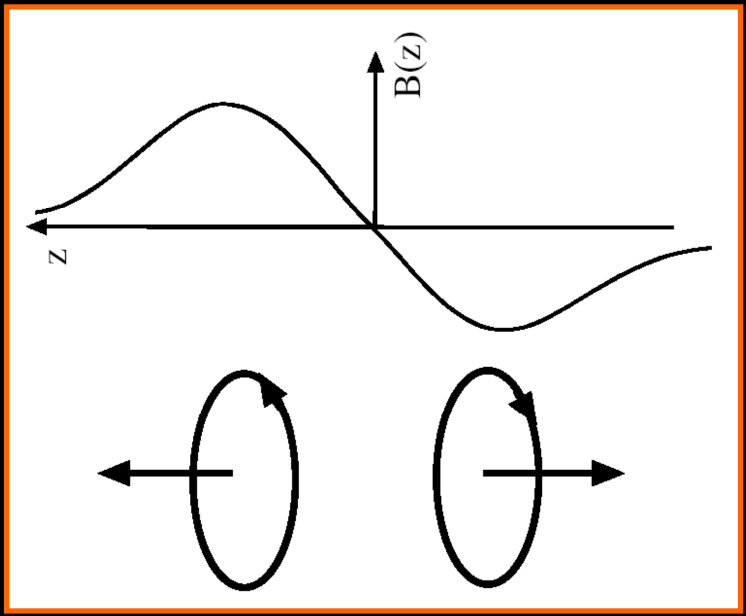
- Role
  - Select excited volumes, encode spatial information
- Important components and parameters
  - Components: gradient amplifiers, gradient coils
  - Parameters: gradient strength, slew rate, homogeneity volume, length
- Three coils that modify the main field as follow:
  - $\mathbf{B} = (B_0 + G_x x + G_y y + G_z z)\,\hat{\mathbf{z}}$
- Impact
  - Increased imaging speed/efficiency (+)
  - Improved velocity and diffusion enconding
  - Short TE/TR

X, Y And Z Gradient Coils



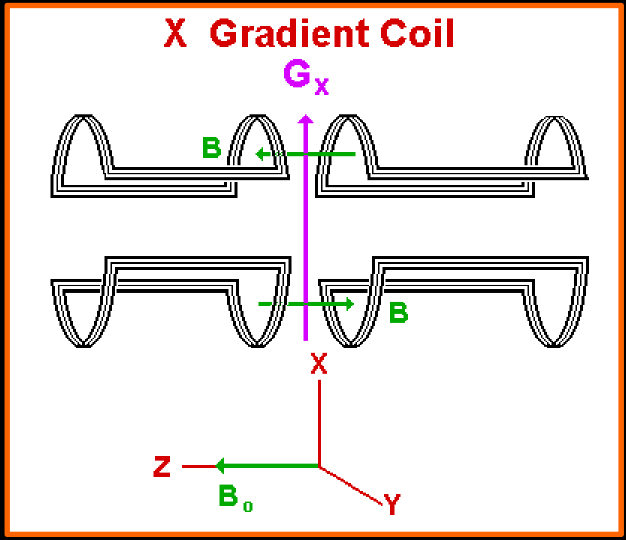
Z-Gradient

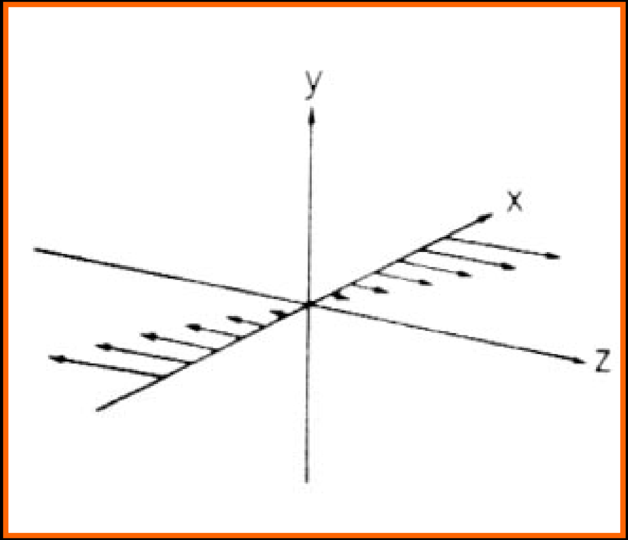


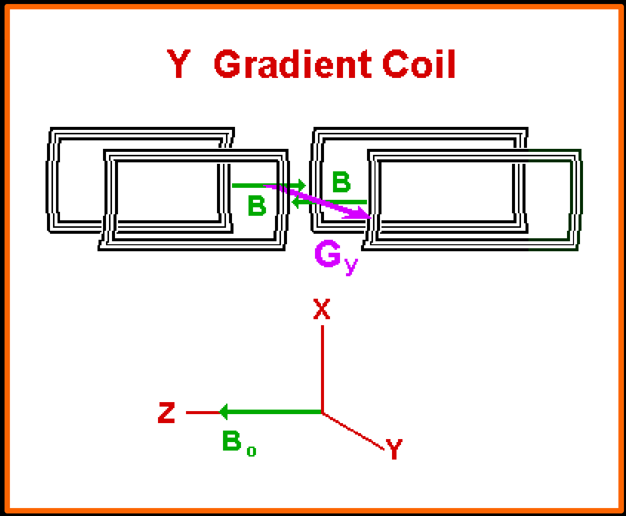


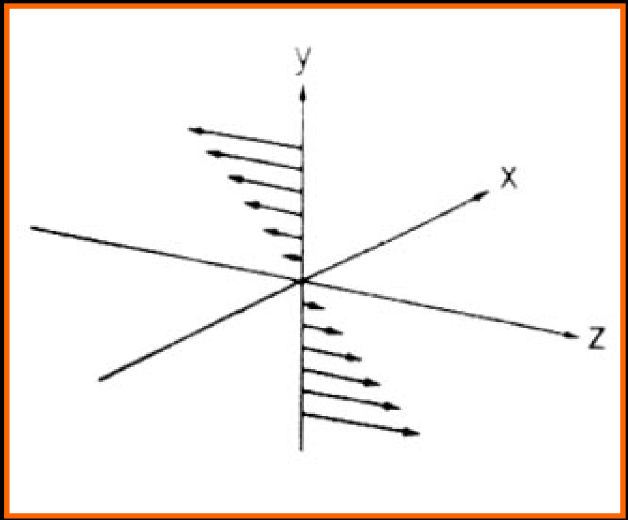
Anti-Helmholtz Coil

X- And Y-Gradient









Specifications for Gradient Coils

- Maximum gradient amplitude
  - 1-6 Gauss/cm
- Switching times
  - 0.1-1.0 ms
- Slew Rates
  - 5-250 mT/m/msec
- FDA limit
  - 40 T/s

RF coils

RF coils are required to produce the B_1 field that excites the spins (known as 'transmission') and also to detect the emitted signal from the spins (known as 'reception').

An important feature of RF coils that distinguishes them from the gradient and main magnet coils is that they produce and detect ***time-varying*** magnetic fields

To do so efficiently, the RF coils must be designed to resonate at the Larmor frequency

Separate RF coils can be used for transmission and reception, or the same coil can serve both purposes. Coils designed for both transmission and reception are called 'transmit/receive' coils

Clinical systems have an integrated 'body coil' inside the casing of the scanner. They also interface to a wide variety of specialized coils designed for imaging different parts of the anatomy.

One reason for using specialized coils has to do with SNR considerations

SNR considerations in RF coil design

An important property of an RF coil is the SNR can be achieved

The ***noise*** depends on the effective resistance of the system, which includes contributions from the electronics, the sample being imaged, and the coil itself.

At high fields, the sample noise is dominant.

The amount of noise from the sample depends on the total volume of tissue within the sensitivity range of the coil

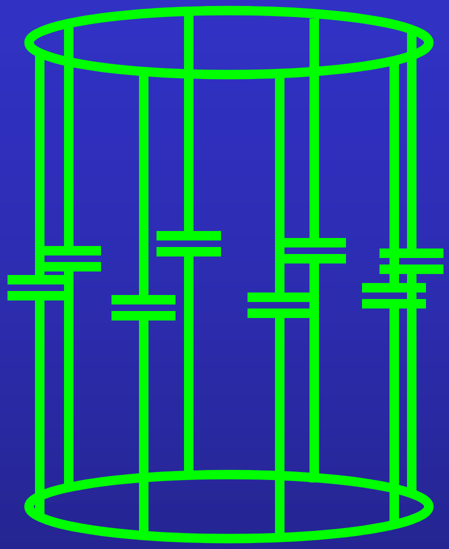
The amount of ***signal***, however, depends only on the volume of the slice or slab being excited

An optimal coil is therefore one whose range of sensitivity is limited to the volume of interest (the tissue being excited)

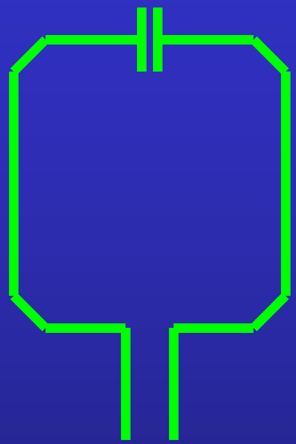
Coil design thus depends on the particular application. The coil sensitivity should be high in the volume of interest and low everywhere else. In practice, this means that the coil fits relatively snugly around the anatomy of interest.

RF coil designs

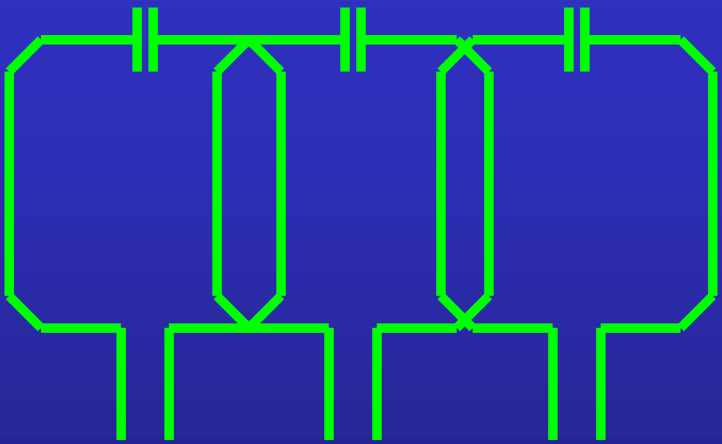
birdcage

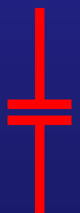


surface coil



phased array





signifies a capacitor

Birdcage coils

The birdcage coil is a quadrature coil, meaning that it can generate and detect circularly polarized (rotating) RF fields. This involves producing both x- and y-components of the field and alternating their amplitudes in the appropriate phase

The birdcage coil has a fairly uniform profile over the volume it encloses. This means that it can produce a relatively homogeneous B_1 field, and its detection sensitivity is also uniform

It should be noted, however, that at very high fields (> 1.5T) the wavelength of the RF fields become comparable to the size of the human body, and it becomes more difficult to obtain good homogeneity

The integrated body coil is a birdcage design

Surface and phased array coils

Surface coil

Surface coils are designed to be placed close to the body surface

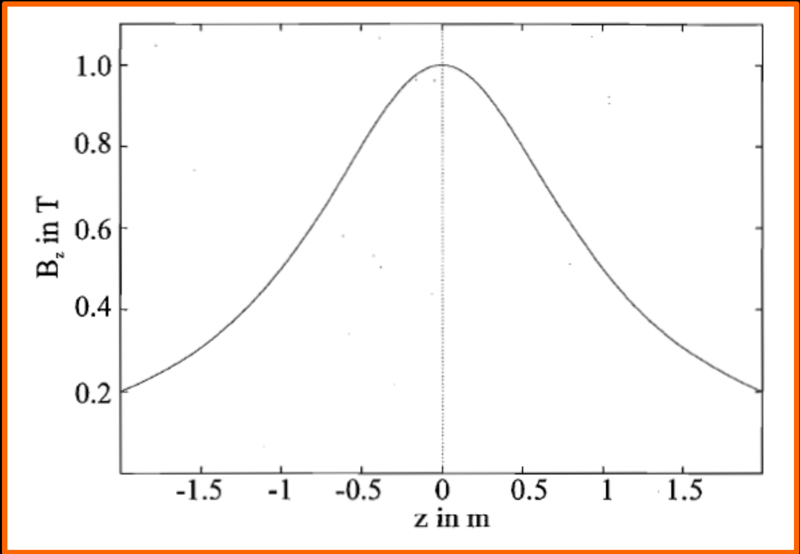
Their depth of sensitivity is roughly equal to their radius, so they provide high SNR for superficial structures.

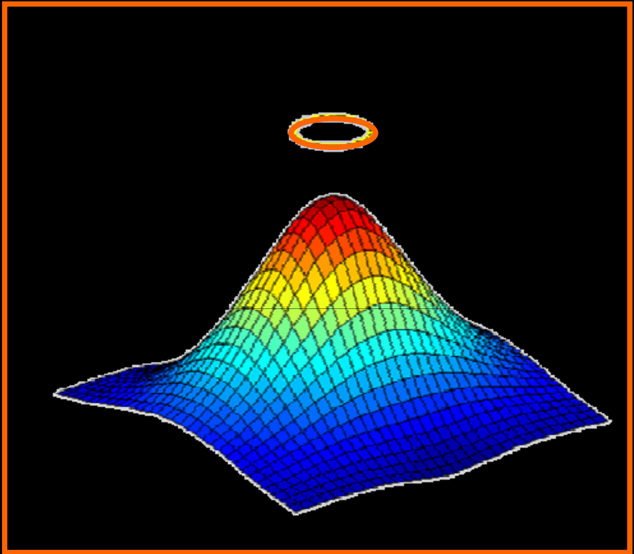
In clinical imaging, they are typically receive-only coils; Transmission is by the body coil.

Sensitivity Profile of Surface Coils

An elementary calculation can be made to find the on-axis field:

$$B_z(\rho = 0) = \frac{\mu I a^2}{2(a^2 + (z - s)^2)^{3/2}}$$





Maximum SNR at depth *d* is obtained with loop of radius: $a = \frac{d}{\sqrt{5}}$

Surface and phased array coils

Surface coil

Surface coils are designed to be placed close to the body surface

Their depth of sensitivity is roughly equal to their radius, so they provide high SNR for superficial structures.

In clinical imaging, they are typically receive-only coils; Transmission is by the body coil.

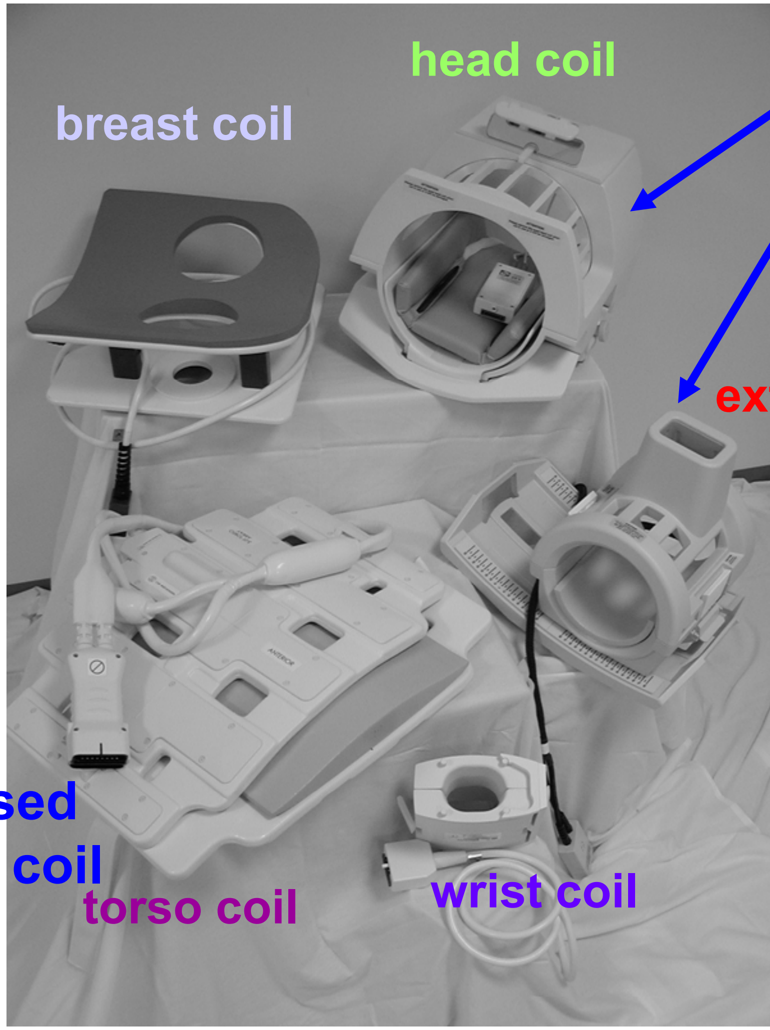
Phased array coil

A phased array is a set of surface coils whose signals are combined. Like surface coils, they are typically receive-only coils

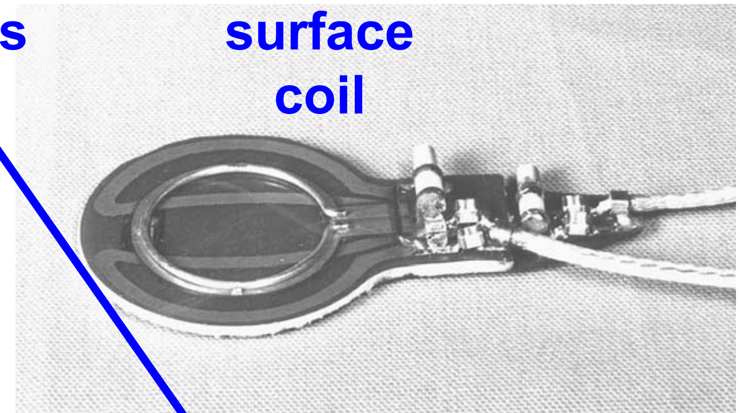
They offer the high SNR of small individual coils, but can be used for imaging larger regions such as the spine.

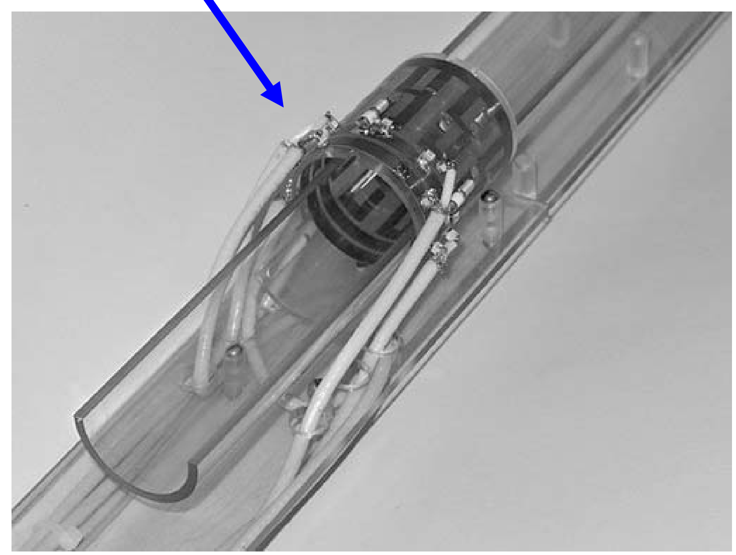
They can also be used for parallel acquisition (invented by Dr Dan Sodickson)

RF coils



coils for clinical imaging





coils for small-animal imaging

128-Element Cardiac Array

Front Back

Courtesy of Larry Wald (MGH, Harvard)

Outline

- Review of MR Physics
- MRI hardware
  - Magnet
  - Gradient coils
  - RF coils
- Data acquisition and image reconstruction

How is an image produced?

Earlier we discussed the origin of the MR signal and how it may be manipulated to produce different types of signal contrast

We saw that the origin of the MR signal involves:

- Polarization of spins by a static B_0 field in the z direction
- Excitation of spins by a rotating B_1 field in the x-y plane
- Detection of the emitted signal by a receiver coil

We also saw that the emitted signal could be sensitized to tissue-dependent properties such as relaxation times to achieve signal contrast among different tissues and lesions

To produce an image, however, we need to know **where** the signal originates, and know it **with high resolution**

This is not possible using just the main magnet and the RF excitation and receiver coils, however, since they encompass the entire body (or body part) of interest

Imaging: the solution

A solution to the problem of mapping the spatial distribution of the MR signal was invented by Paul Lauterbur and Peter Mansfield, for which they won the Nobel Prize in 2003.

They observed that the frequency of precession is a very precise measure of the ***local magnetic field*** at the site of the spins

Therefore, by introducing magnetic field gradients, the frequency could be used to identify the ***position*** of the spins

Magnetic field gradients alter the precession frequency of the spins in a spatially-dependent manner

They are used in two different ways to produce an image:

- Selective excitation

  When applied ***during*** excitation, magnetic field gradients ensure that only certain spins are excited

- Spatial encoding

  When applied ***after*** excitation, magnetic field gradients can be used to encode spatial information in the signal via the spins' frequency and phase

Magnetic field gradients

Ideally, the magnetic field gradients should alter the ***amplitude*** of the B_0 field in a ***linear*** manner, without affecting its direction

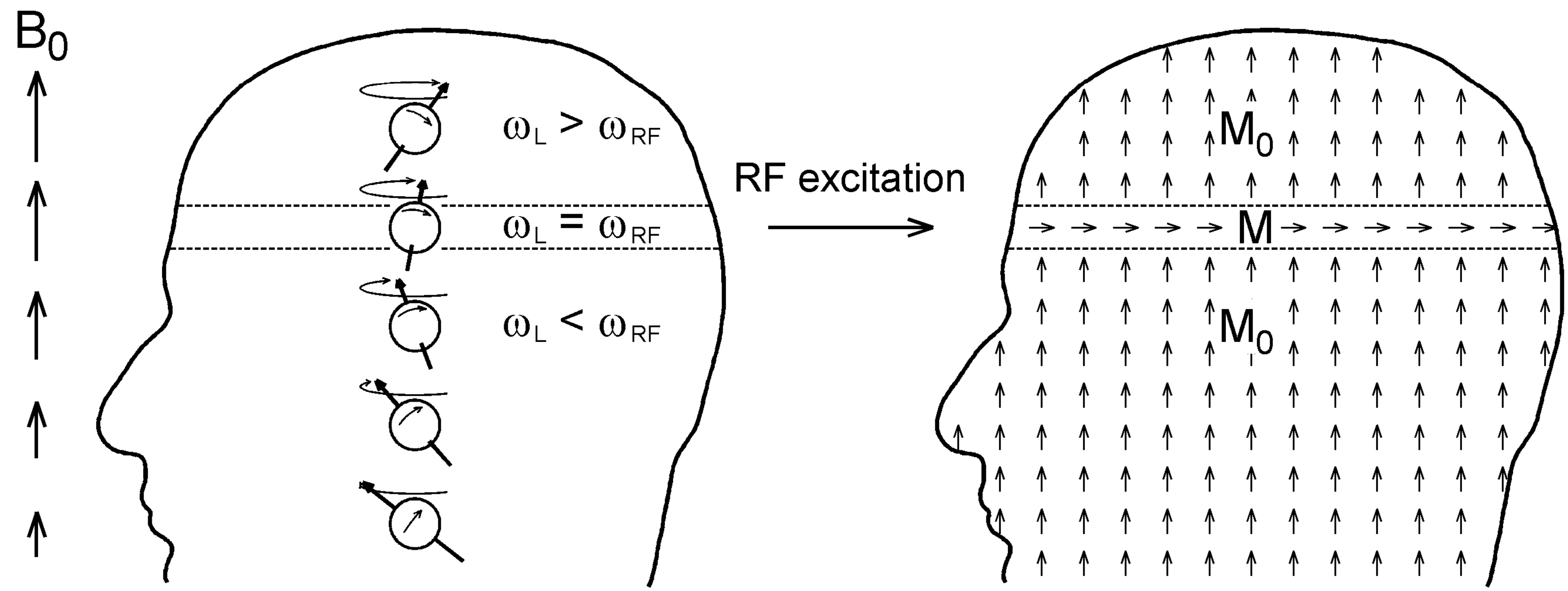
Gradients are required in all 3 directions, and can be written as

$$G_x = \frac{\partial B_z}{\partial x}$$

$$G_y = \frac{\partial B_z}{\partial y}$$

$$G_z = \frac{\partial B_z}{\partial z}$$

Slice-selective excitation



A magnetic field gradient is applied during the RF excitation pulse

(Note that in the above diagram the gradient is applied in the same direction as B_0, but it can in practice be along any direction)

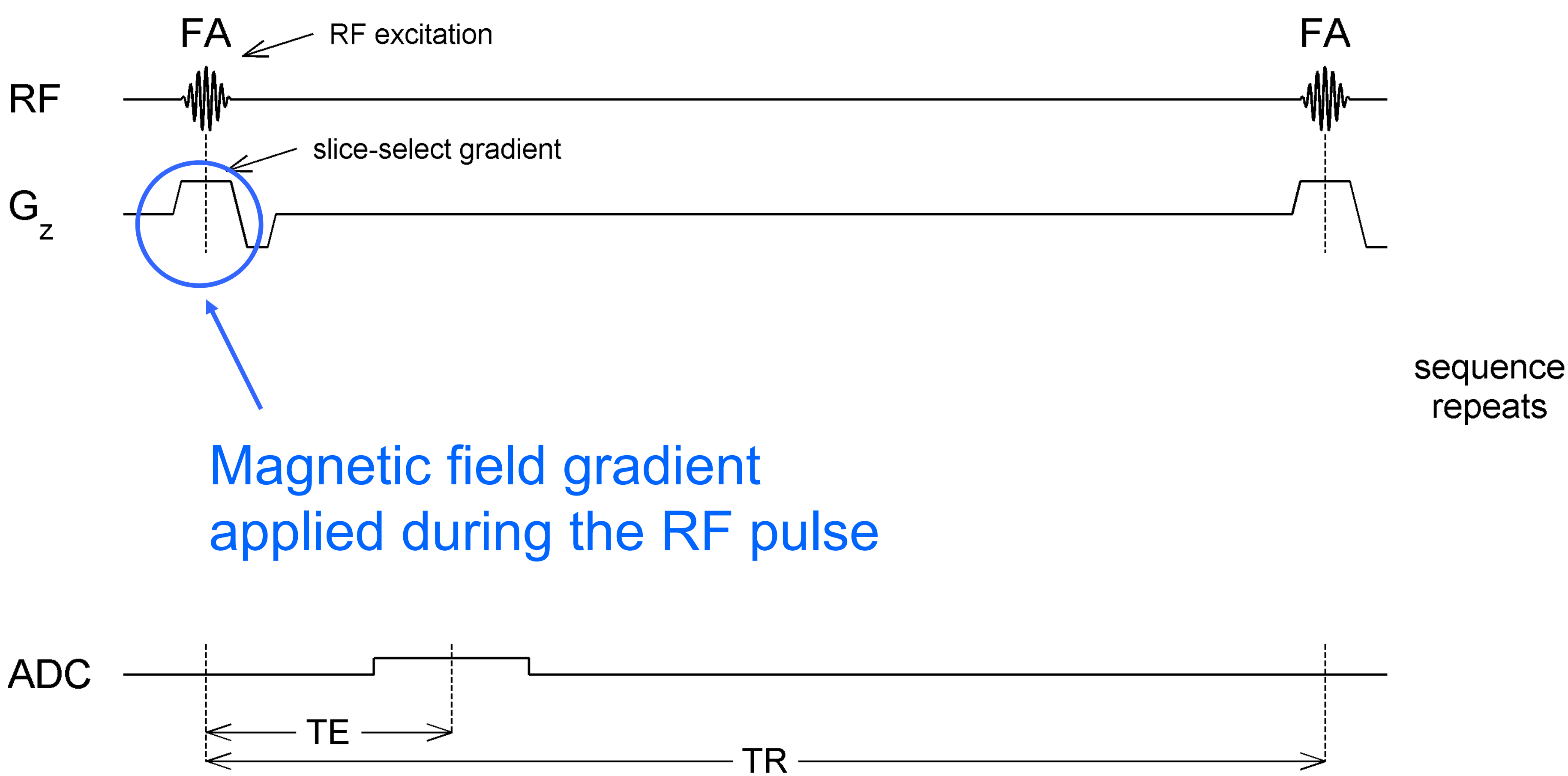
The gradient alters the Larmor frequency ω_L of the spins along the direction of the gradient

Only those spins whose Larmor frequency equals the frequency of the RF pulse $\omega_L = \omega_{RF}$ will be excited

Such spins lie in a ‘slice’ of tissue perpendicular to the gradient

Pulse sequence with slice-selective excitation

By applying a magnetic field gradient during the RF pulse, we ensure that spins are excited only within a desired slice.

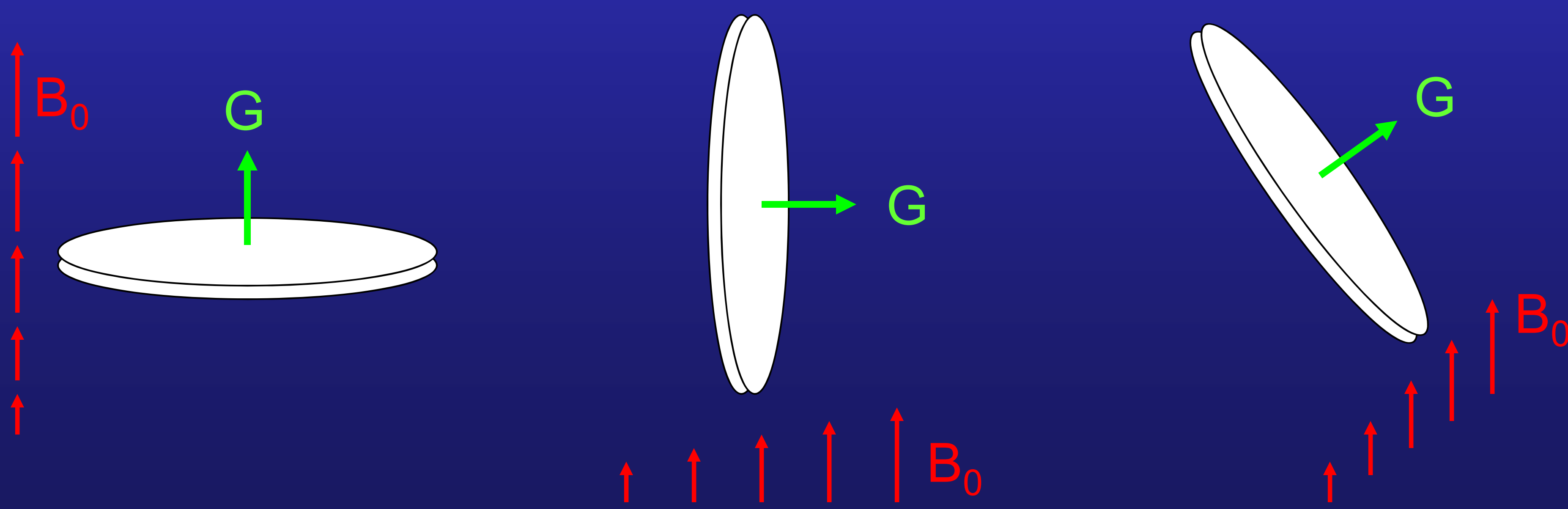


Prescribing the imaging slice

An imaging slice can be prescribed in any plane and with any thickness

Orientation of the slice

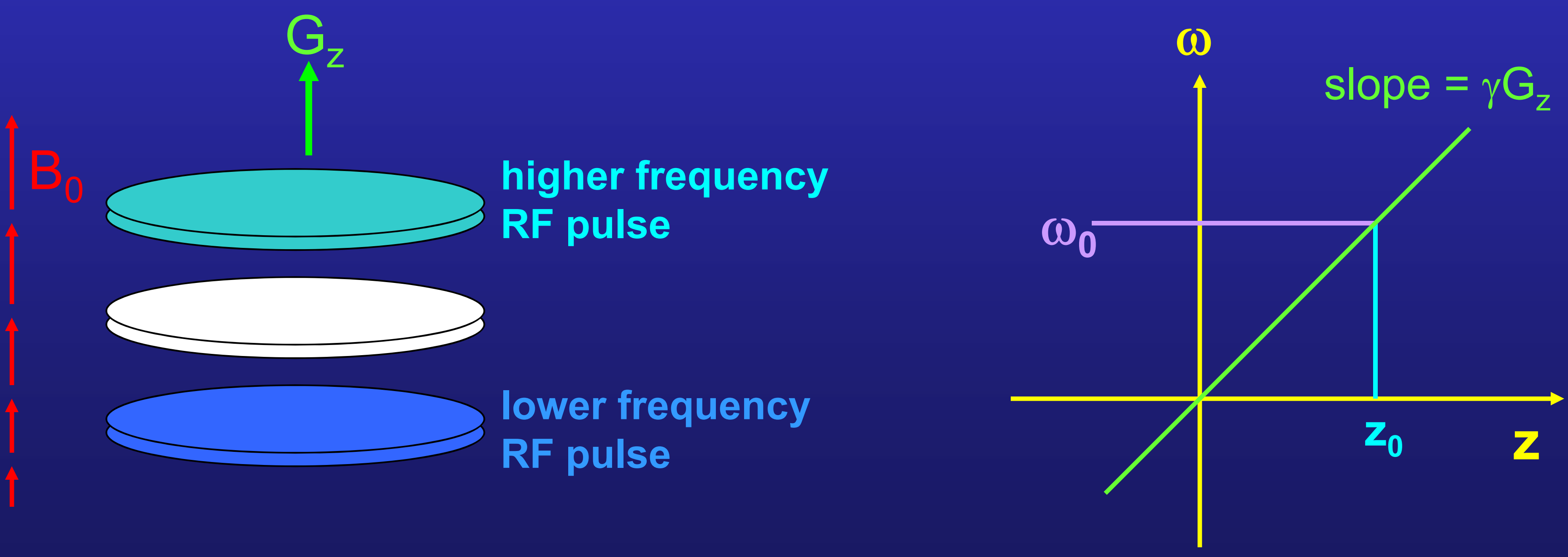
The plane of the slice is perpendicular to the magnetic field gradient. We can choose a gradient G in any direction (including oblique directions) to excite a slice of spins in any desired plane



axial slice **sagittal/coronal slice** **oblique slice**

Offset of the slice from isocenter

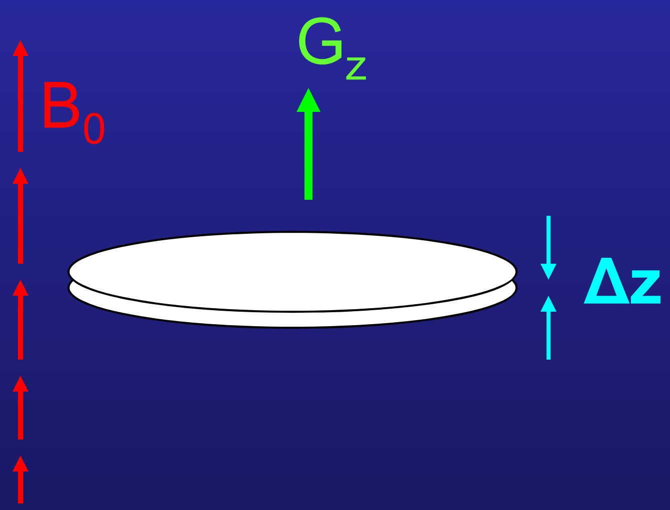
The location of the imaging plane along the direction of the slice-select gradient is controlled by the frequency of the RF pulse

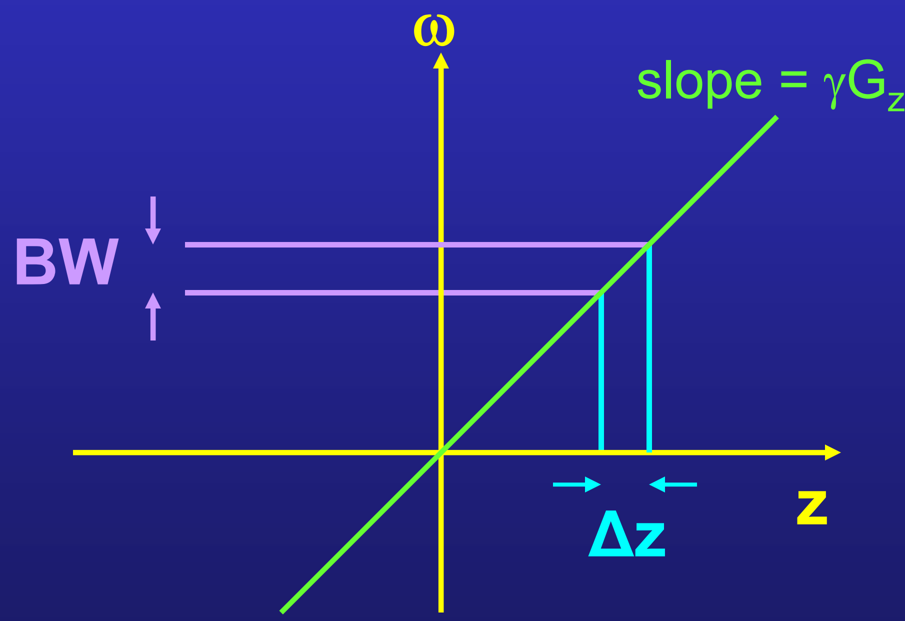


Slice thickness

In practice, any RF pulse of finite duration will have a range of frequencies, called its ‘bandwidth’ (BW).

The thickness of the slice depends on both the bandwidth and the gradient strength. It can be controlled by adjusting either of these parameters





Thin slices are used for 2D imaging, and thick slabs for 3D imaging

Spatial encoding

Following selective excitation, all the spins within a chosen slice are excited

The net signal will therefore consist of contributions from all the spins in that slice

To form an image we need a way to determine the ***spatial distribution*** of the signal within the slice

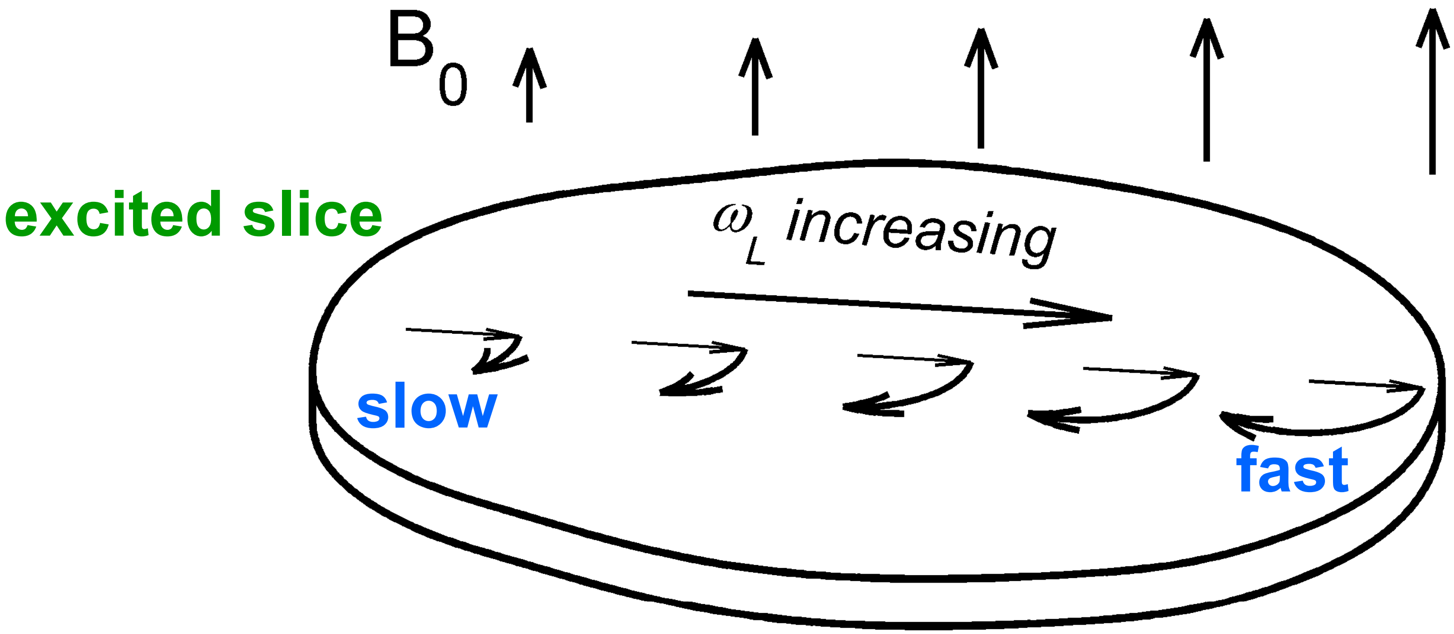
This is done by applying magnetic field gradients in the imaging plane ***after*** the RF excitation. The gradients make the excited spins at different points in the slice precess at different rates

This provides a way to encode spatial information into the signal

If a gradient is applied during the period of data acquisition, then the signal contributions from spins at different points along the direction of the gradient will have different frequencies

This is known as ***'frequency encoding'***, since the location of the spins is 'encoded' in the frequency of their emitted signals

Frequency encoding



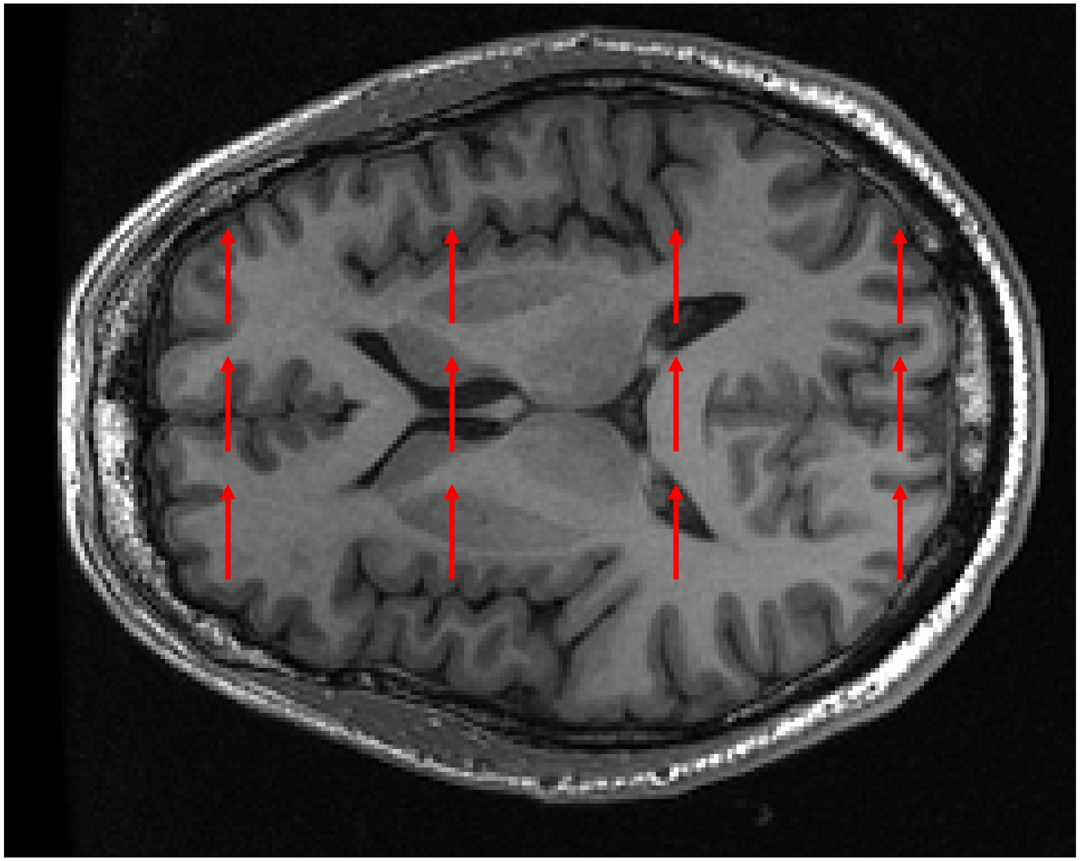
A magnetic field gradient is applied during the data acquisition

The gradient alters the Larmor frequency ω_L of the spins in a spatially-dependent manner

The frequency of the signal emitted by each spin will therefore depend on its location along the direction of the gradient

The frequency thus provides a ‘label’ to identify the spins’ location

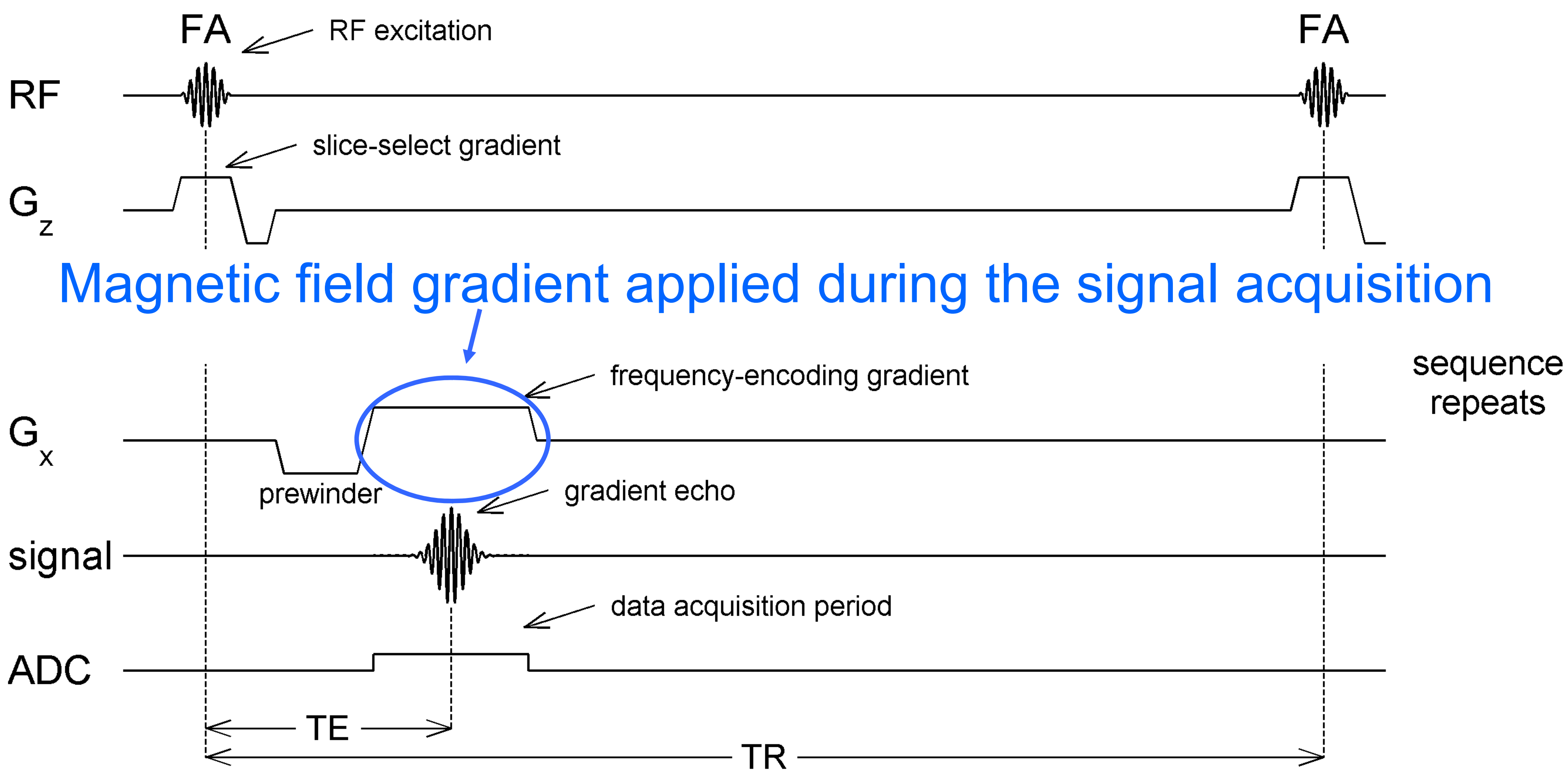
Frequency encoding



frequency-encoding direction

Pulse sequence with frequency encoding

By applying a magnetic field gradient during the signal acquisition we can distinguish the position of spins along one direction within the excited slice by their different frequencies



Decoding the spatial information

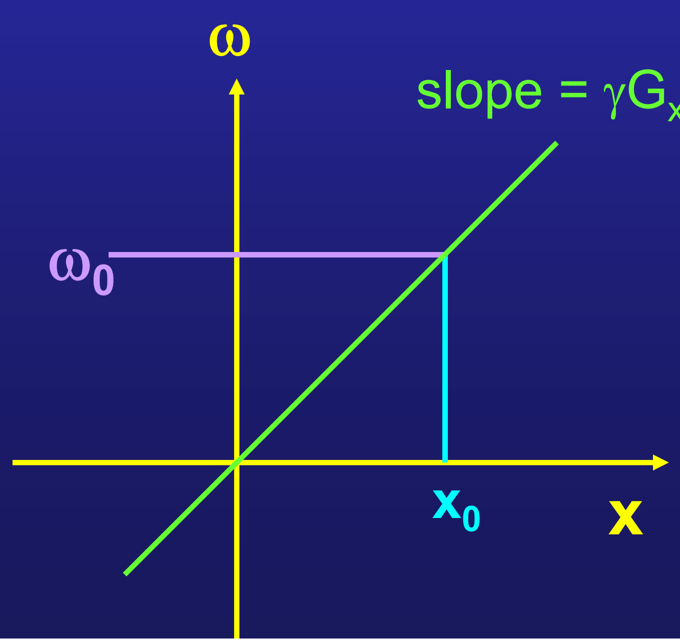
The frequency-encoding gradient makes the spins at different positions emit signals with different frequencies

The net signal therefore consists of a sum of contributions with different frequencies, each of which originates from a different position along the direction of the frequency-encoding gradient

In other words, the ***spatial distribution*** is given by the ***frequency spectrum*** of the signal

To calculate the spatial distribution we apply an inverse Fourier transform to the signal timecourse s(t)

$$\text{IFT}\{\, s(t) \,\} = S(\omega) = S(\gamma G_x x)$$



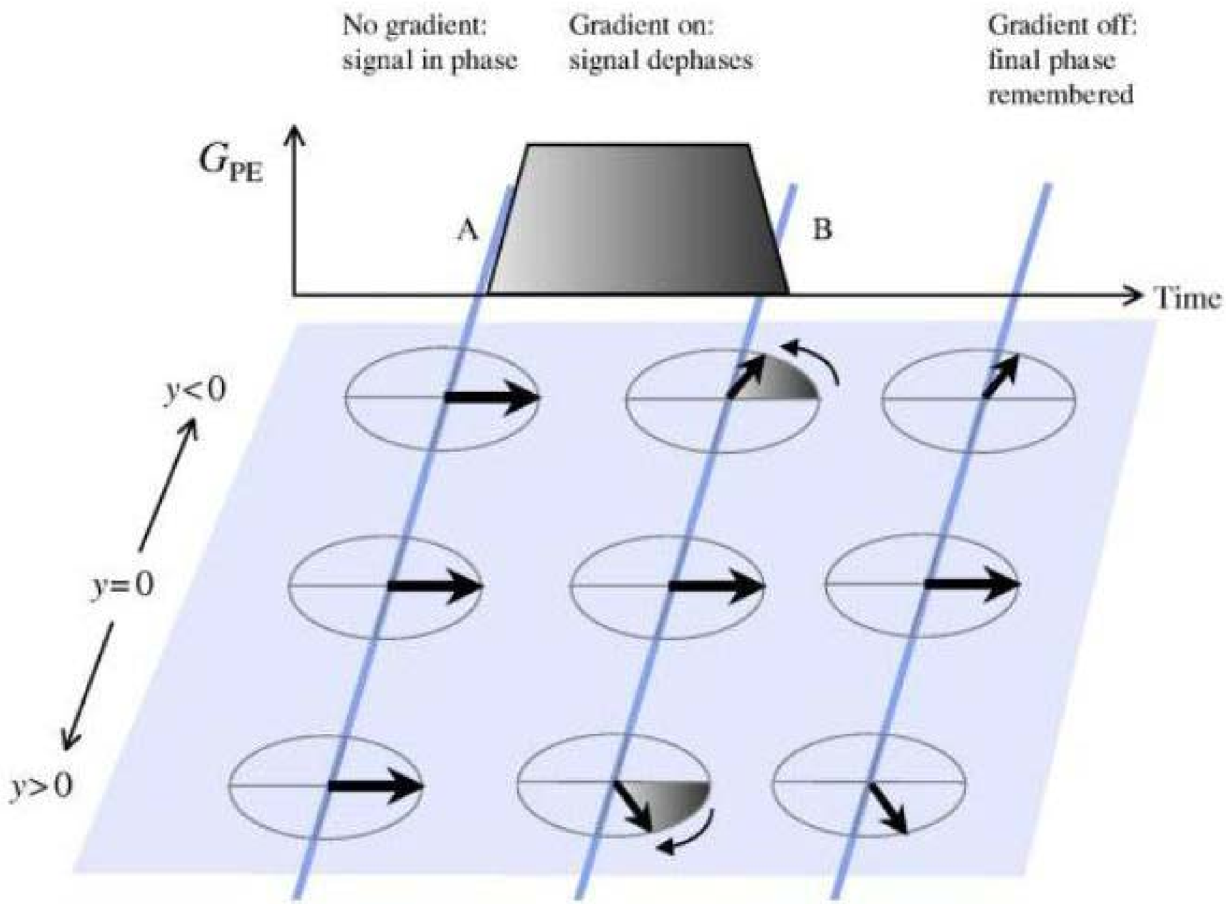
Spatial encoding in the remaining direction

Frequency encoding provides spatial information along only one direction (the direction of the frequency-encoding gradient)

To reconstruct an image, however, we need to determine the spatial distribution of the signal in two dimensions

There are various ways to do this, but the most commonly used is the mechanism of ***phase encoding***

Phase encoding is used in combination with frequency encoding to resolve the locations of the spins in both in-plane directions

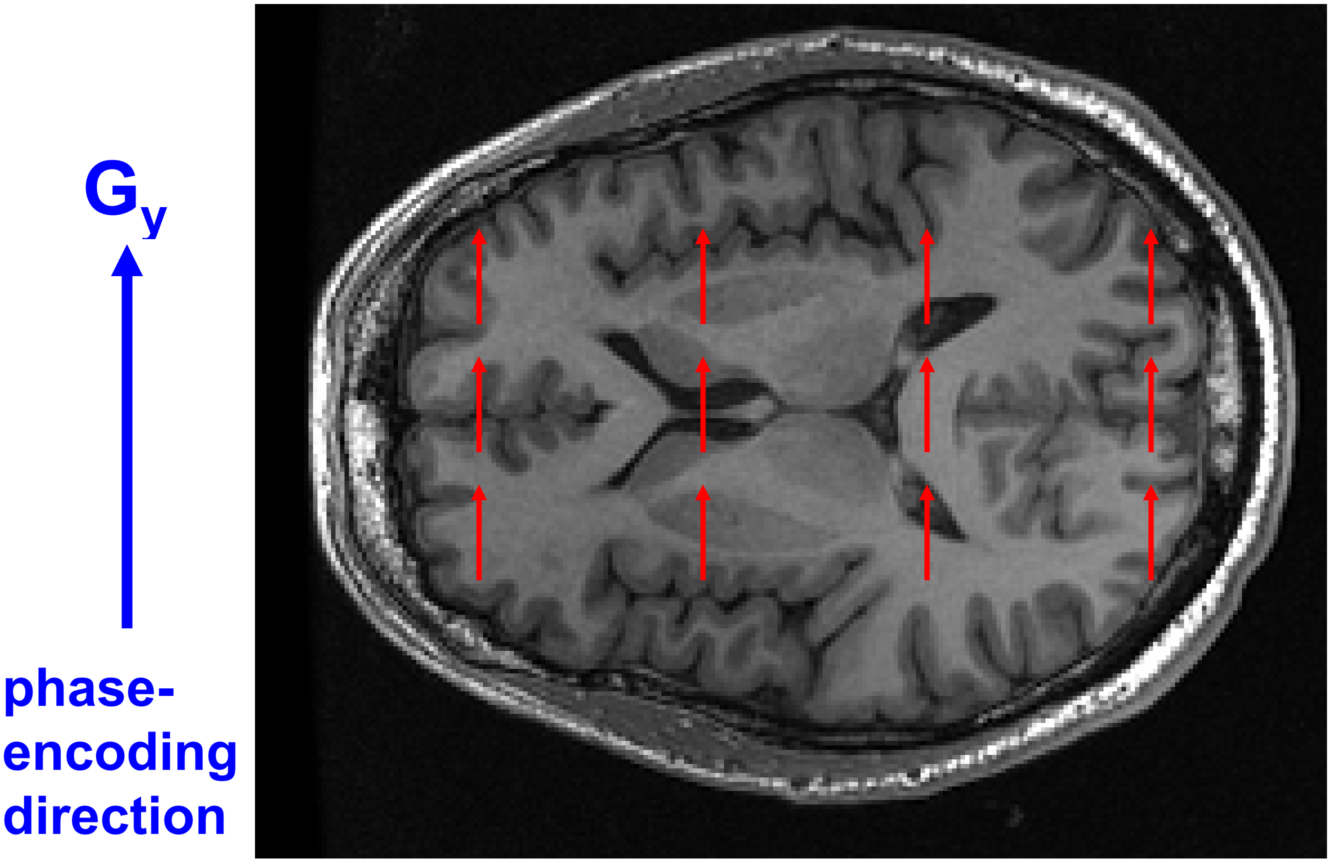


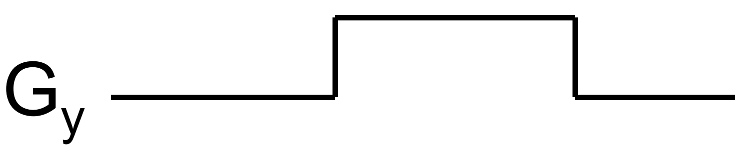
A magnetic field gradient is applied in the remaining direction for a short period ***after*** excitation but ***before*** data acquisition

The gradient imparts a spatially-varying phase shift to the spins

During the subsequent data acquisition period, the spins along any line in the phase-encoding direction will precess with identical frequencies but different phases

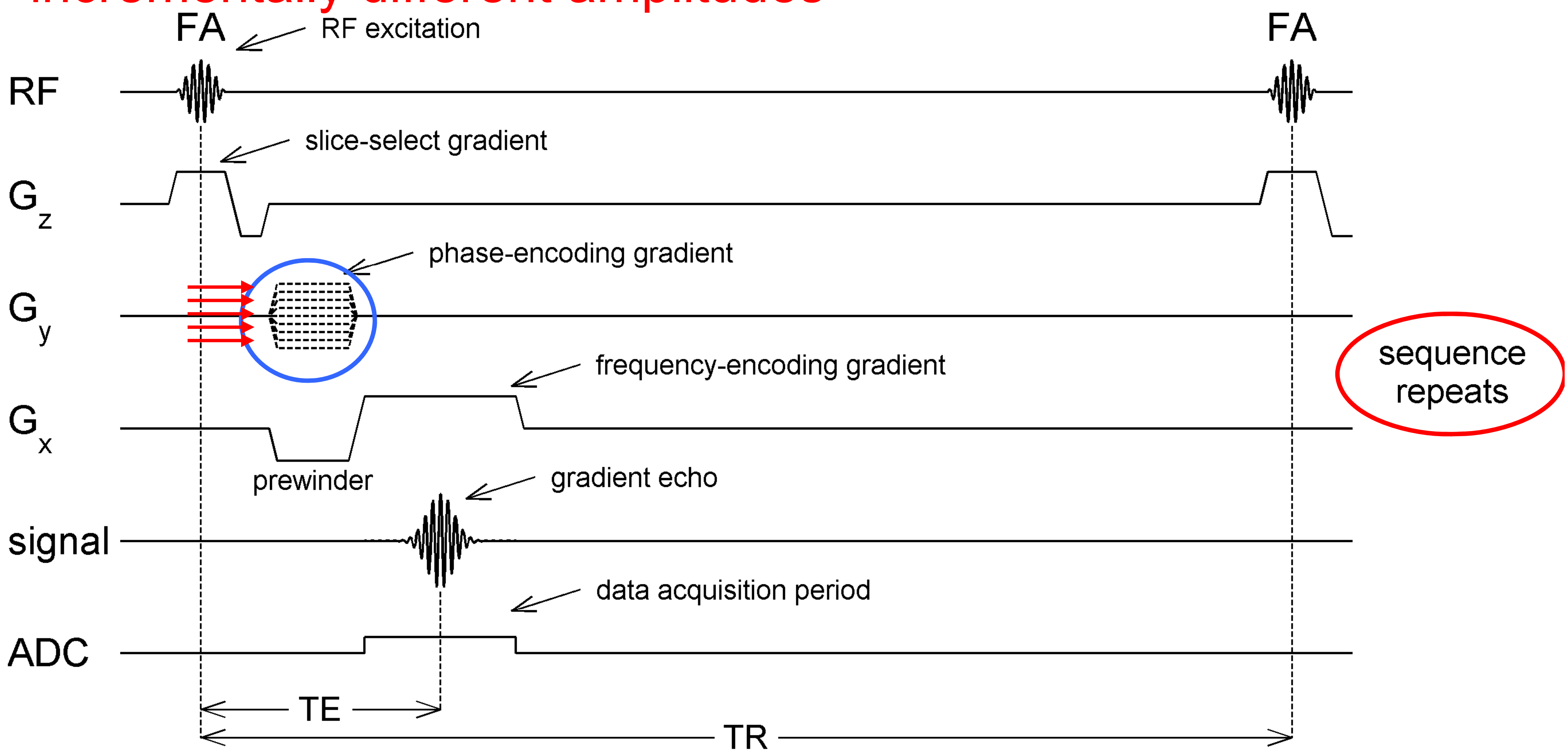
Phase encoding





Pulse sequence with phase encoding

The phase-encoding gradient is applied along the remaining direction (y) after RF excitation but before signal acquisition,
To determine the signal distribution along y we need to repeat the sequence many times with phase-encoding gradients of incrementally different amplitudes



Equivalence between phase and frequency encoding

The fact that the ***phase change*** with successive acquisitions determines the location of the spins makes phase encoding ***mathematically equivalent*** to frequency encoding.

Frequency specifies how the phase changes with time. So:

In ***frequency encoding***, the phase change from one instant of ***time*** to the next identifies the location of the source

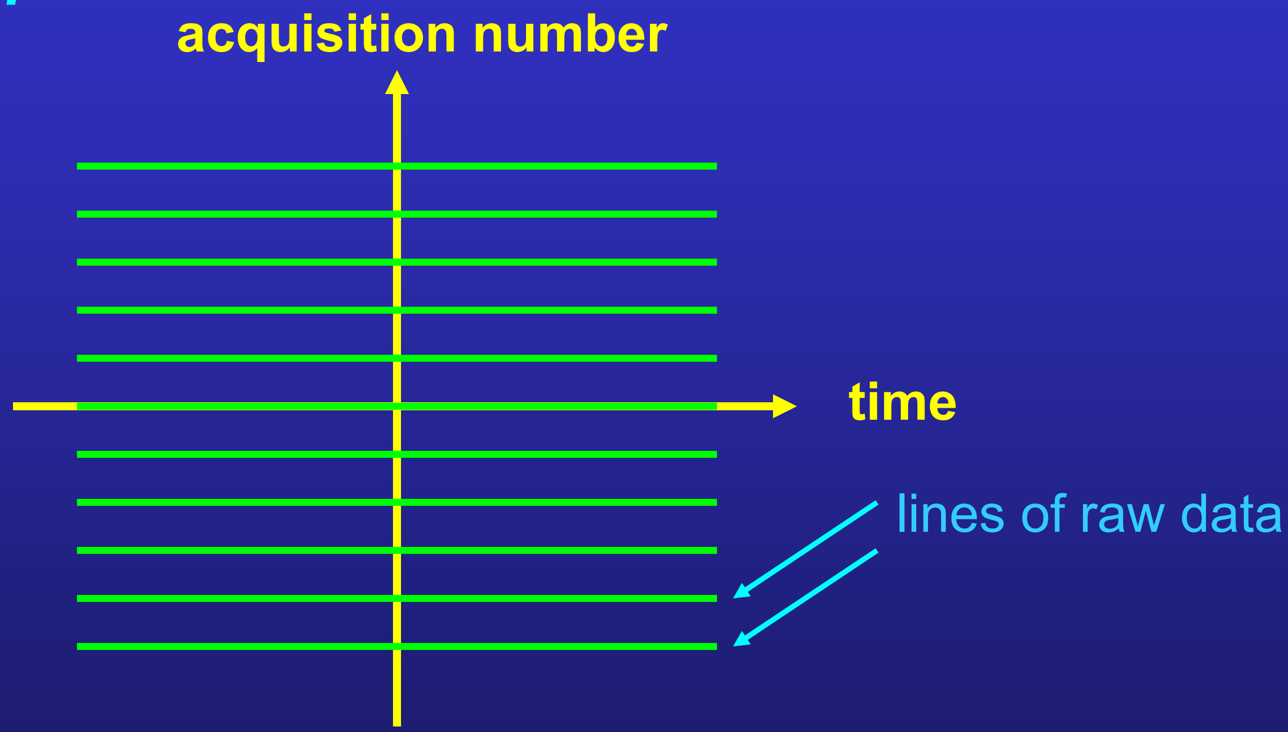
In ***phase encoding***, the phase change from one ***acquisition*** to the next identifies the location of the source

Just as an inverse Fourier transform of the signal over time s(t) gives the spatial distribution in the frequency-encoding direction, an inverse Fourier transform of the signal across successive acquisitions gives the distribution in the phase-encoding direction

It turns out that an inverse 2D Fourier transform of the entire data set gives the spatial distribution of the signal in two dimensions.

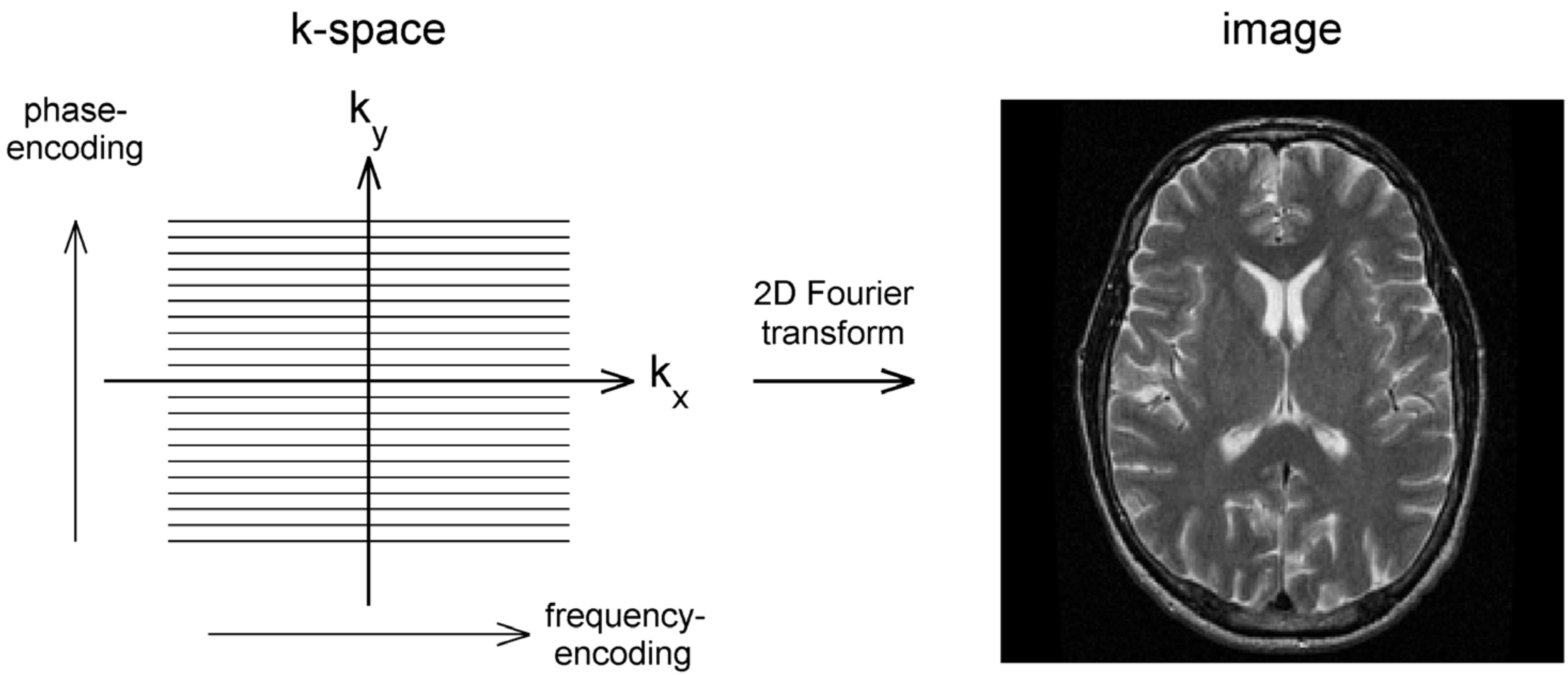
Image reconstruction in 2D

It is useful to think of the raw signal from each successive acquisition as being recorded as lines in a two dimensional space, which we call ***k-space***



A 2D inverse Fourier transform of this data then provides the spatial distribution of the signal. In general the signal distribution is complex-valued, and the magnitude is taken to produce the image

Image reconstruction

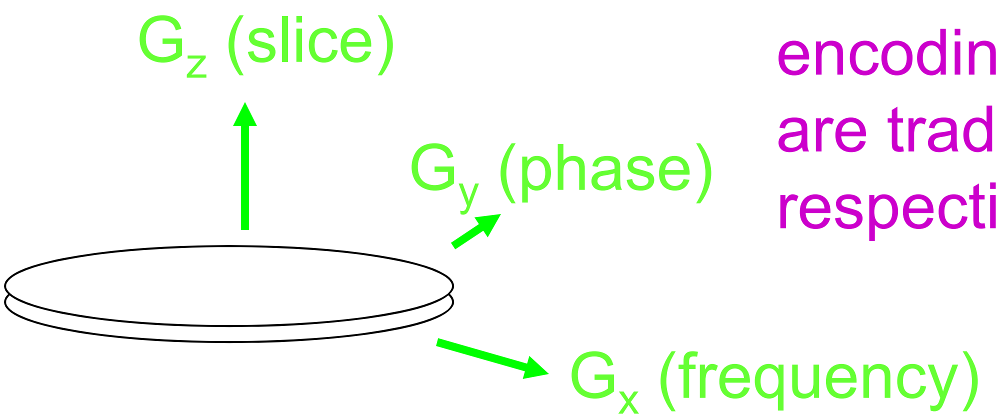


The relation between k_x and the time and between k_y and the acquisition number will be derived when we introduce the mathematical formalism

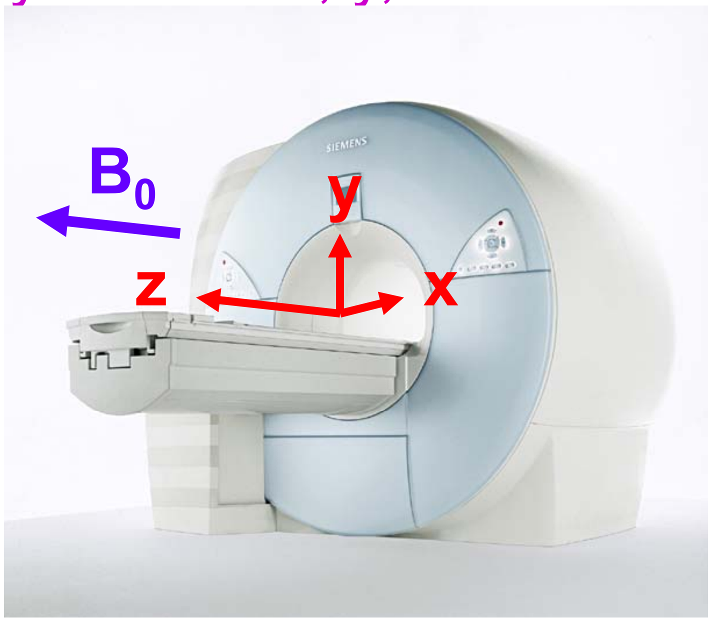
Summary so far…

Phase encoding and frequency encoding are used in combination to produce an image in two dimensions.

The same slice of tissue is excited repeatedly and the signal is sampled as a function of time after each excitation. The frequency-encoding gradient remains constant with each repetition, but the phase-encoding gradient is incremented from one run to the next



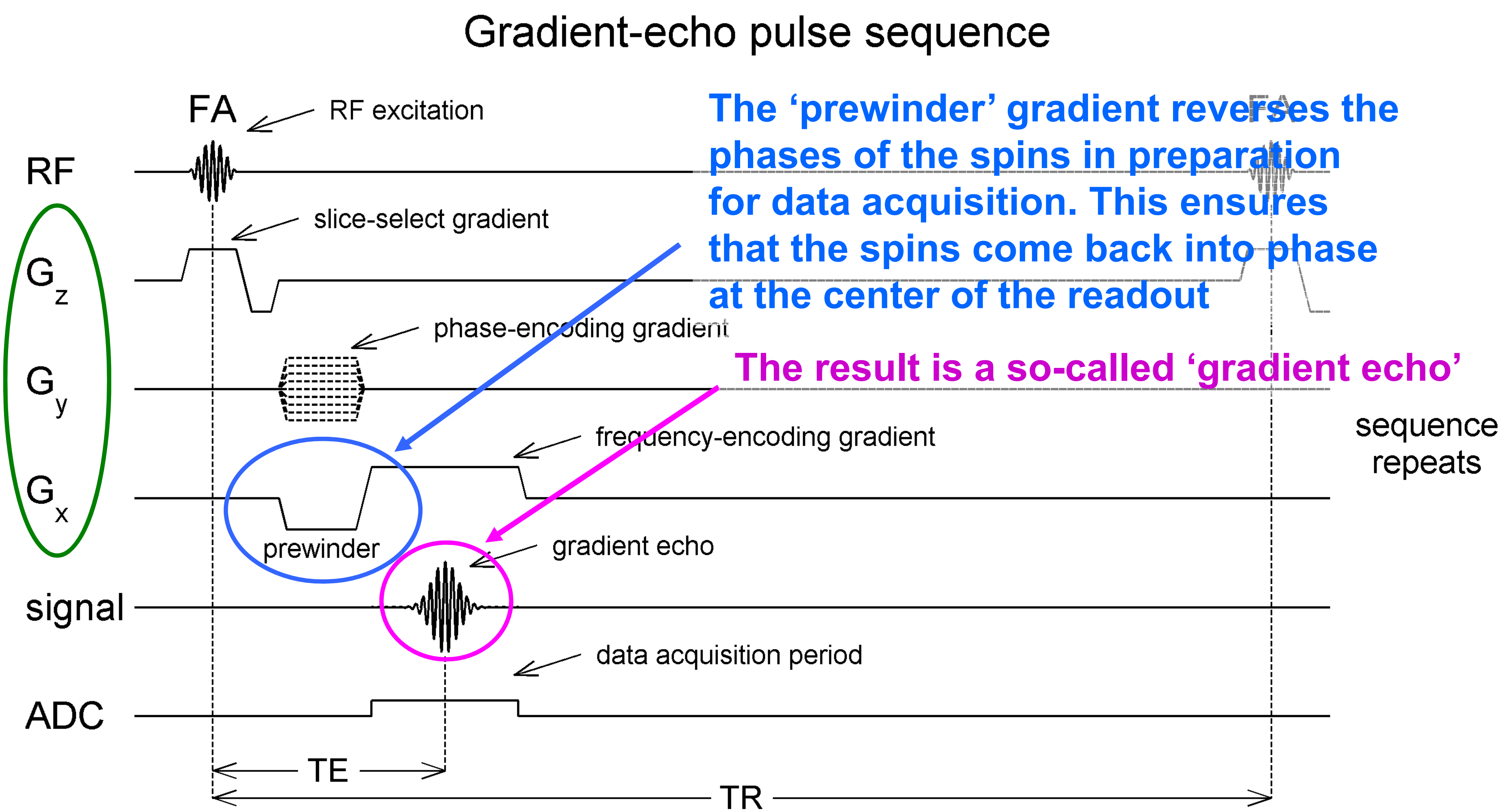
The frequency-encoding, phase-encoding and slice-select directions are traditionally labeled x, y, and z respectively



However this ‘logical’ coordinate system should not be confused with the ‘physical’ coordinate system of the scanner

When we sketch a pulse sequence, we often depict the gradients using the ‘logical coordinate system’

The sequence we have produced here is known as a ‘gradient echo’ sequence



Mathematical formalism

The mathematical description of frequency and phase encoding is simplified if we represent the x- and y-components of the transverse magnetization as real and imaginary parts of a complex number

$$M_{\perp} = M_x + iM_y$$

If the precession frequency ω_0 is constant the evolution of the transverse magnetization at a point **r** can then be written as

$$M_{\perp}(\mathbf{r},t) = M_{\perp}(\mathbf{r},0)e^{-t/T_2(\mathbf{r})}e^{-i\omega_0 t}$$

The processes of frequency and phase encoding however involve the application of magnetic field gradients, which make the precession frequency change in both space and time

$$\omega(\mathbf{r},t) = \omega_0 + \Delta\omega(\mathbf{r},t) \quad \text{where} \quad \Delta\omega(\mathbf{r},t) = \gamma\Delta B(\mathbf{r},t)$$

The transverse magnetization then evolves according to

$$M_{\perp}(\mathbf{r},t) = M_{\perp}(\mathbf{r},0)e^{-t/T_2(\mathbf{r})}e^{-i\omega_0 t}\exp\left(-i\int_0^t \Delta\omega(\mathbf{r},t')\mathrm{d}t'\right)$$

Measured signal

The measured signal s(t) is a sum of contributions S(x,y,t) from all points within the excited slice. The contribution from a given point depends on many factors, including the local transverse magnetization, the local coil sensitivity, and various constants, such as the slice thickness and the receiver gains

Since the signal s(t) we measure is demodulated (i.e. it has the fast oscillation at frequency ω_0 removed), it can be expressed as

$$s(t) = \int S(x,y,0)\, e^{-t/T_2(x,y)} \exp\left(-i\int_0^t \Delta\omega(x,y,t')\,\mathrm{d}t'\right)\mathrm{d}x\,\mathrm{d}y$$

If we define

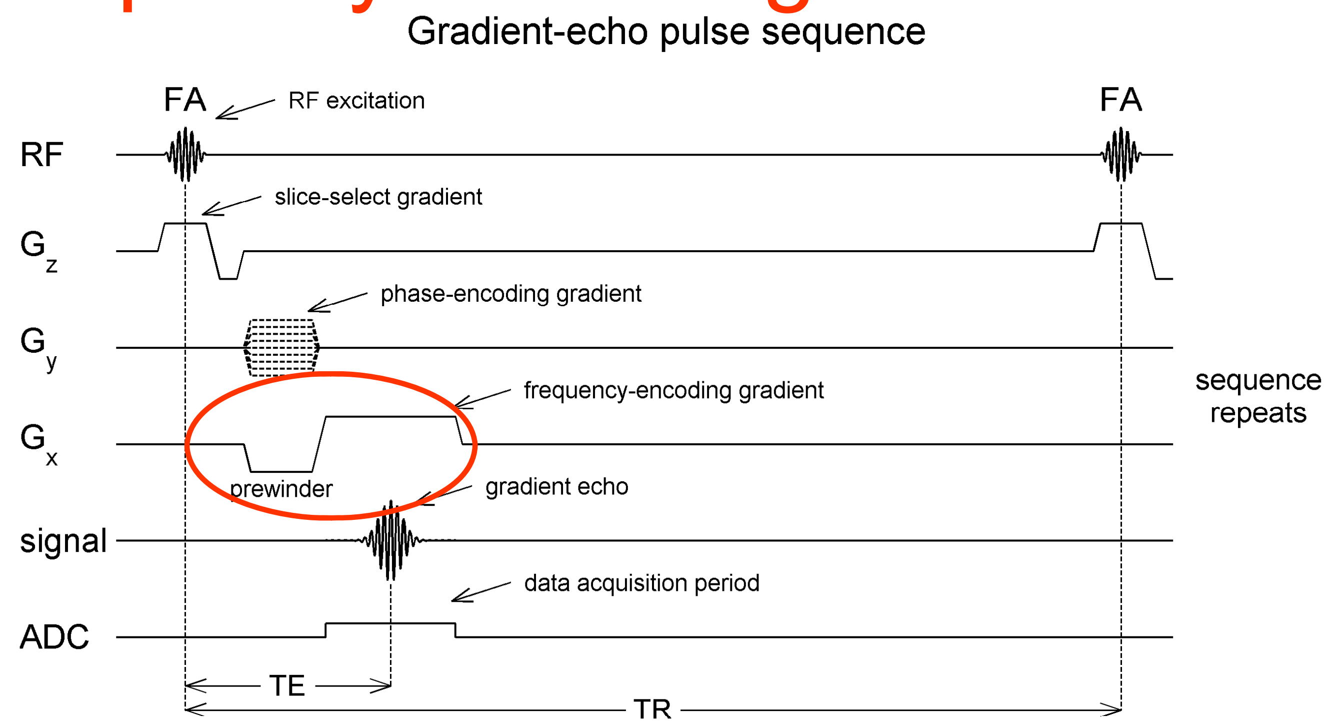
$$S(x,y) = S(x,y,0)\, e^{-\mathrm{TE}/T_2(x,y)}$$

where TE is the center of the acquisition, and if we neglect the effects of relaxation ***during*** the acquisition period, we obtain

$$s(t) = \int S(x,y) \exp\left(-i\int_0^t \Delta\omega(x,y,t')\,\mathrm{d}t'\right)\mathrm{d}x\,\mathrm{d}y$$

Frequency-encoding

In frequency encoding, a gradient is applied in the x direction



If we neglect other sources of frequency offset, such as chemical shift and magnetic susceptibility differences, the accumulated phase (including the effects of the 'prewinder') can be written as

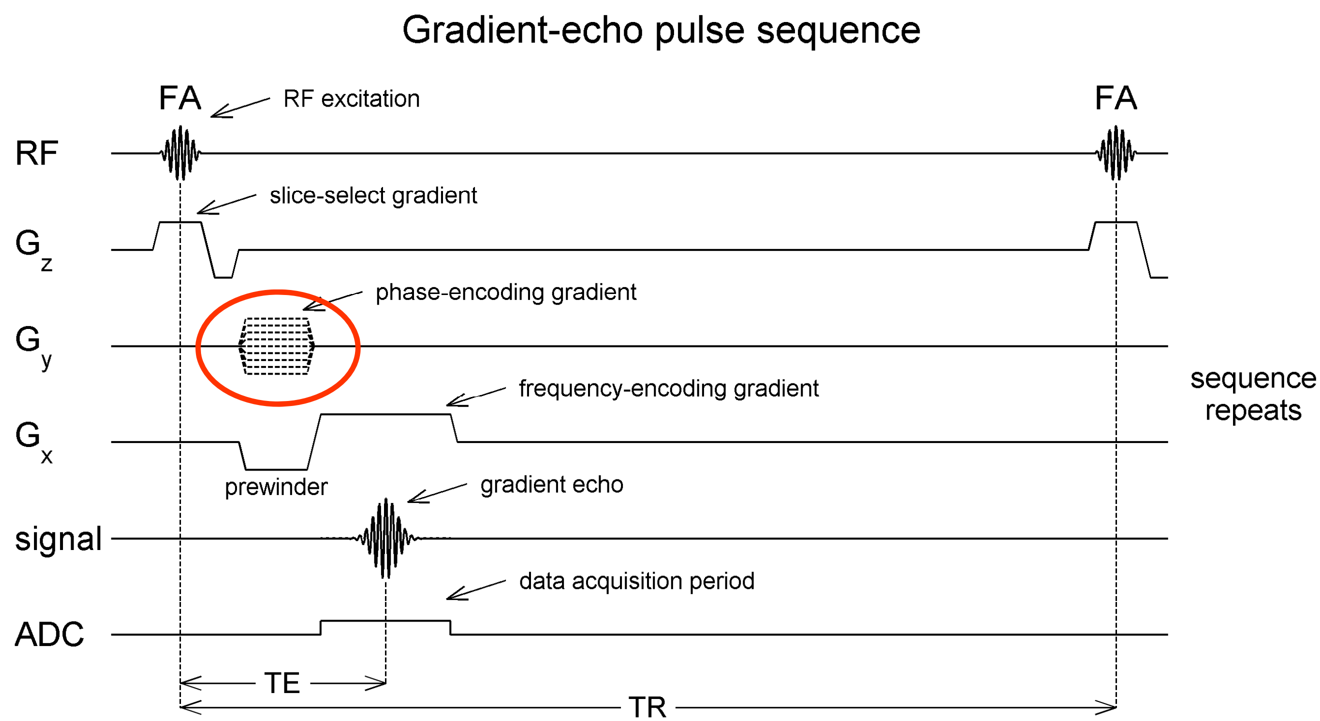
$$\int_0^t \Delta\omega(\mathbf{r},t')\mathrm{d}t' = \gamma G_x x(t-\mathrm{TE}) = 2\pi k_x(t)x$$

where

$$k_x(t) = \frac{\gamma}{2\pi} G_x(t-\mathrm{TE})$$

Phase-encoding

In phase encoding, a gradient is applied in the y direction prior to data acquisition



The gradient varies from run to run, and produces a phase

$$\int_0^t \Delta\omega(\mathbf{r},t)\mathrm{d}t = \gamma G_y^{(n)} y \tau = 2\pi k_y^{(n)} y \quad \text{where} \quad k_y^{(n)} = \frac{\gamma}{2\pi} G_y^{(n)} \tau \ ,$$

$G_y^{(n)}$ is the gradient during the nth run, and τ is the duration of the gradient pulse

k-space and the raw signal

The raw signal from the nth repetition can therefore be written as

$$s_n(t) = s\left(k_x(t), k_y^{(n)}\right)$$

where $s(k_x, k_y) = \int S(x, y) \exp\left[-i2\pi\left(k_x x + k_y y\right)\right] \mathrm{d}x\mathrm{d}y$

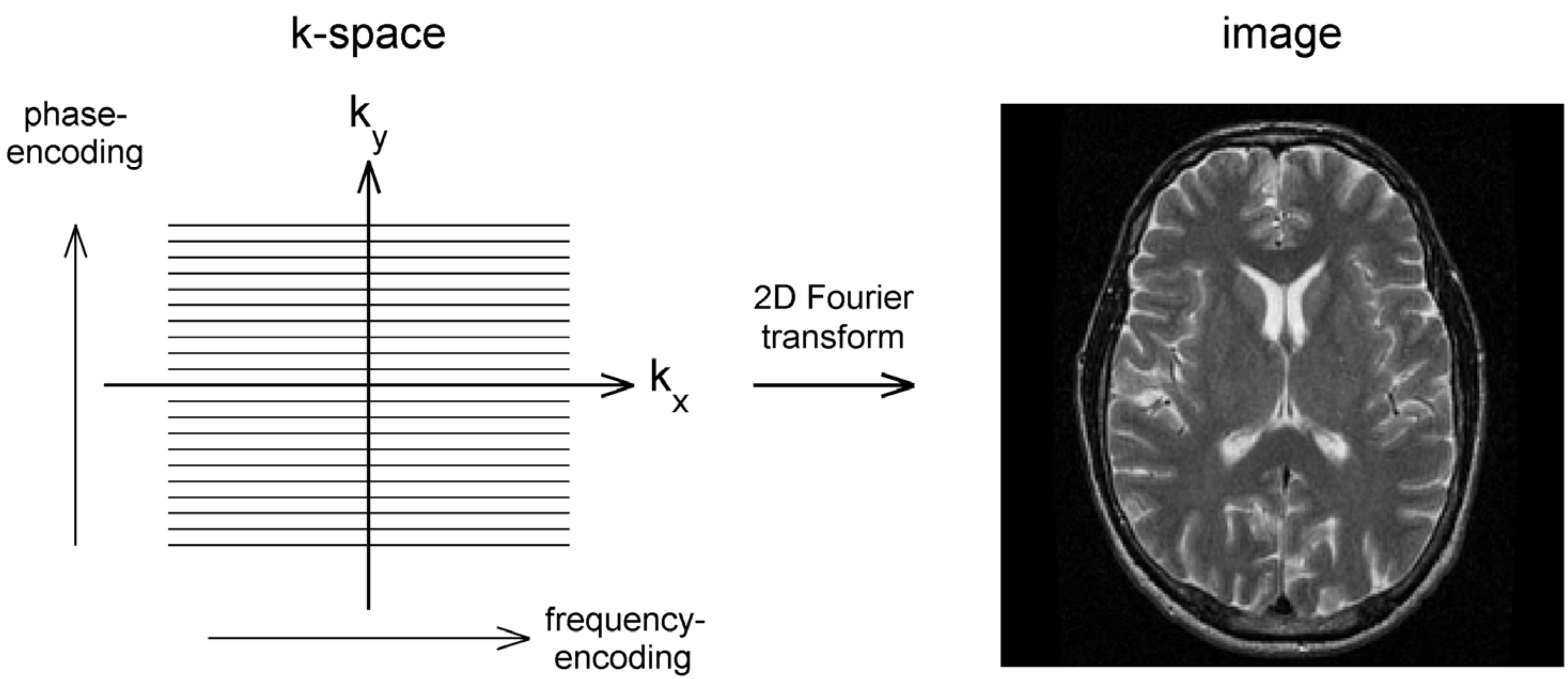
We can therefore interpret the signal from the nth repetition as sampling a line of data in k-space, where k-space is the Fourier domain of the image space.

Once we have sampled sufficient data in k-space, we can perform a 2D inverse Fourier transform to recover the signal distribution in image space

$$S(x, y) = \int s(k_x, k_y) \exp\left[i2\pi\left(k_x x + k_y y\right)\right] \mathrm{d}k_x \mathrm{d}k_y$$

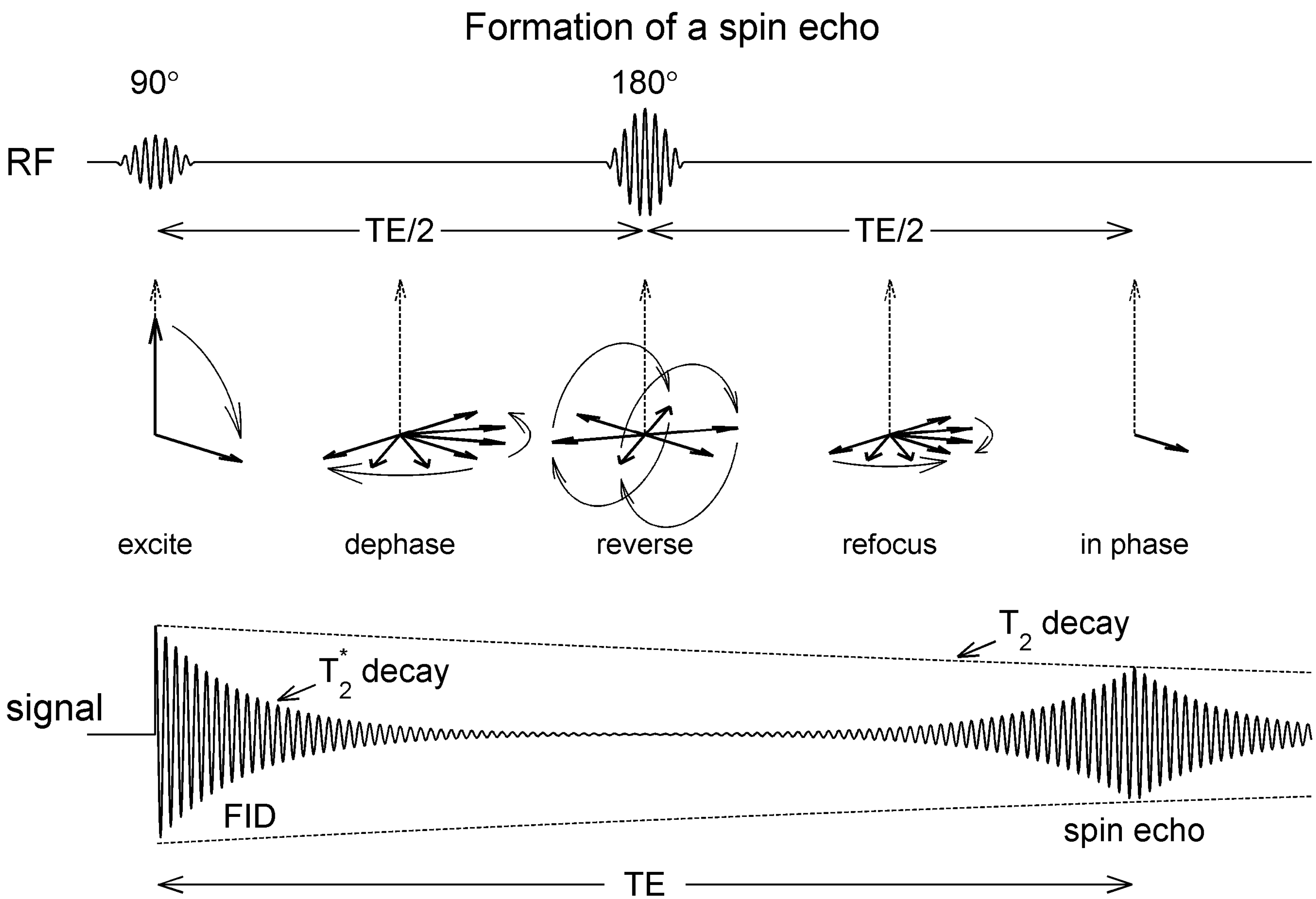
The concept of k-space is a useful one for many reasons. Among others, it allows us to understand other sampling strategies besides conventional frequency and phase encoding

Image reconstruction

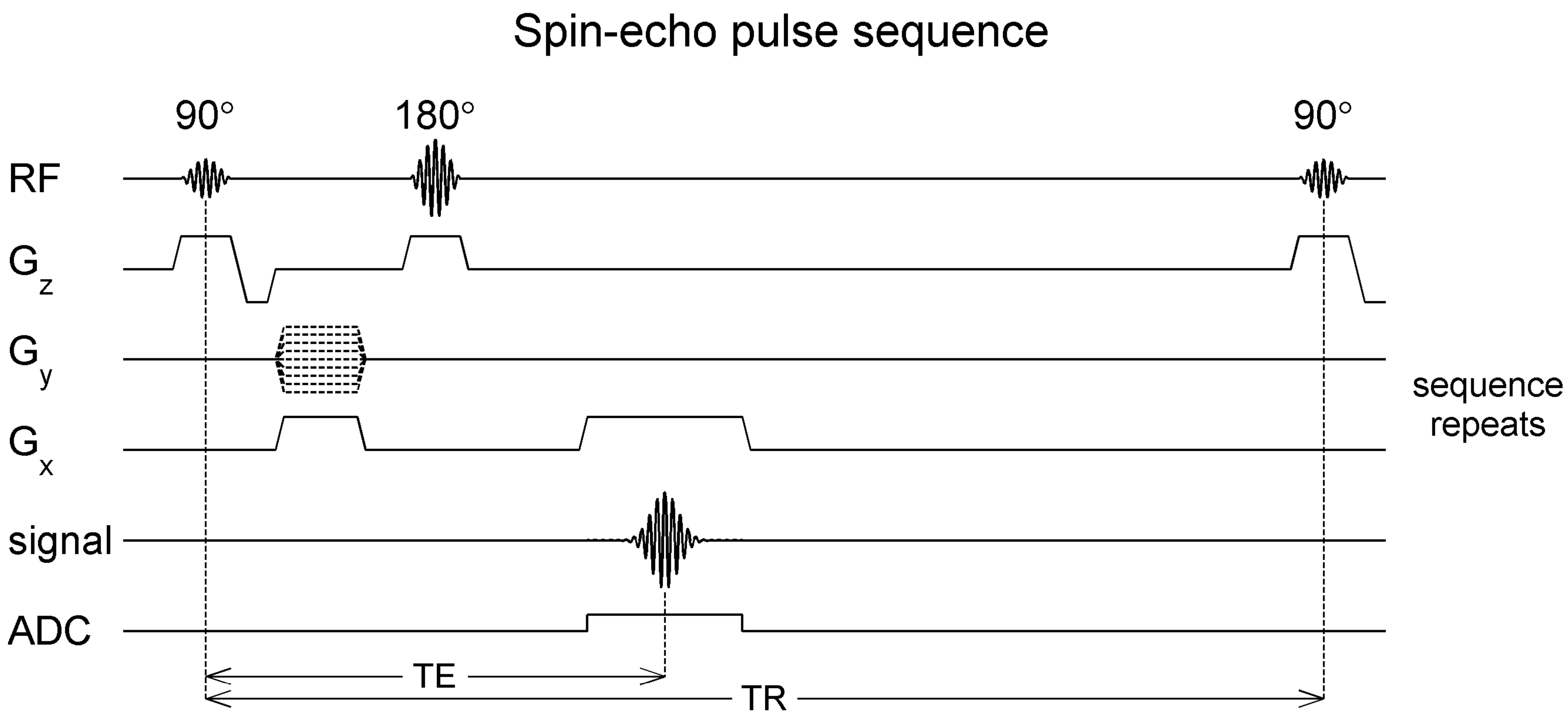


During the next lecture we will discuss in more detail how the sampling of k-space affects the field of view and resolution of the image

The same imaging strategies can be applied to other types of pulse sequence. For example, we can add imaging gradients to the spin echo sequence below:



The result is a spin-echo imaging sequence.



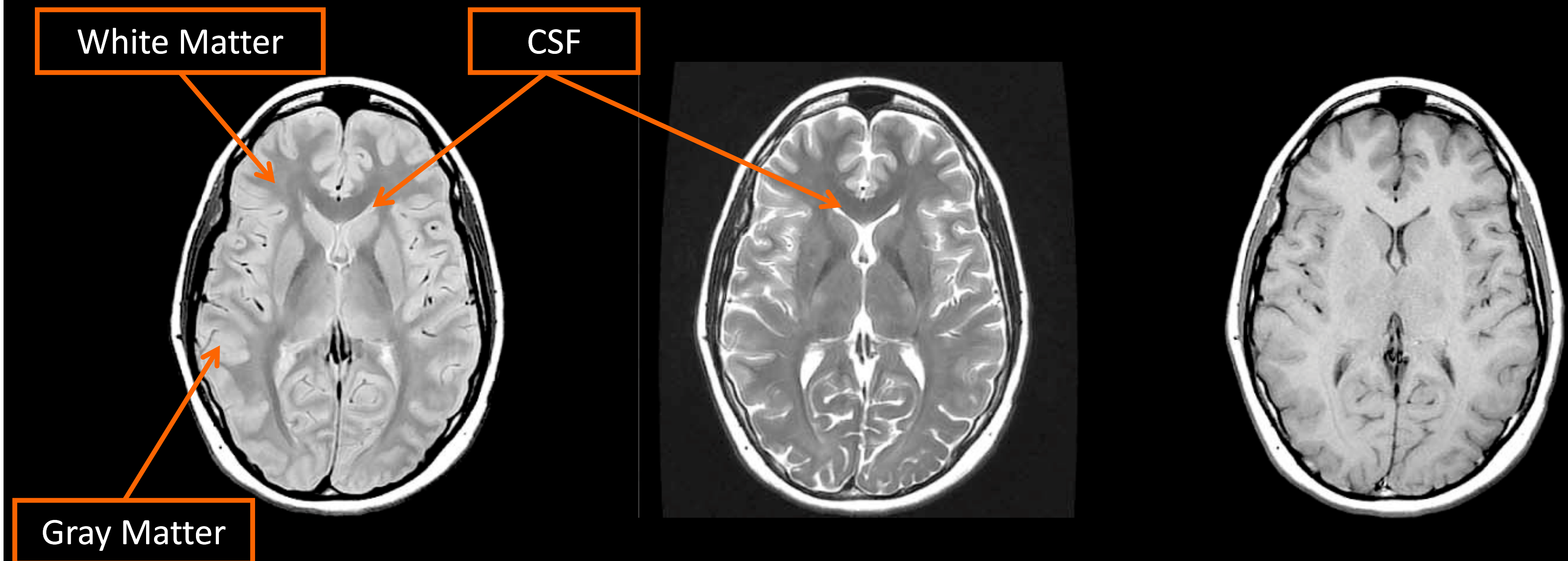
With appropriate choices of TE and TR we can use this sequence to produce T2- and T1-weighted images.

Recap of T_1, T_2, PD Weighting

- Different tissues vary in T_1, T_2 and PD (proton density)
- The pulse sequence parameters can be designed so that the captured signal magnitude is mainly influenced by one of these parameters
- Pulse sequence parameters
  - Tip angle
  - Echo time TE
  - Pulse repetition time TR

T_1, T_2, PD Weighting: Example

	PD	T_2 (ms)	T_1 (ms)
White Matter	0.61	67	510
Gray Matter	0.69	77	760
CSF	1.00	280	2650



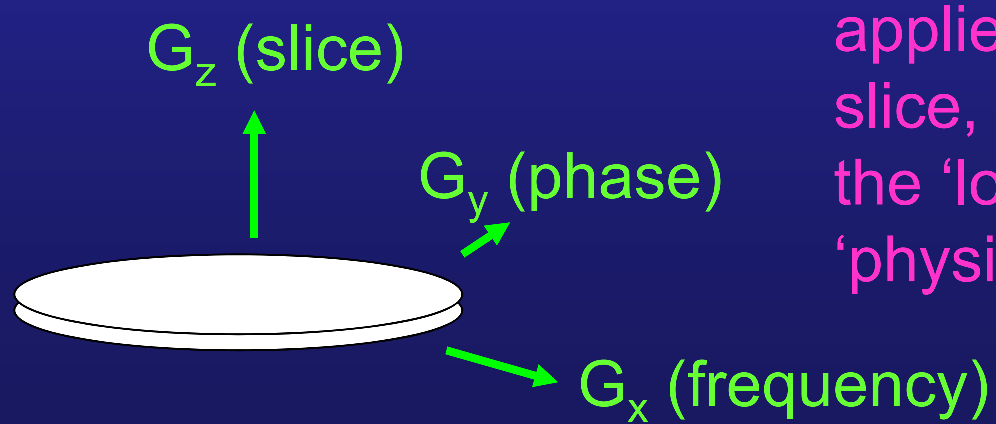
PD-weighted T_2-weighted T_1-weighted

Recap of spatial encoding

To produce an image we first excite a single slice of tissue using a slice selective excitation.

The detected signal is then a sum of contributions from ***all*** the spins in the slice, and to reconstruct an image we need to identify the contributions from each point.

This is achieved by encoding spatial information into the signal, by means of magnetic field gradients



The spatial encoding gradients are applied in the plane of the imaging slice, i.e. the x and y directions in the ‘logical’ (as opposed to ‘physical’) coordinate system

Recap of mathematical formalism

The gradients alter the precession frequency of the spins in a spatially dependent manner.

As time advances, the spins therefore accumulate a spatially dependent phase offset, and the net signal from all the points in the slice can be approximated by

$$s(t) = \int S(x,y) \exp\left(-i \int_0^t \Delta\omega(x,y,t')\,\mathrm{d}t'\right) \mathrm{d}x\,\mathrm{d}y$$

where S(x,y) is the magnitude of the signal contribution from point (x,y) at time TE (the center of the data acquisition period)

$$S(x,y) = S(x,y,0)\,e^{-\mathrm{TE}/T_2(x,y)}$$

This takes into account relaxation that has occurred between the excitation and the center of the readout. However we have neglected relaxation that occurs between the beginning and the end of the readout.

Recap of assumptions

The technique of spatial encoding however relies on a second assumption, namely that:

The only source of frequency offset is the imaging gradients.

Under this assumption, we can write

$$s(t) = \int S(x, y) \exp\left(-i\gamma \int_0^t \mathbf{G}(t') \cdot \mathbf{r} \, \mathrm{d}t' \right) \mathrm{d}x \mathrm{d}y$$

where **r** and **G** are the position and gradients written in vector form, i.e. (x,y) and (G_x, G_y) respectively

This assumption ignores effects of chemical shift and magnetic susceptibility differences. When those effects are present, they cause misregistration in the image, as was discussed previously

K-space

The previous equation can be expressed in the form

$$s(t) = \int S(x, y)\exp\left[-i2\pi\left(k_x(t)x + k_y(t)y\right)\right]\mathrm{d}x\mathrm{d}y$$

where

$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(t')\mathrm{d}t'$$

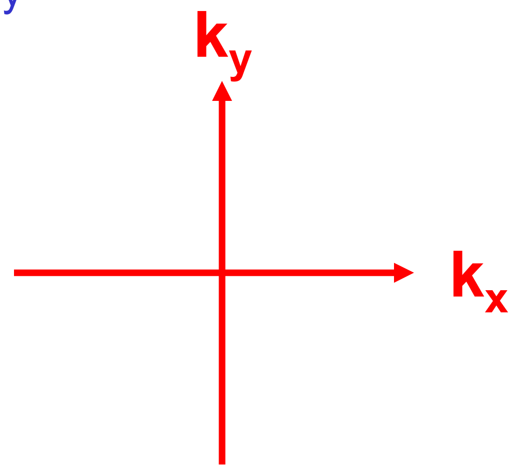
$$k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(t')\mathrm{d}t'$$

Note that the top equation has the form of a Fourier transform.

We can therefore interpret the signal acquisition as sampling data in a space known as ‘k-space’, which is the Fourier domain of physical space

Sampling k-space

We can think of k-space as a two dimensional space with coordinates k_x and k_y.



The gradients determine a trajectory ($k_x(t)$,$k_y(t)$) through this space, according to the earlier equations

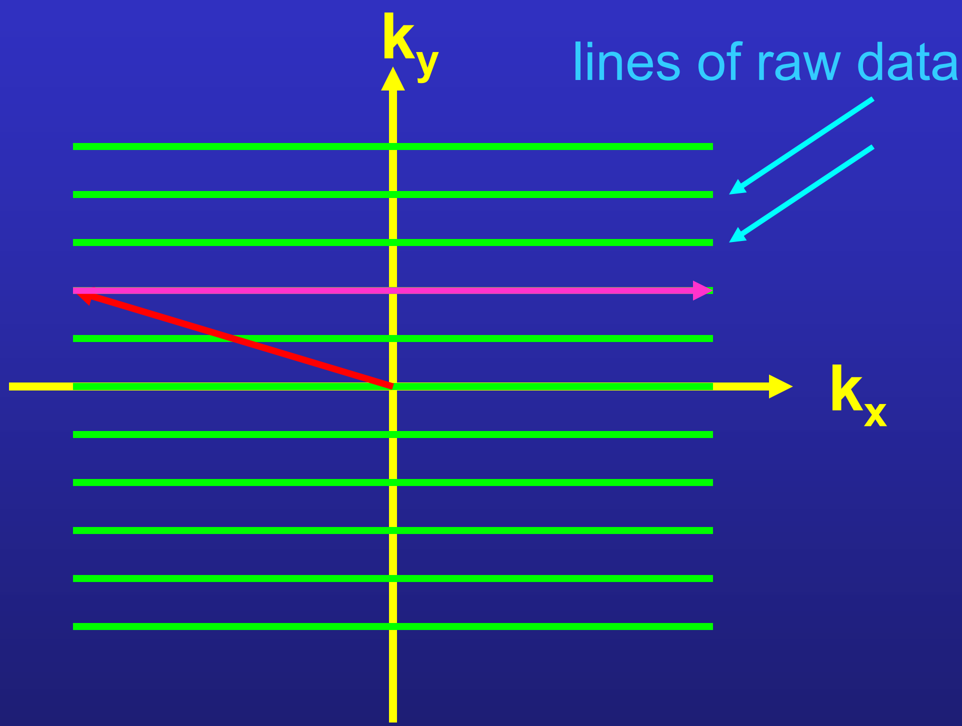
$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(t')\mathrm{d}t' \qquad k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(t')\mathrm{d}t'$$

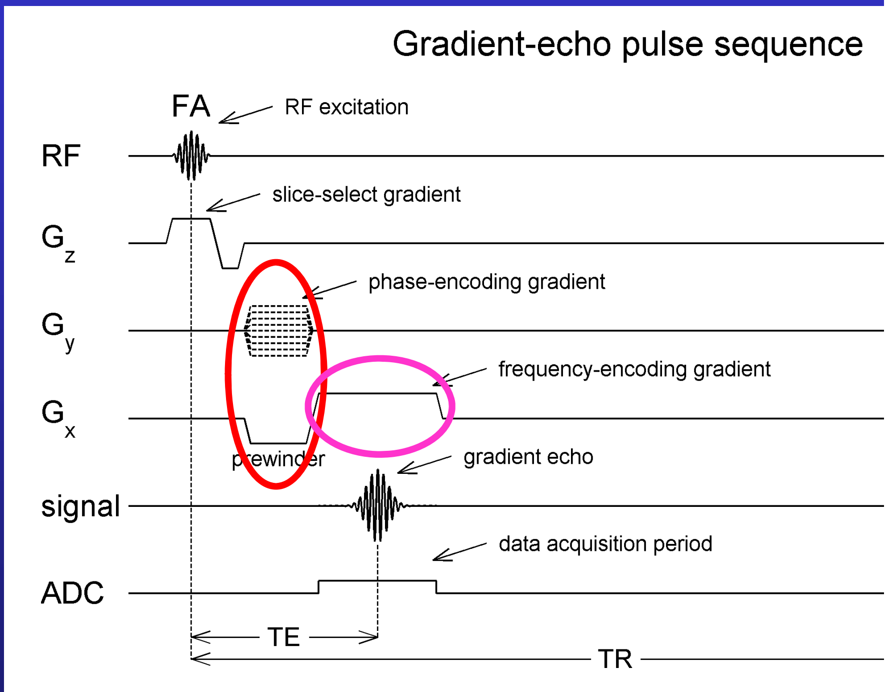
At points along the trajectory we sample the value of

$$s(k_x, k_y) = \int S(x, y)\exp\left[-i2\pi\left(k_x x + k_y y\right)\right]\mathrm{d}x\mathrm{d}y$$

Cartesian sampling

The most commonly used spatial encoding method is frequency- and phase-encoding. This corresponds to sampling lines in k-space. During each repetition (TR period) we sample a new line.



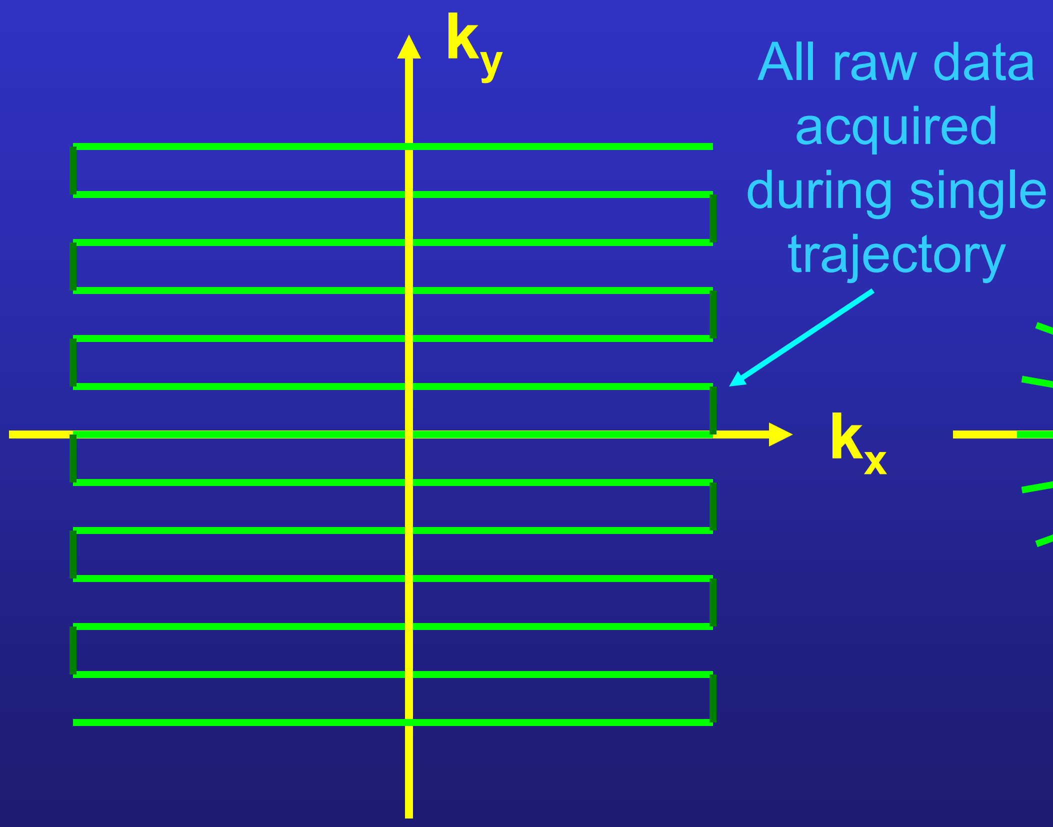


The phase-encoding gradient G_y and the prewinder of the frequency-encoding gradient G_x take us to the beginning of the line

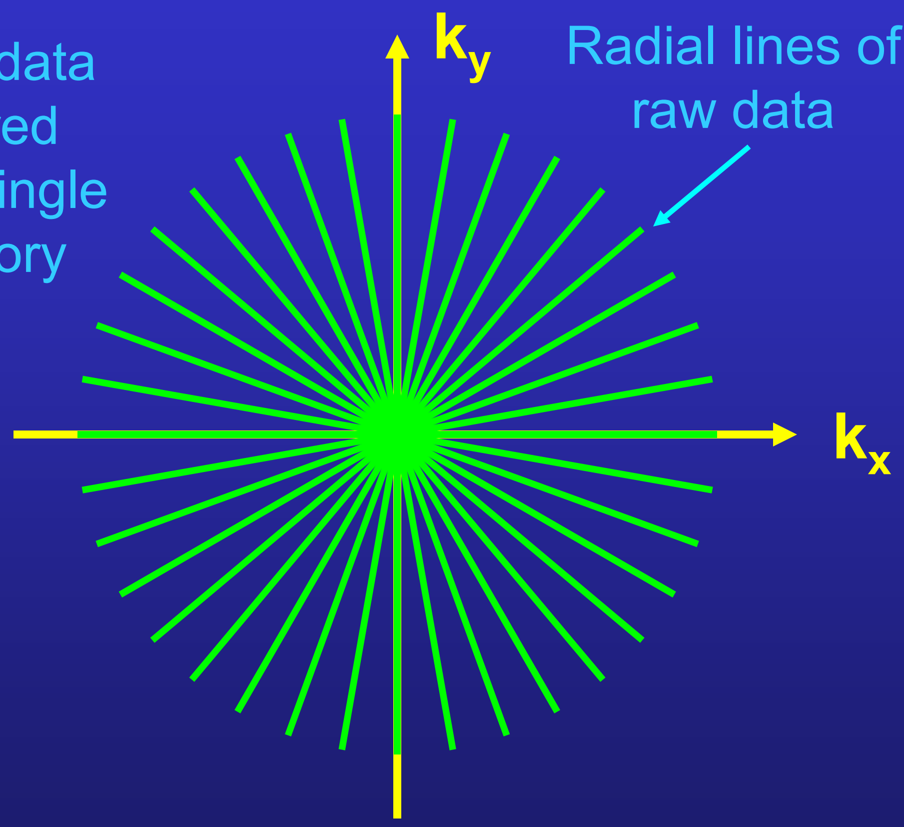
During the readout period only G_x is applied and we move along the line, acquiring data along the way

Alternate sampling schemes

Some of the power of the k-space concept lies in the fact that it helps us to understand other sampling schemes very easily



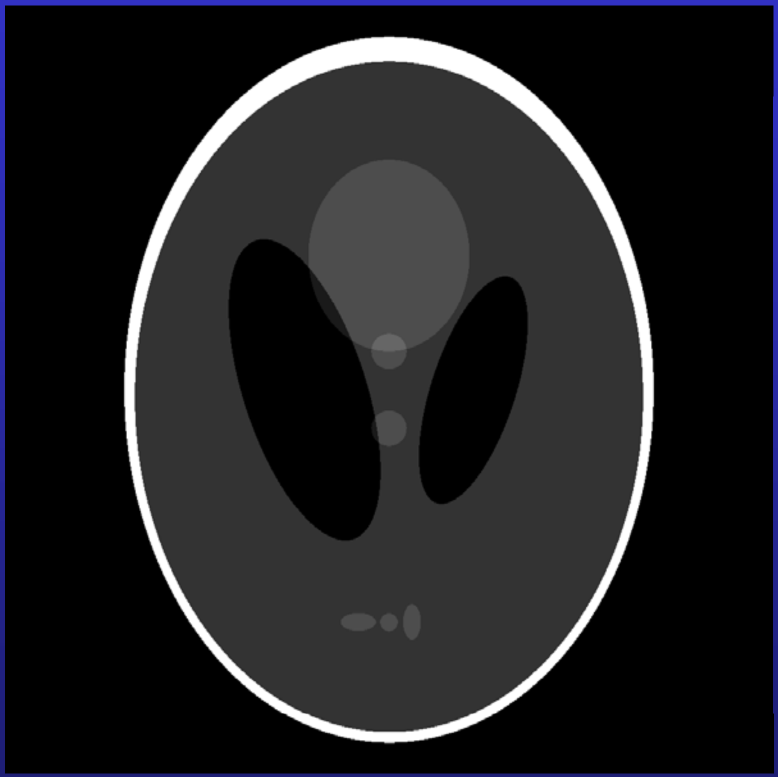
Echo planar imaging



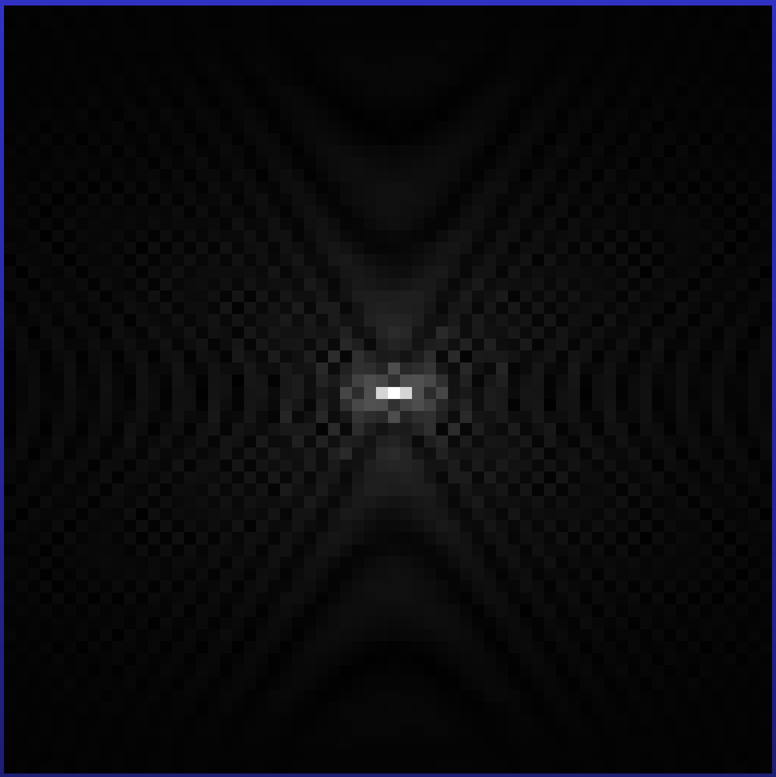
Radial sampling

Data acquisition

Phantom
in physical space



Raw data
in k-space



64x64

(absolute value of data,
windowed by factor of 2)

Note that the data values are highest near the center of k-space

Recovering the spatial distribution

The k-space data $s(k_x,k_y)$ is related to the spatial distribution of the signal S(x,y) by a 2D Fourier transform

$$s(k_x,k_y)=\int S(x,y)\exp\left[-i2\pi\left(k_x x+k_y y\right)\right]\mathrm{d}x\mathrm{d}y$$

The spatial distribution S(x,y) can therefore be recovered from the k-space data by means of an inverse 2D Fourier transform

$$S(x,y)=\int s(k_x,k_y)\exp\left[i2\pi\left(k_x x+k_y y\right)\right]\mathrm{d}k_x\mathrm{d}k_y$$

Since the signal S(x,y) is related to the transverse magnetization it has two quadrature components. And as in the case of the transverse magnetization, the components are typically represented as real and imaginary parts of a complex number.

In most cases we are interested only in the ***magnitude*** of the signal, and not its phase. The images typically displayed to the MRI console are ***magnitude images***. However it is possible to display the phase information.

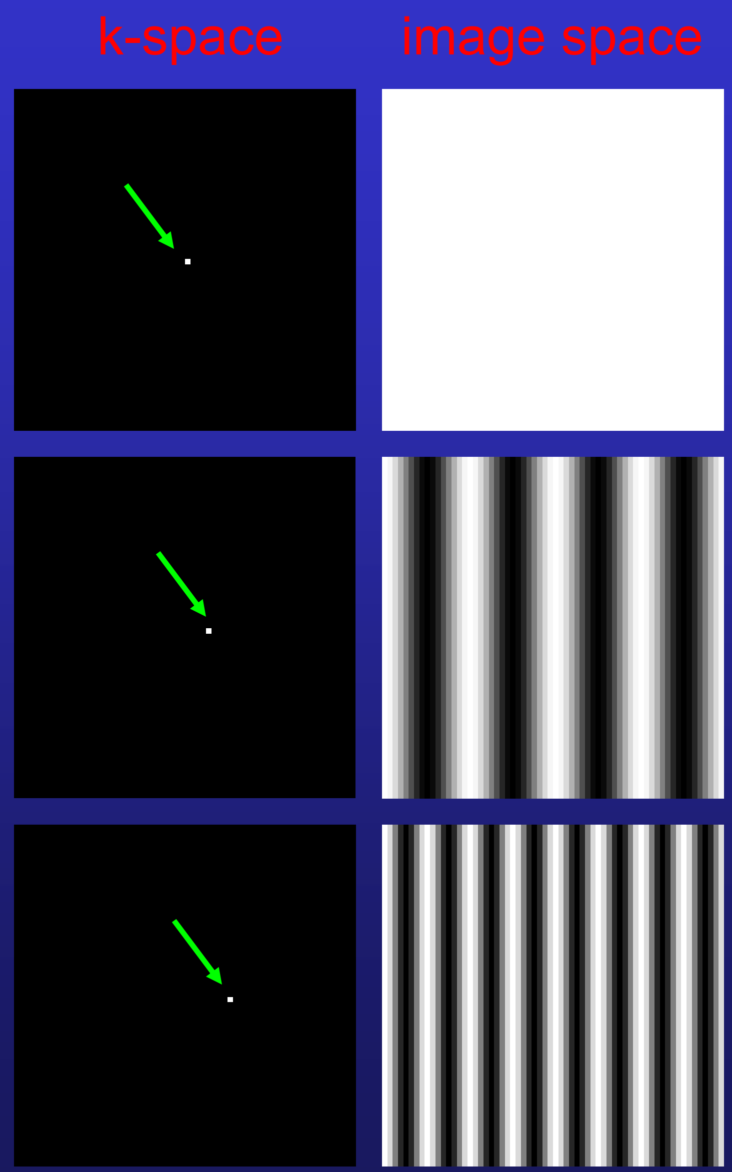
K-space as spatial frequency domain

Since k-space is the Fourier domain of the physical space, k-space data can be interpreted as the spatial frequency components of the object.

The center of k-space ($k_x = 0$, $k_y = 0$) corresponds to uniform intensity

A point with $k_x > 0$ and $k_y = 0$ corresponds to an oscillating intensity in the x direction

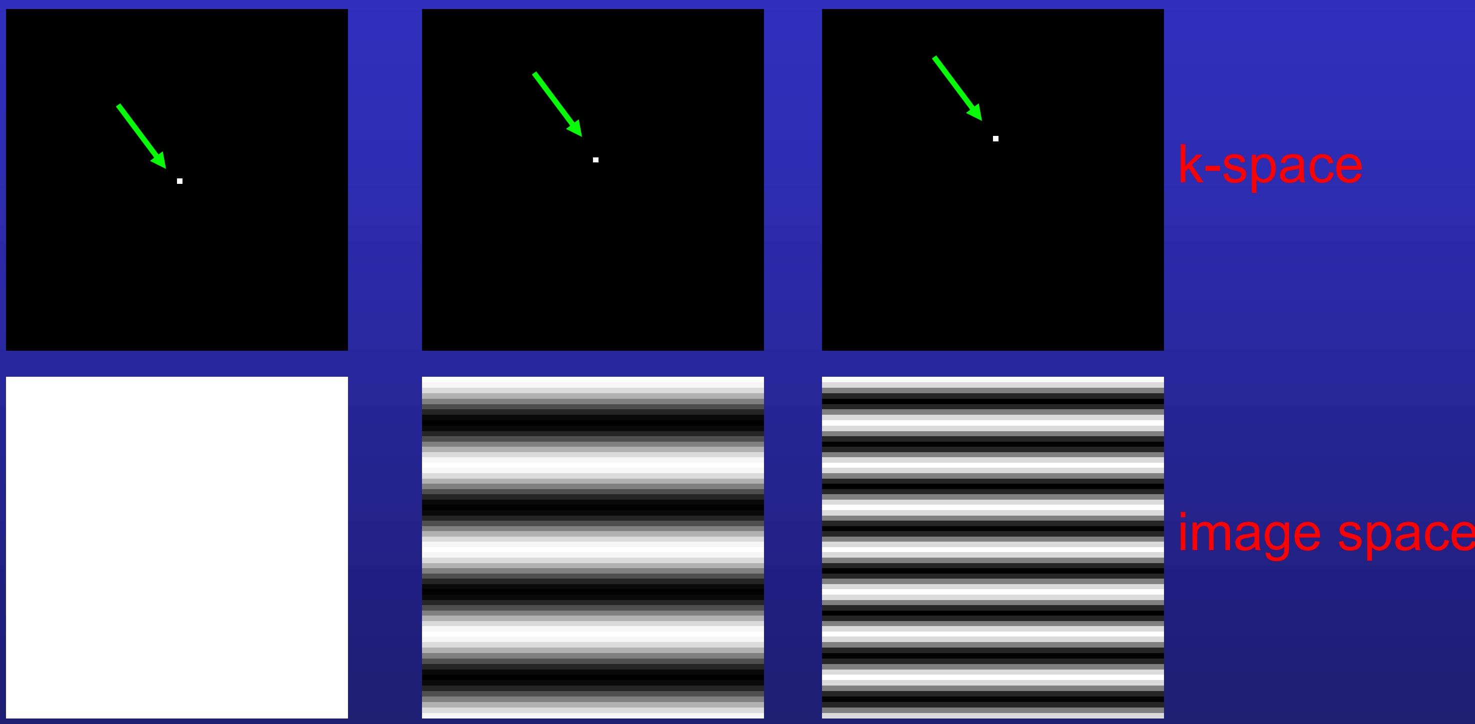
For larger k_x the spatial frequency is higher



Spatial Encoding With Gradients

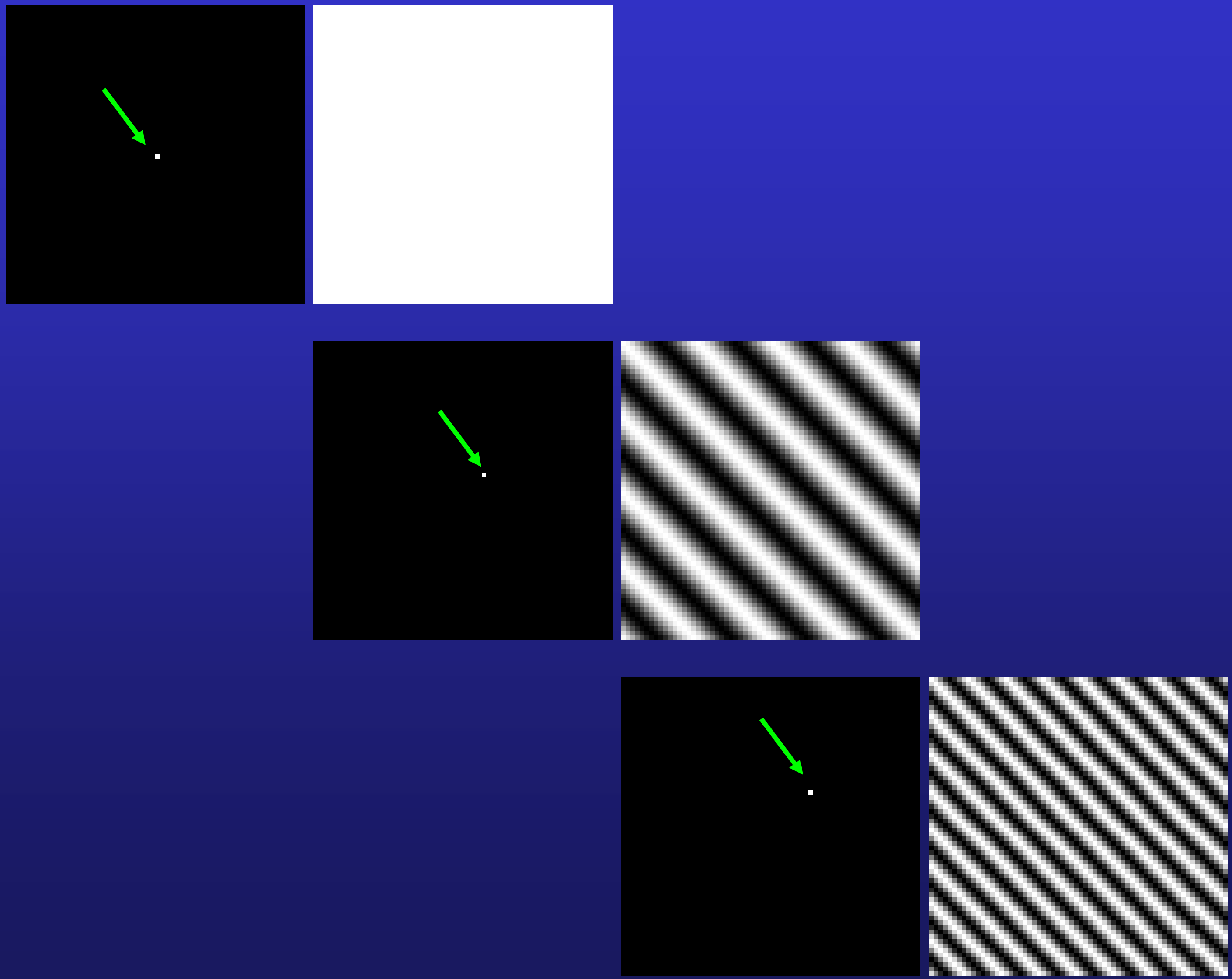
Spatial frequencies continued

Points with $k_x = 0$ and $k_y \neq 0$ corresponds to spatial frequencies in the y direction



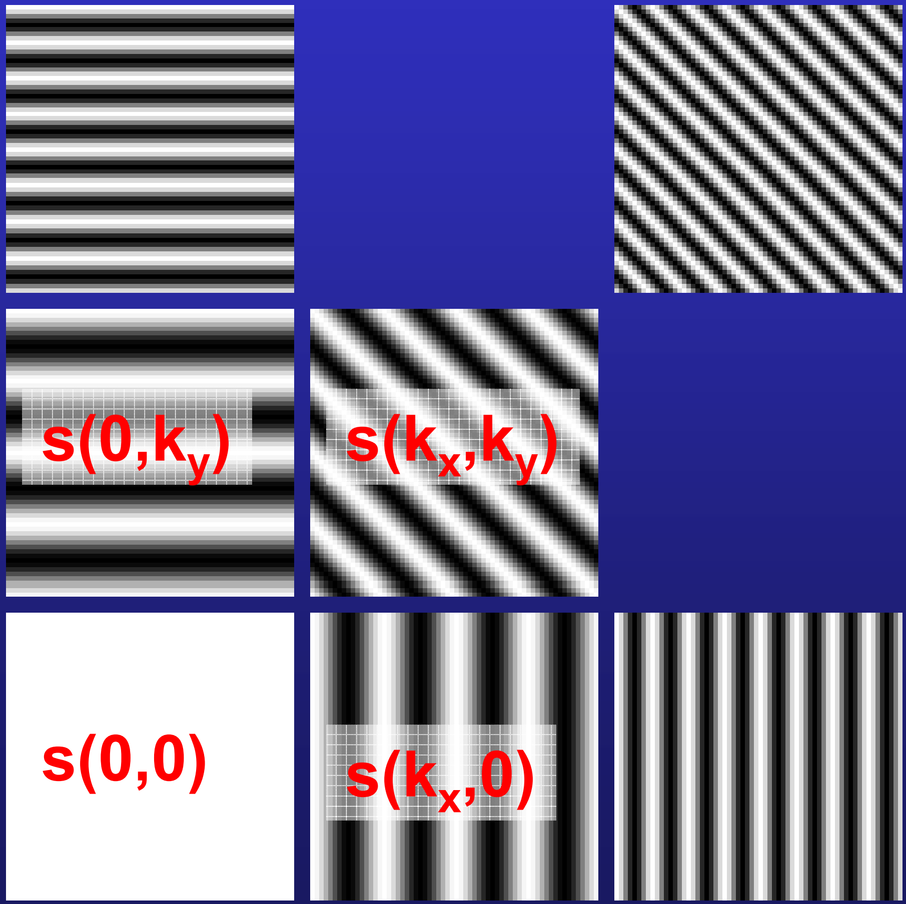
Spatial frequencies continued

Points with $k_x \neq 0$ and $k_y \neq 0$ correspond to spatial frequencies in the other directions



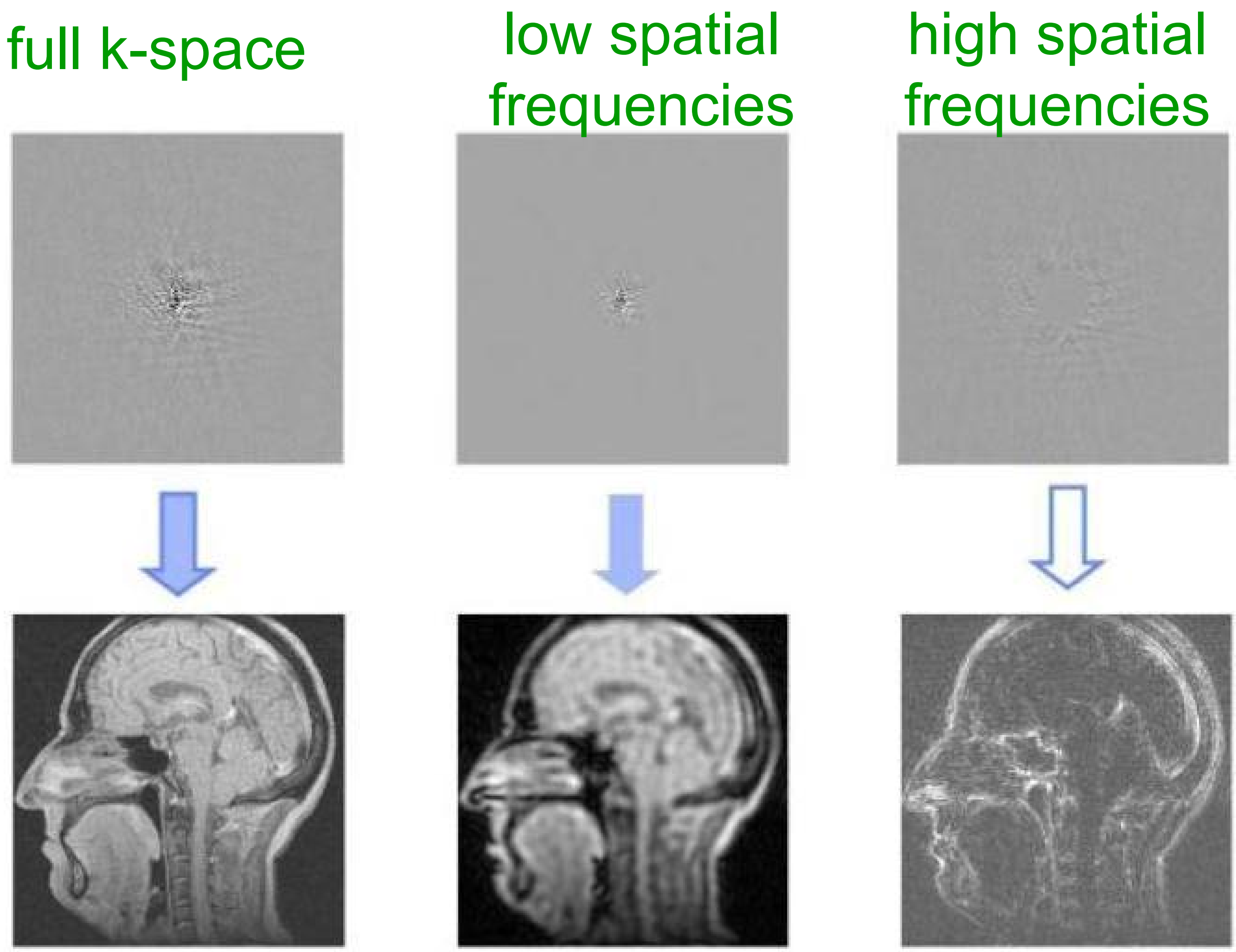
Spatial frequencies of an object

The signal distribution from a sample can be thought of as a sum of many different spatial frequency components with a certain characteristic complex weighting factors. The complex weightings $s(k_x,k_y)$ are just the k-space distribution.

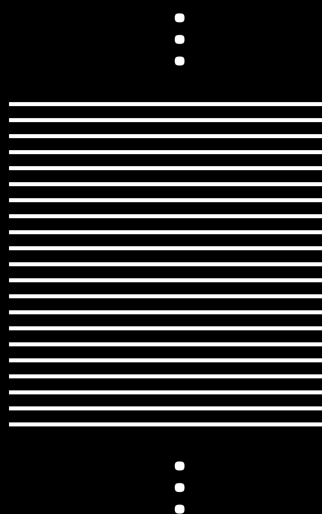


Spatial frequency components of an image

Low spatial frequency components capture the overall signal intensity and shading. Higher spatial frequency components describe the fine structure and edges of an object.



Standard MR Image Acquisition



Mathematical description of image reconstruction

For simplicity we shall treat the mathematical description of image reconstruction in one dimension. The extension to two dimensions is trivial, but unnecessarily complicates the equations.

K-space is usually sampled more coarsely in the phase-encoding direction, so the implications of discrete sampling are more evident in that direction. We will therefore consider the y-direction.

In 1D the pair of equations relating the k-space data and the signal distribution along the y direction are

$$s(k_y) = \int S(y) \exp[-i2\pi k_y y] \mathrm{d}y$$

$$S(y) = \int s(k_y) \exp[i2\pi k_y y] \mathrm{d}k_y$$

The image is reconstructed from the discretely sampled data

$$\hat{S}(m\Delta y) = \frac{1}{N} \sum_{n=-N/2}^{N/2-1} s(n\Delta k_y) \exp[i2\pi n\Delta k_y m\Delta y]$$

where –N/2 < m < N/2 -1

Any questions?

Acknowledgments

- I am extremely grateful to Prof. Pippa Storey (NYU, Radiology) who provided most of the slides used in this presentation

Homework

- Reading:
  - Prince and Links, Medical Imaging Signals and Systems, Chap. 13
  - Note down all the corrections for Ch. 12 on your copy of the textbook based on the provided errata (see Course website or book website for update)
- Problems
  - P13.1
  - P13.2
  - P13.3
  - P13.4 (except part d)
  - P13.10

See you next week!