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PIC simulations for extreme laser intensities

Martin Jirka

FNSPE, Czech Technical University in Prague

The Extreme Light Infrastructure ERIC

Contents

How to turn light into matter?
Electron-positron pair creation

Electron-positron pair creation <u>out of the vacuum</u>

Schwinger* (Sauter**) limit field required $E_S = \dfrac{m_e c^2}{e \lambda\!\!\!^{-}_C} = 1.3 \times 10^{18}\,\dfrac{V}{m}$

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- **a) In vacuum**
  - In Schwinger field, spontaneous creation of electron-positron pairs from the nonlinear vacuum is possible.
  - Required intensity ≈ 10^{29} W/cm^2
  - But today‘s record*** is only ≈ 10^{22} W/cm^2
  - Collision of multiple laser beams lowers the threshold

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- **b) In laser-matter interaction**
  - If Schwinger field is not reachable in the laboratory frame, it will be very close in the boosted frame of highly-accelerated electrons.
  - Interaction of a laser with electrons → emitted photons interact with laser field → photons annihilate, electron-positron pairs are created

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Ultra-intense lasers
New generation of lasers

Lasers are planning to achieve the intensity > 10^{23} W/cm^2

10 PW

- ELI-Beamlines (1.5 kJ, 150 fs)
- ELI-NP (200 J, 20 fs)
- Vulcan (300 J, 30 fs)
- APOLLON-10P (150 J, 15 fs)
- SIOM-10PW (300 J, 30 fs)

100 PW and 200 PW

- OMEGA EP OPAL (2 kJ, 20 fs)
- XCELS (12 beams, 300-400 J, 20-30 fs)
- GEKKO EXA (2 kJ, 10 fs)

* T. Zh. Esirkepov *et al.*, *European Astronomical Society Publications Series* **58**, 7–22 (2012)

Ultra-intense lasers
Why do we need them?

Such powerful lasers will allow studying*:

1) Laboratory astrophysics
Processes that are present in astrophysics environments (e.g. pulsars, active galactic nuclei).


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2) Electron-positron pair creation
The lowest threshold process in the photon-photon interaction.

Controls the energy emitted in gamma-ray bursts .

3) New physics
Understanding the mechanism of vacuum breakdown.

* T. Zh. Esirkepov *et al.*, *European Astronomical Society Publications Series* **58**, 7–22 (2012)

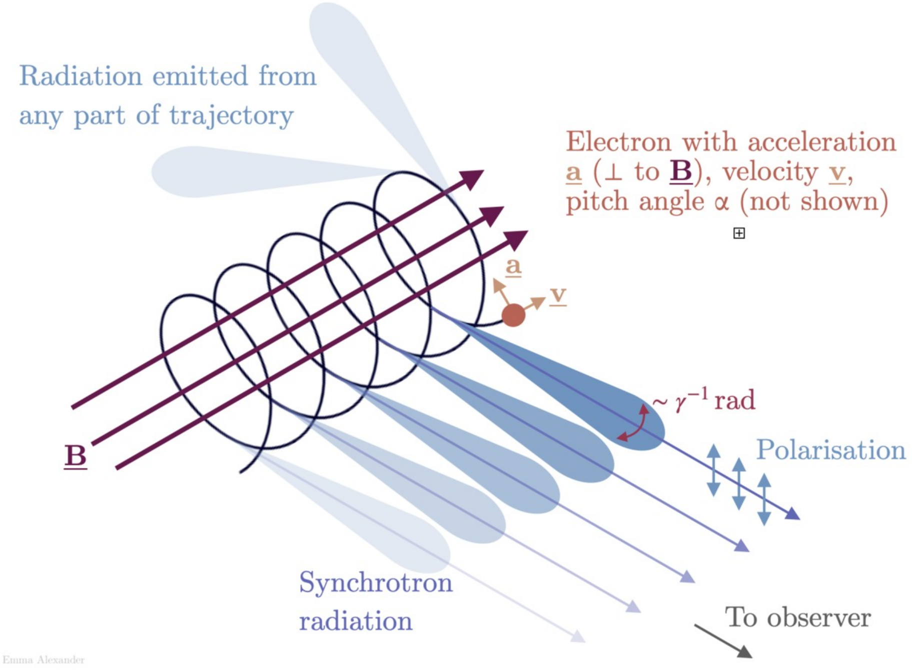
Contents

Radiation reaction
Synchrotron-like radiation

Radiation reaction
When a charged particle is accelerated in the strong electromagnetic field, it loses energy by emitting photons.[*,**]

Synchrotron radiation

- Electromagnetic radiation emitted when a charged particle travels in curved paths.
- Particle moves in a constant external magnetic field.



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[*] J. Larmor, *Mathematical, Physical and Engineering Sciences* **190**, 205–493 (1897); [**] Ya. B. Zel'dovich, *Soviet Physics Uspekhi* **18**, 79–98 (1975); [***] P.A.M. Dirac. *Mathematical, Physical and Engineering Sciences* **167**, 148–169 (1938);

Radiation reaction
Equation of motion

Radiation reaction

When a charged particle is accelerated in the strong electromagnetic field, it loses energy by emitting photons.*,**

Weak electromagnetic fields

- The radiation reaction force can be neglected.
- The motion of the particle can be described by the Lorentz force

$$\frac{d\boldsymbol{p}}{dt} = q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right)$$

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Strong electromagnetic fields

- Laser intensity > 10^{22} W/cm^2 (today's record 10^{23} W/cm^2)
- The radiation reaction force $\mathbf{G}$ has to be taken into account ***

$$\frac{d\mathbf{p}}{dt} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) + \mathbf{G}$$

---

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Equations of motion with $\boldsymbol{G}$

Lorentz-Abraham-Dirac
P.A.M. Dirac. *Mathematical, Physical and Engineering Sciences* **167**, 148–169 (1938)

Landau-Lifshitz
L.D. Landau and E.M. Lifshitz. The classical theory of fields. Pergamon Press 1975.

Ford-O’Connell
G.W. Ford and R.F. O’Connell, *Phys. Lett. A* **157**, 217–220 (1991)

Mo-Papas
T.C. Mo and C.H. Papas, *Physical Review D* **4**, 3566–357 (1971)

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$$\boldsymbol{G} = \frac{2}{3} q\,\tau_e\,\gamma\left(\dot{\boldsymbol{E}} + \boldsymbol{v}\times\dot{\boldsymbol{B}}\right) - \frac{2}{3}\frac{q}{E_S}\left[\left(\frac{\boldsymbol{v}}{c}\cdot\boldsymbol{E}\right)\boldsymbol{E} - c\,\boldsymbol{B}\times\left(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}\right)\right] - \frac{2}{3}\frac{q}{E_S}\gamma^2\left[\left(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}\right)^2 - \left(\frac{\boldsymbol{v}}{c}\cdot\boldsymbol{E}\right)^2\right]\frac{\boldsymbol{v}}{c}\ , \tau_e = \frac{r_e}{c}$$

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Radiation reaction
Particle-in-cell loop

- Lorentz force is replaced by Landau-Lifshitz equation.

$$\boldsymbol{F}_{RR}=\frac{2}{3}q\,\tau_e\,\gamma\left(\dot{\boldsymbol{E}}+\boldsymbol{v}\times\dot{\boldsymbol{B}}\right)-\frac{2}{3}\frac{q}{E_S}\left[\left(\frac{\boldsymbol{v}}{c}\cdot\boldsymbol{E}\right)\boldsymbol{E}-c\,\boldsymbol{B}\times\left(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}\right)\right]-\frac{2}{3}\frac{q}{E_S}\gamma^2\left[\left(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}\right)^2-\left(\frac{\boldsymbol{v}}{c}\cdot\boldsymbol{E}\right)^2\right]\frac{\boldsymbol{v}}{c}$$

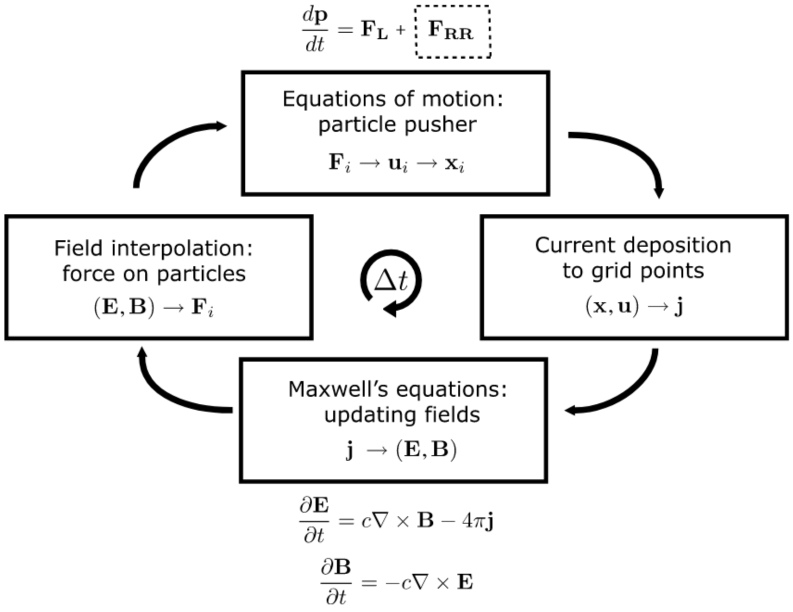


Fig. 8. Main PIC loop. $\mathbf{x}_i$, $\mathbf{u}_i$, $\mathbf{F}_i$—particle position, generalised velocity $\mathbf{u}_i=\gamma\mathbf{v}_i$ and force acting on the ith particle; $\mathbf{j}$—total current; $\mathbf{E}$, $\mathbf{B}$ the electric and the magnetic field respectively.

Source: M. Vranic et al., Computer Physics Communications **204**, 141-151 (2016)

Radiation reaction
Particle-in-cell loop

- **Example**: Electron energy evolution with and without radiation reaction (RR) during the collision with counter-propagating laser pulse

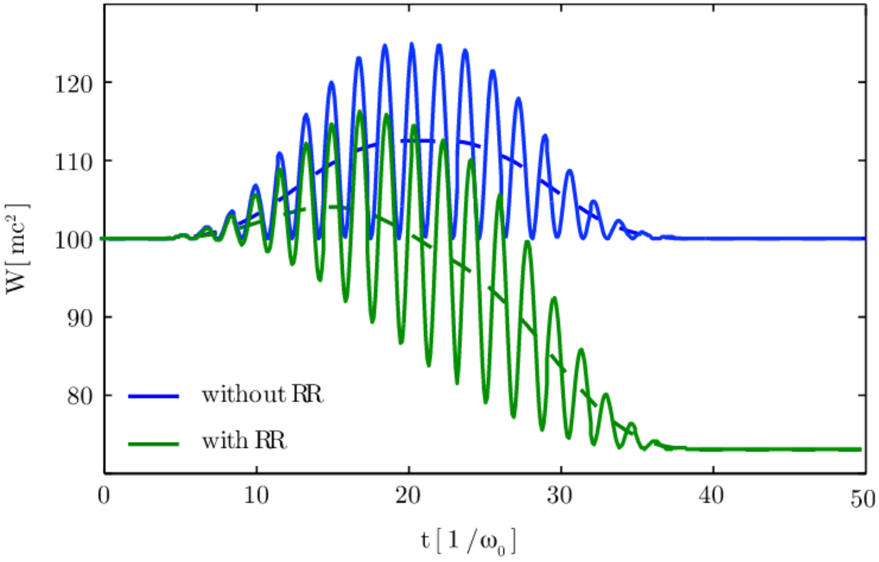


Fig. 4. Particle energy over time in the field of linearly (solid line) and circularly (dashed line) polarised laser pulse.

Source: M. Vranic et al., Computer Physics Communications **204**, 141-151 (2016)

Radiation reaction
Parameter χ_e

Characterizes the probability of photon emission by the electron.*

Compares the magnitude of the electric field ***E*** in the electron’s rest frame to the Schwinger field E_S.

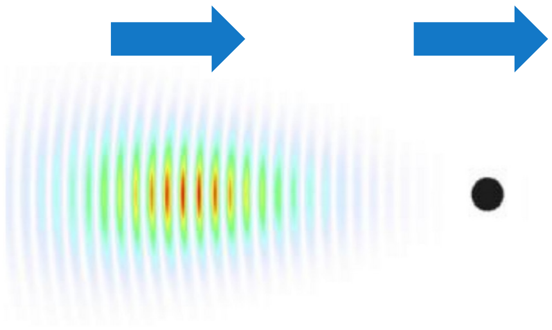
$$\chi_e = \frac{\gamma}{E_S}\sqrt{(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})^2 - \left(\frac{\boldsymbol{v}}{c}\cdot\boldsymbol{E}\right)^2}$$

* V.I. Ritus. *J. Russ. Laser Res.* **6**, 497–617 (1985); ** T. Zh. Esirkepov *et al. Phys. Lett. A* **379**, 2044-2055 (2015); *** A.R. Bell *et al.*, *Phys. Rev. Lett.* **101**, 200403 (2008)

Radiation reaction
Laser-electron interaction geometry

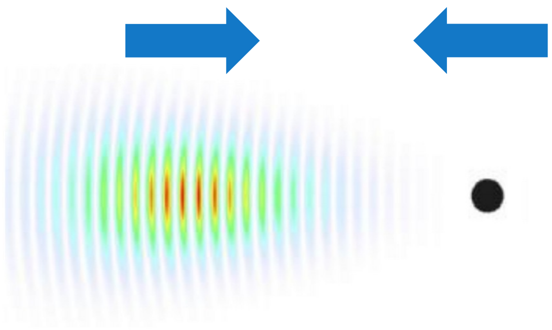
The parameter χ_e depends on the interaction setup: on mutual orientation of ***E*** (laser) and ***p*** (electron).*

Co-propagating case



$$\chi_e^{\rightarrow\rightarrow} \approx \frac{1}{2\,\gamma}\frac{\|\boldsymbol{E}\|}{E_S}$$

Counter-propagating case



$$\chi_e^{\rightarrow\leftarrow} \approx 2\,\gamma\,\frac{\|\boldsymbol{E}\|}{E_S}$$

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* S. S. Bulanov *et al.*, *Phys. Rev. A* **87**, 062110 (2013)

Radiation reaction
Classical description of emission*

For $\chi_e \ll 1$

- The emission is a continuous process of losing particle’s energy.
- The trajectory of the emitting particle is a smooth function of time.
- Particle motion can be described by the Landau-Lifshitz eq.
- The typical energy of the emitted photon $\hbar\,\omega_\gamma = 0.44\,\chi_e\,\gamma\,m_e c^2$

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* C. P. Ridgers *et al.*, *J. Comput. Phys.* **260**, 273–285 (2014)

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What if $\chi_e \approx 1$**?**

- The emitted photon carries out a significant fraction of the electron’s energy.
- A recoil experienced by the emitting particle has to be taken into account and can not be neglected.
- **Quantum description of radiation reaction is needed.**

* C. P. Ridgers *et al.*, *J. Comput. Phys.* **260**, 273–285 (2014)

Radiation reaction
QED description of emission*

- **For** $\chi_e \geq 1$
- The process of emission is probabilistic, and thus the electron motion is stochastic.
- The QED process is simulated using a Monte Carlo algorithm:

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   4) Photon emission rates g depends on the local values of the electromagnetic fields and the particle's energy

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   5) When $\tau = \tau_{\mathrm{em}}$ is met the particle emits

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   4) Photon emission rates g depends on the local values of the electromagnetic fields and the particle's energy
   5) When $\tau = \tau_{\mathrm{em}}$ is met the particle emits
   6) Emitted photon is added to the simulation
   7) Photon energy is randomly sampled from photon emission spectra.
   8) Velocity of the emitting electron is reduced.

* C. P. Ridgers *et al.*, *J. Comput. Phys.* **260**, 273–285 (2014)

Radiation reaction
Classical vs. quantum regime*

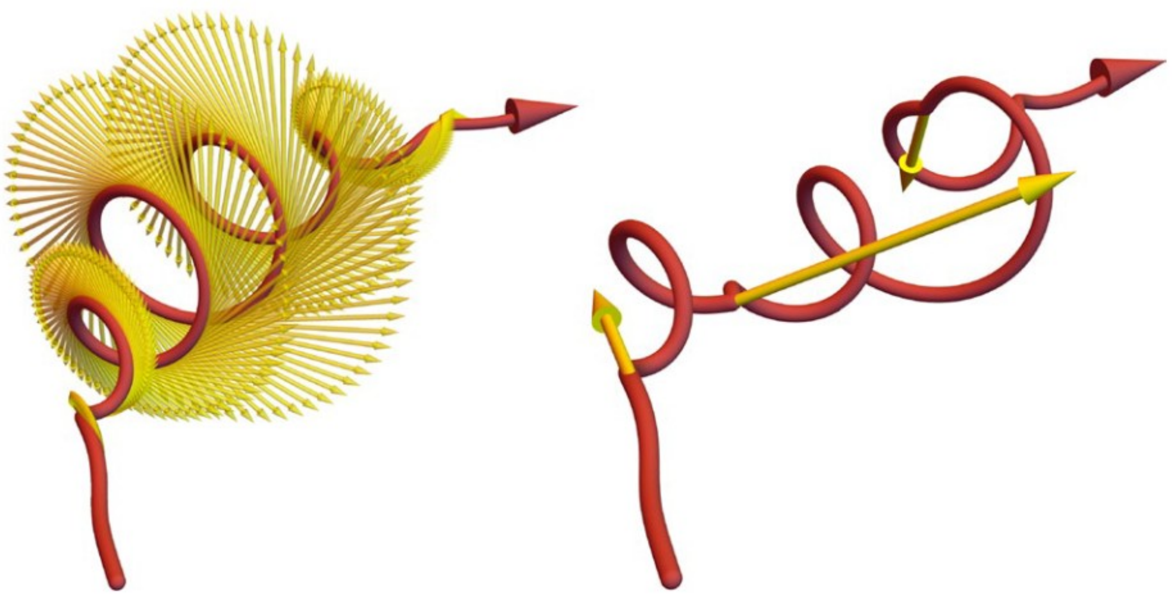

Fig. 4 In the classical picture, radiation reaction is a continuous drag force that arises from the emission of very many photons that individually have vanishingly low energies (left). In the quantum regime, however, the electrons emits a finite number of photons, any or all of which can exert a significant recoil on the electron. The probabilistic nature of emission leads to radically altered electron dynamics, with implications for laser–matter interactions beyond the current intensity frontier. From Blackburn (2015)

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* T. Blackburn, *Reviews of Modern Plasma Physics* **4**, 5 (2020)

Radiation reaction
Semiclassical numerical method

- **Numerical scheme that can model quantum processes:**

  - Particles (electrons, positrons and photons) move classically in between point-like QED interactions

  - The rates and spectra for the individual QED interactions are calculated for the equivalent interaction in a constant, crossed field

* T. Blackburn, *Reviews of Modern Plasma Physics* **4**, 5 (2020)

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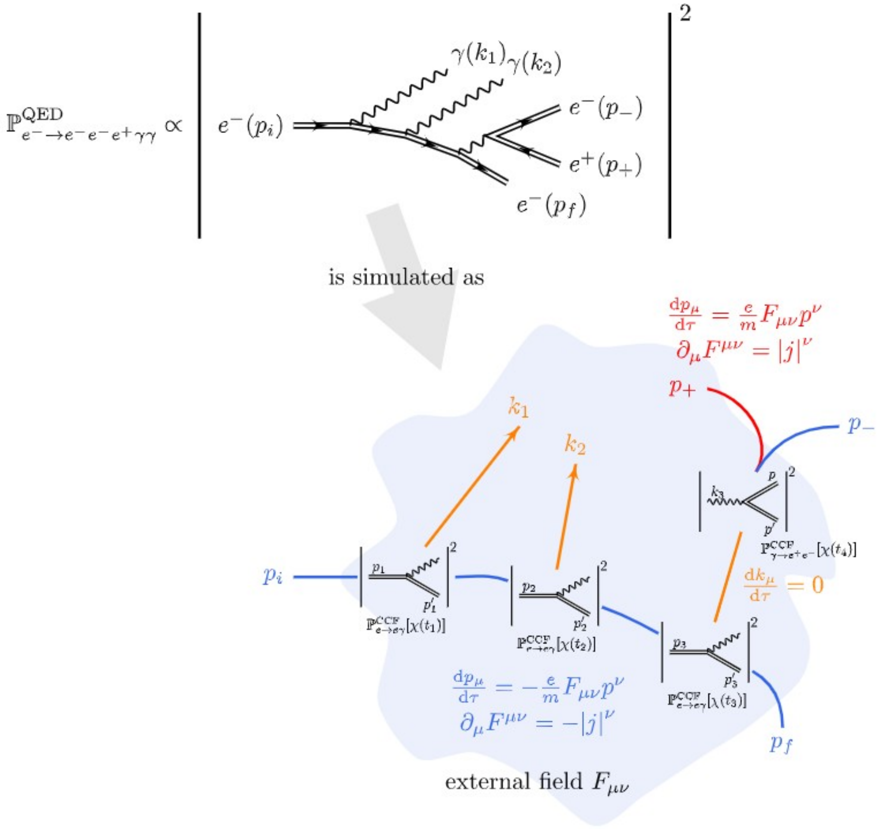


Fig. 5 A general strong-field QED interaction, featuring the emission and creation of multiple photons and electron–positron pairs, is simulated 'semiclassically' by breaking it down into a chain of first-order processes (electrons, photons and positrons in blue, orange and red, respectively). Between these point-like, instantaneous events, the particles follow classical trajectories guided by the Lorentz force:

* T. Blackburn, *Reviews of Modern Plasma Physics* **4**, 5 (2020)

Radiation reaction
Implementation

- Photon emission is considered by including the Monte Carlo algorithm in the PIC as a new step at the end of each PIC loop
- Each particle can only emit once in a time-step Δt
- There is a finite probability, that a particle should emit multiple times over the time step duration.
- Required timestep:

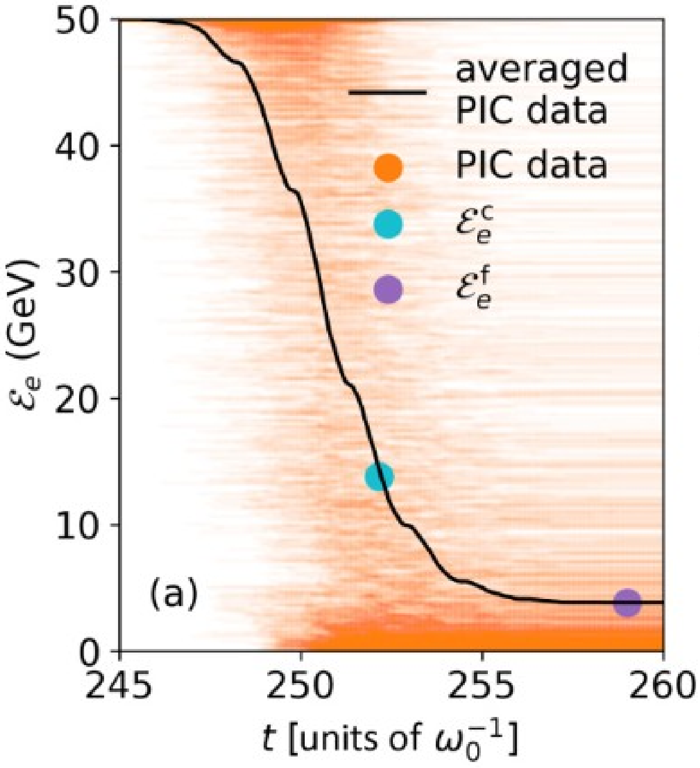
$$\Delta t \ll \frac{5.24 * 137}{2\sqrt{3}\, m_e c^2 h\, Max\left(E/E_S, cB/E_S\right)}$$

- The constraint for pair creation is an order of magnitude less stringent than that for photon production.

* C. P. Ridgers *et al.*, *J. Comput. Phys.* **260**, 273–285 (2014)

Radiation reaction
QED description of emission*

- **Example:** 50 GeV electrons lose energy in the interaction with a counter-propagating laser pulse of intensity 10^{24} W/cm^2.*



* M. Jirka et al., Phys. Rev. A **103**, 053114 (2021)

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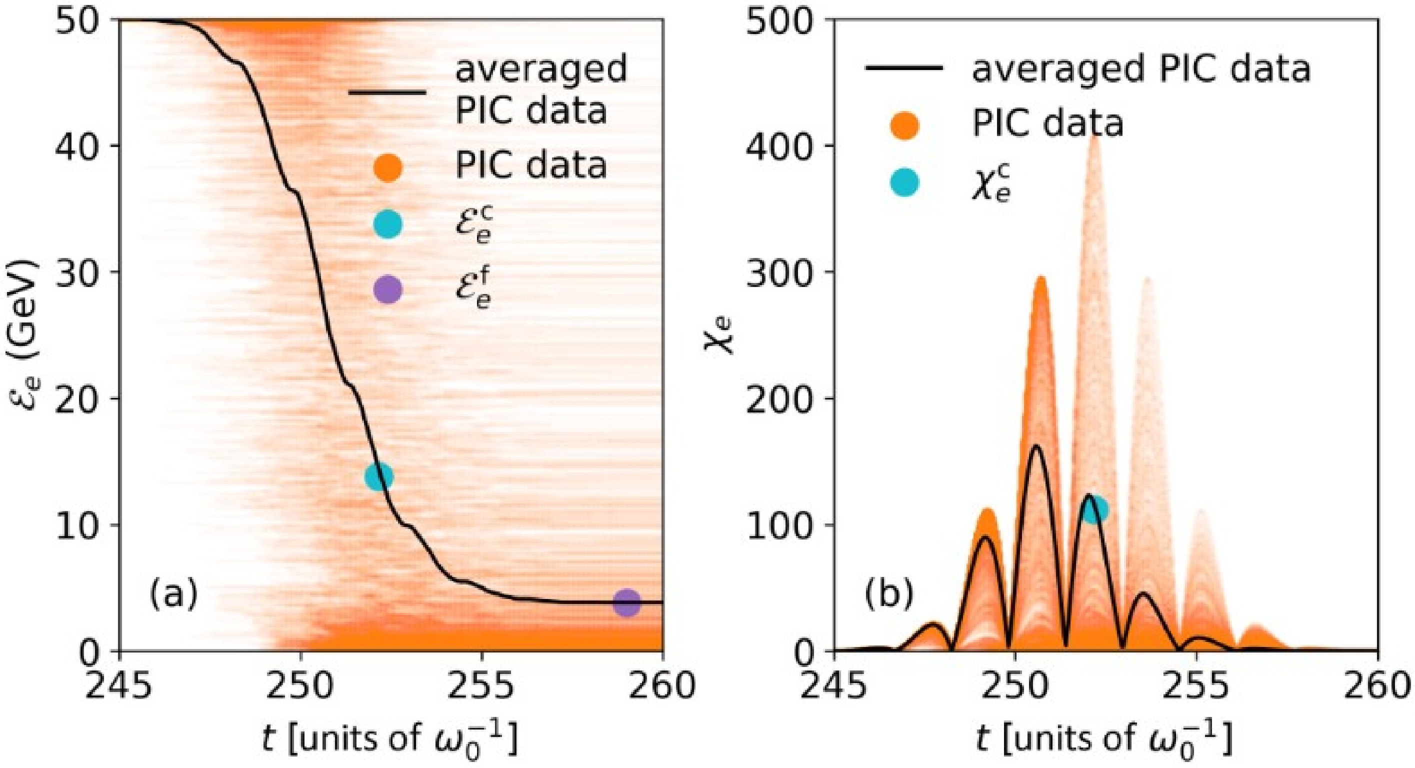

* M. Jirka et al., Phys. Rev. A **103**, 053114 (2021)

Radiation reaction
Validity of the emission model*

- **The presented approach rests on two approximations:**

* C. P. Ridgers *et al.*, *J. Comput. Phys.* **260**, 273–285 (2014)

Radiation reaction
Validity of the emission model*

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    - It holds for $a_0 = qE/\omega m_e c \gg 1$, i.e. for strong laser fields (however, low energy photons have larger formation length, corrections needed)
    - The macroscopic laser fields are thus treated as static during the QED interactions
    - This assumption also means that back-reaction effects are neglected
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- Moreover, the perturbative QED theory is valid up to $\chi_e < 1600$

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Radiation reaction
Summary of the emission model*

$\chi_e \ll 1$ Radiation reaction can be considered as a continuous process of losing particle energy (Landau-Lifshitz eq.).**

$\chi_e \sim 1$ The emitted photon carries out a significant fraction of the electron’s energy, a recoil cannot be neglected.***

$\chi_e \geq 1$ Radiation reaction has to be modeled as a step-like quantum proces (the process of emission is probabilistic, and thus the electron motion is stochastic).**

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* V.I. Ritus. *J. Russ. Laser Res.* **6**, 497–617 (1985); ** T. Zh. Esirkepov *et al. Phys. Lett. A* **379**, 2044-2055 (2015); *** A.R. Bell *et al.*, *Phys. Rev. Lett.* **101**, 200403 (2008)

Radiation reaction
Summary of the emission model*

- **Example:** Quantum description causes broadening of the electron spectra. This feature can be used as a signature of QED photon emission in experiments.*

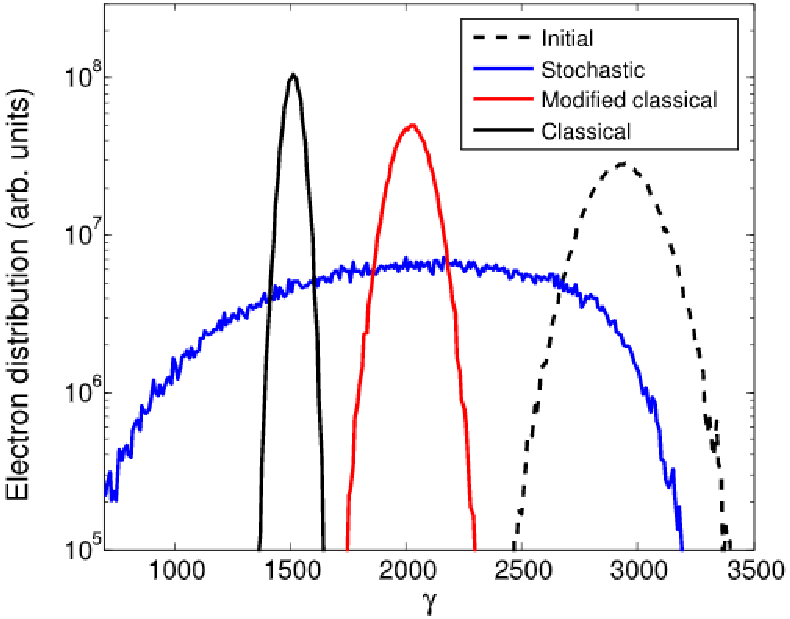


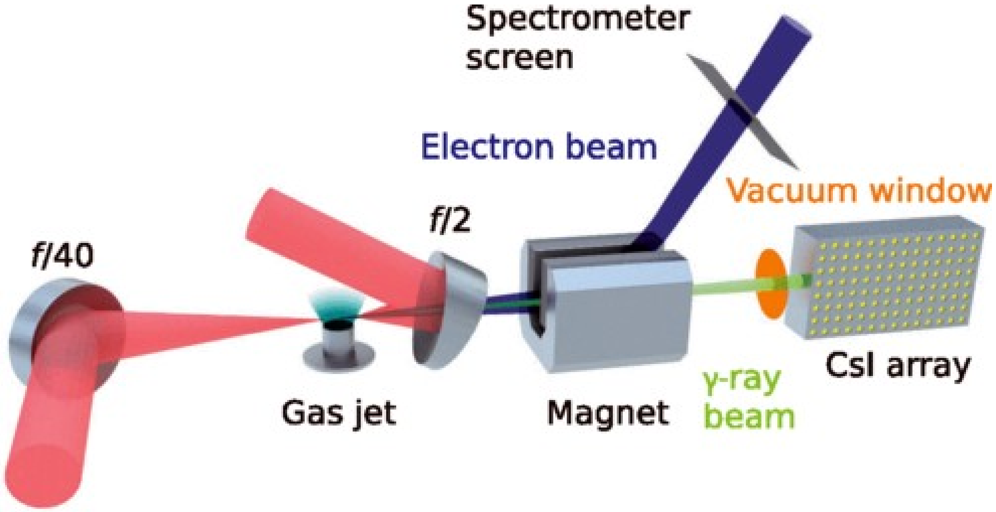
FIGURE 2. Electron energy distribution, 10.5 fs after collision of the electron bunch with the laser pulse, compared to initial distribution using the stochastic, modified classical and classical emission operators.

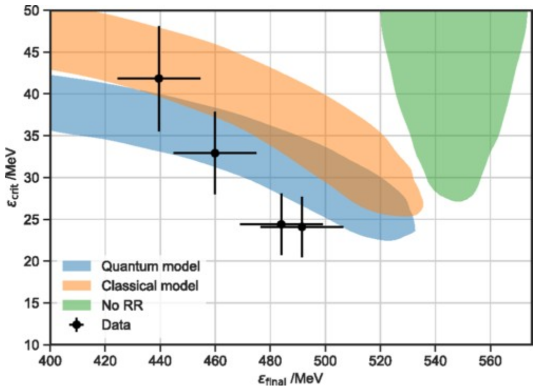
*C. Ridgers at el., J. Plasma Phys. **83**, 715830502 (2017)

Radiation reaction
Experimental evidence

All-optical setup: electrons are accelerated by the laser.

An energy loss of electron spectrum has been measured after the collision that is correlated with the measured signal of photons.





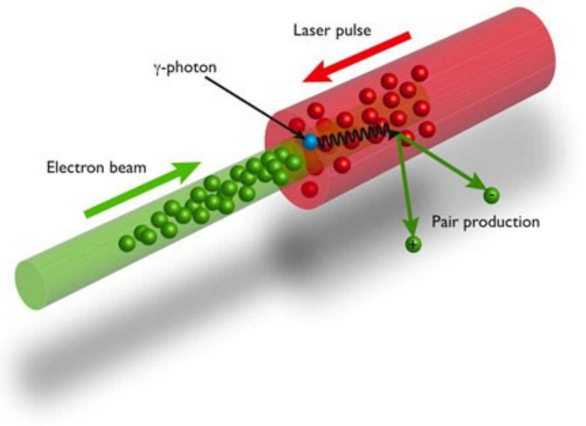
J. Cole et al. PRX 8 (2018), p. 011020
K. Poder et al. PRX 8 (2018), p. 031004

Contents

Pair production
Breit-Wheeler process*

Pair generation is accessible in electron collision with the extreme-intensity lasers:

- 1. Emitted photon interacts with number of laser photons
- 2. They can be converted into a single electron-positron pair



The production of electron-positron pairs in electron-laser interactions.
(Courtesy: Mattias Marklund)

G. Breit and J. A. Wheeler, *Physical Review* **46**, 1087–1091 (1934)

Pair production
Breit-Wheeler process

Linear process

- Two energetic photons collide (annihilate) and then a new electron and a new positron are created

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Non-linear (multiphoton) process

- A photon collides (annihilates) with *n* laser photons

1. **Step:** High-energy electron emits a photon (radiation reaction).

Electron + strong EM field → a photon is emitted

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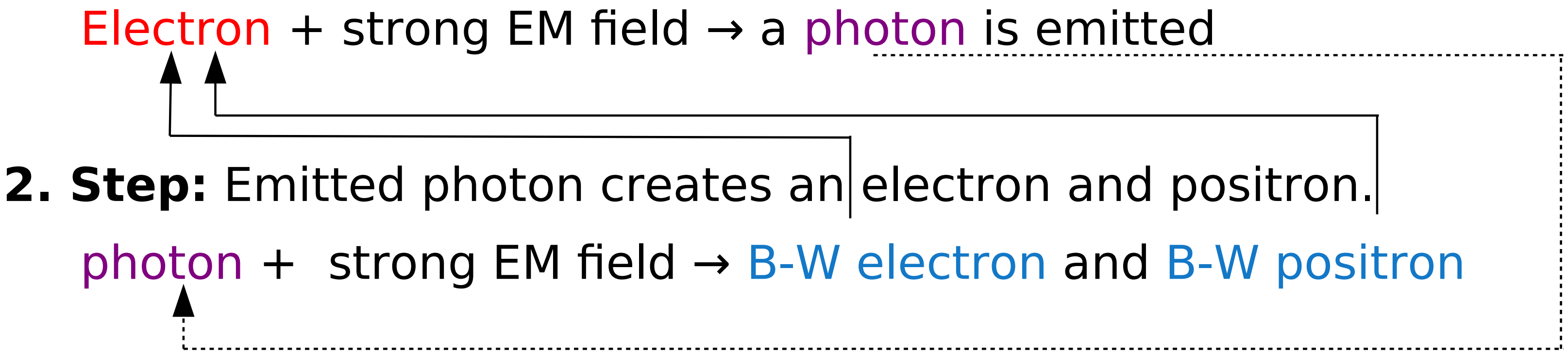

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Cascade:

B-W electron and B-W positron radiate additional photons.

G. Breit and J. A. Wheeler, *Physical Review* **46**, 1087–1091 (1934)

Pair production
Parameter χ_γ

Characterizes the probability of pair production by the emitted photon. *

$$\chi_\gamma = \frac{\hbar\,\omega_\gamma}{m_e c^2 E_S}\sqrt{\left(\boldsymbol{E}+\frac{c^2}{\omega_\gamma}\boldsymbol{k}_{\boldsymbol{\gamma}}\times\boldsymbol{B}\right)^2-\left(\frac{c}{\omega_\gamma}\boldsymbol{k}_{\boldsymbol{\gamma}}\cdot\boldsymbol{E}\right)^2}$$

Condition for cascade pair production

$$\chi_e,\chi_\gamma > 1$$

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* V.I. Ritus. *J. Russ. Laser Res.* **6**, 497–617 (1985)

Pair production
QED model of pair production*

- Pair creation Is simulated using a Monte Carlo algorithm:

1) Probability of pair creation after a photon traverses a distance of optical depth τ_{em}: $P(t) = 1 - e^{-\tau_{em}}$.
2) Each photon is assigned an optical depth at which it creates an electron-positron pair by the following procedure:
   1) P is assigned pseudo-random value between 0 and 1
   2) Equation for P is then inverted to yield τ_{em}
   3) Photon optical depth τ evolves according to $\tau(t) = \int_0^t g[\chi_\gamma(t')]dt'$
   4) Electron-positron pair production rate g depends on the local values of the electromagnetic field and the photon’s energy
   5) When the condition $\tau = \tau_{\text{em}}$ is met, the electron-positron pair is created
   6) The emitted electron and positron are added to the simulation, photon is removed from the simulation
   7) Photon energy is randomly divided between the electron and the positron

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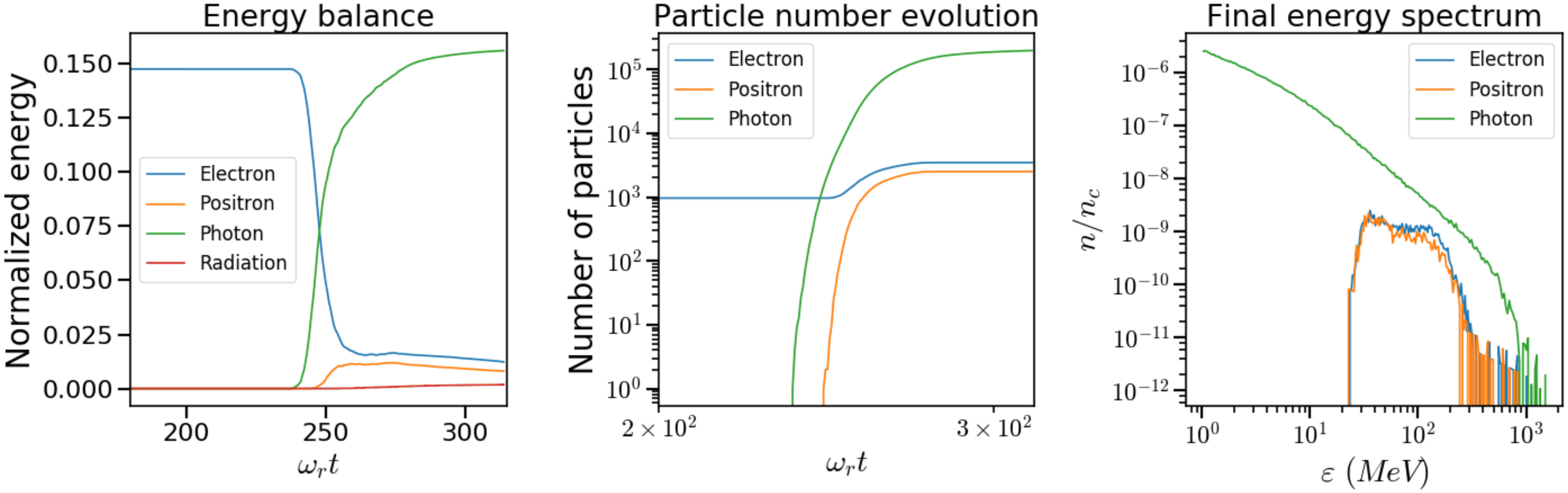
Pair production
Cascade pair production

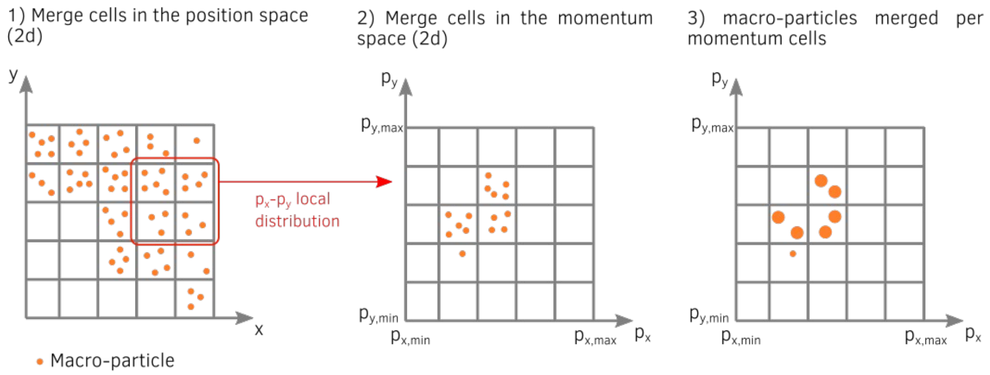
- **Example:**
  - 4 GeV electrons interacts with a laser pulse of intensity 10^{23} W/cm^2 ($\chi_e = 10$)
  - A huge number of photons, electrons and positrons can be created during in the QED cascade =>computationally demanding



Source: https://smileipic.github.io/Smilei/Understand/multiphoton_Breit_Wheeler.html

Pair production
Particle merging*

- **Merging algorithm**
  - It allows to explore problems that would otherwise not be accessible due to memory limitations.



* M. Vranic et al., Comput. Phys. Commun. 191, 65-73 (2015); Source: https://smileipic.github.io/Smilei/Understand/particle_merging.html

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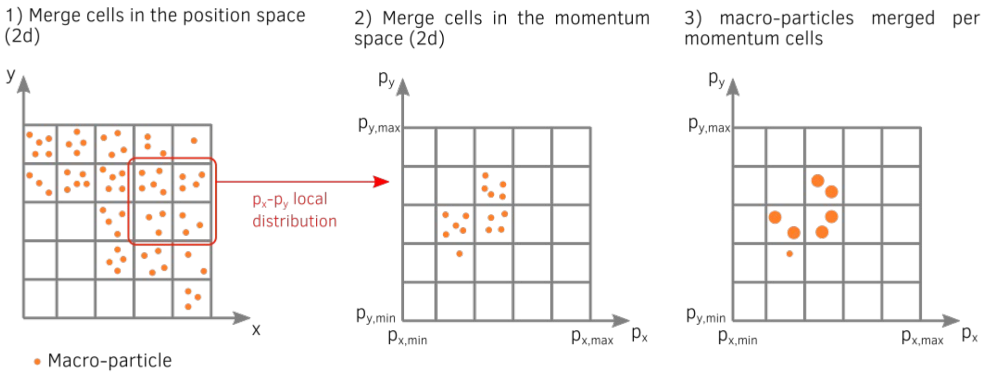

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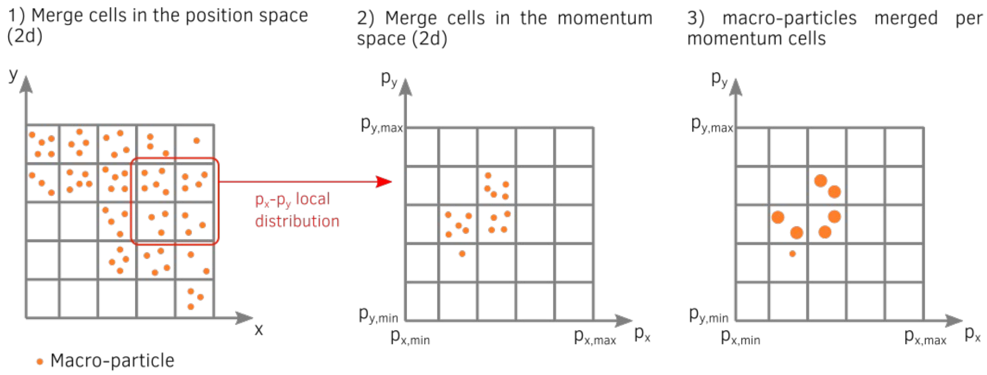

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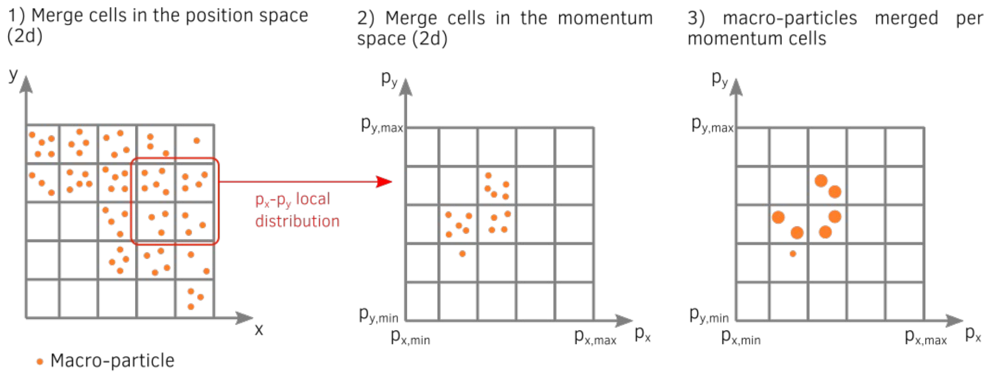

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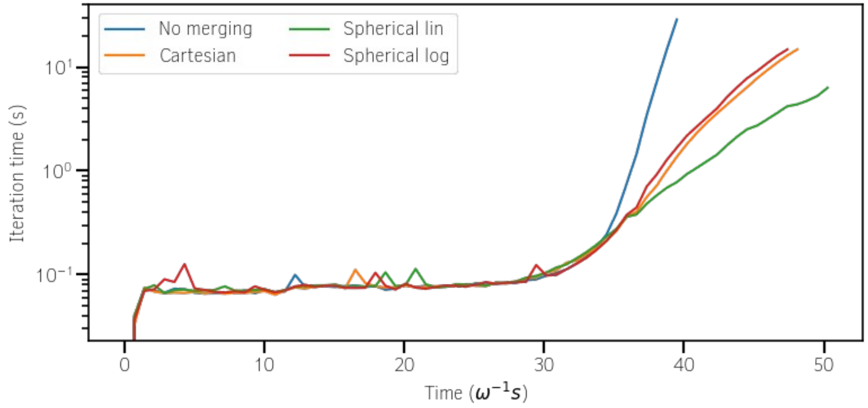

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Contents

Introduction

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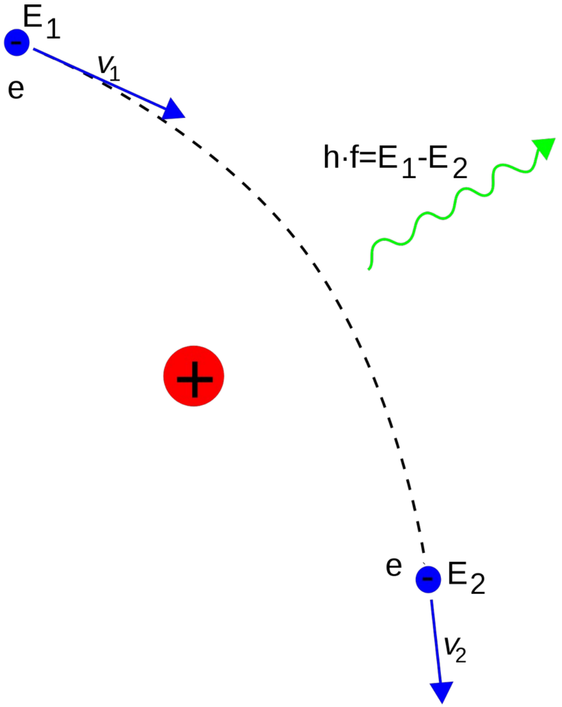
Bremsstrahlung

Examples

Bremsstrahlung emission

- **Bremsstrahlung (braking radiation)**
  Photon emission due to the deceleration of a charged particle when deflected by another charged particle (e.g. electron scattering on the ion or nucleus of neutral atom)

- Dominant for solid density targets made of heavy materials for intensity < 10^{22} W/cm^2



Source: https://en.wikipedia.org/wiki/Bremsstrahlung

Bremsstrahlung emission
Modelling

- **Monte-Carlo particle transport simulations**
  - Electron propagation treated as a random walk process
  - The trajectories of individual electrons are assumed to be mutually independent
  - Target is treated as a static undisturbed bulk of matter
  - This approach is appropriate for bulk targets

* J. Vyskocil et al., Plasma Phys. Control. Fusion **60** (2018) 054013

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- **PIC codes**
  - The assumption that the target is static and undisturbed may be violated due to its significant expansion during the collision with a laser
  - Bremsstrahlung emission should be included directly into the PIC loop

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Bremsstrahlung emission
Modelling

- **Example of implementation into PIC***
  - Energy transfer to the ion neglected.
  - Electron propagating through background matter undergoes random scattering events according to a probability distribution function *P* which is determined by the differential cross section σ of the bremsstrahlung interaction mechanism
  - Minimum energy of the emitted photon must be always specified (integral of σ diverges when photon energy goes to 0)

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2) For each v_e (el. velocity), it compares a random number $x \in \langle 0,1 \rangle$ to the emission probability $P = n_i v_e \sigma \Delta t$

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- If $x < P$:
  - Photon is created and electron momentum is modified

- If P > 1 (e.g. because of a long time step):
  - A photon is emitted
  - P is lowered by 1 for each, until $P < 1$

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Contents

Thank you for your attention.