#### **IPP** INSTITUTE OF PLASMA PHYSICS OF THE CZECH ACADEMY OF SCIENCES

# Fluid transport modelling of the plasma core and edge

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 Výzkum užítečný pro společnosť.

#### Outline

- Basics of fluid turbulent models.
- Simple 2D model.
  - Equations
  - Implementation
  - Results and limitations
- From kinetic to fluid equations to Braginskii equations.
- 3D GBS modell.
  - Inputs / outputs.
  - Implementation of the GBS code.
  - Results and limitations.
- Fluid codes on GPU using Python and JAX.

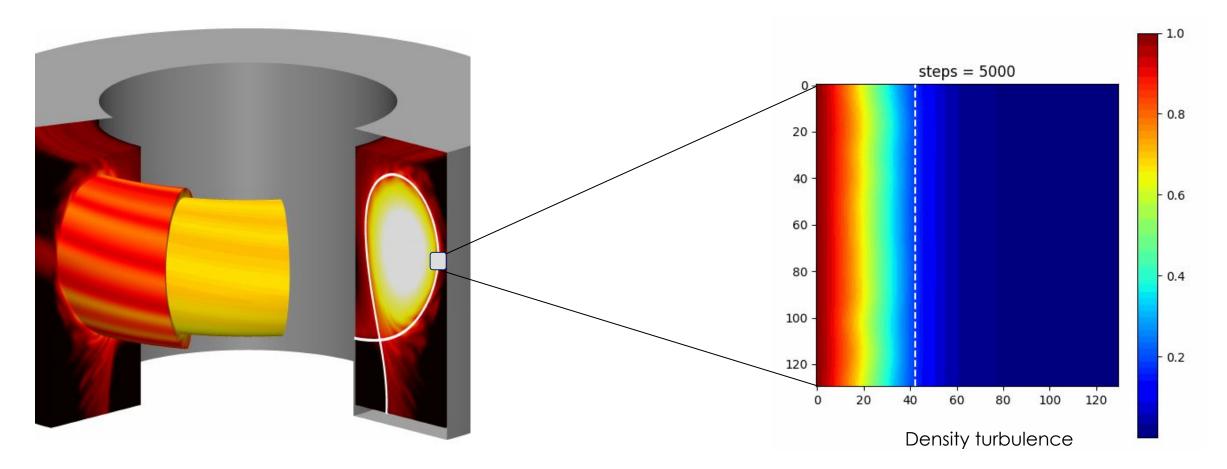
#### Fluid models basics

- From kinetic equation to fluid equations.
- Maxwellian distribution is assumed (high collisionality)!
- Several first moments (up to temperature equations) + closure.
- Much faster compared to kinetic simulations, several 3D models exists (GBS, TOKAM3X, GRILLIX).
- Full-size simulations of medium size machines (COMPASS, TCV, etc).
- Kinetic effects and gyromotions are neglected.
- Describes edge plasma only => unable to simulate core plasma (ITGs, ETGs, TEM neglected).

#### Turbulence v okrajovém plazmatu

Complex 3D model

Simple 2D model





#### Towards simple 2D fluid model



#### **Drift reduced approximation**

- The momentum equation for each charged particle species reduced to an algebraic expression for the fluid drifts in terms of scalar fields.
- To separate the parallel and perpendicular motion.
- To remove fast temporal scales.
- Can be used because the **turbulence** is much **slower** compared to gyro-frequency and much **larger** compared to the gyro-radius.
- Perpendicular motion given by **ExB** drift, **diamagnetic** drift, and **polarization** drift.

$$\boldsymbol{v}_{\perp} = \overbrace{\frac{1}{B}\boldsymbol{b}\times\boldsymbol{\nabla}\phi}^{ ext{ExB drift}} + \overbrace{\frac{1}{qnB}\boldsymbol{b}\times\boldsymbol{\nabla}p}^{ ext{diamagnetic drift}} + \overbrace{\frac{m}{qB}\boldsymbol{b}\times(\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{v}}^{ ext{polarization drift}},$$

Perpendicular transport:

#### Simple 2D model - equations

 $\frac{\partial f}{\partial t} + \nabla \cdot (vf) + \nabla \cdot (\frac{F}{m}f) = C$   $\int_{\frac{\partial n}{\partial t}} \nabla \cdot (nv) = 0,$ Kinetic equation polarization drift diamagnetic drift ExB drift  $\boldsymbol{v}_{\perp} = \overbrace{\frac{1}{B}\boldsymbol{b} \times \boldsymbol{\nabla} \phi}^{\mathbf{1}} + \overbrace{\frac{1}{anB}\boldsymbol{b} \times \boldsymbol{\nabla} p}^{\mathbf{1}} + \overbrace{\frac{m}{aB}\boldsymbol{b} \times (\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{v}}^{\mathbf{1}},$ Density equation  $\boldsymbol{\nabla} \cdot \boldsymbol{v}_{E} = \overbrace{\boldsymbol{\nabla}(\frac{1}{B}) \cdot \boldsymbol{b} \times \boldsymbol{\nabla} \phi}^{(1)} + \overbrace{\frac{1}{B} \boldsymbol{\nabla} \times \boldsymbol{b} \cdot \boldsymbol{\nabla} \phi}^{(2)} = \mathcal{C}(\phi), \quad \text{Temperature equation} \quad \frac{3}{2}n(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla})T + nT\boldsymbol{\nabla} \cdot \boldsymbol{v} + \boldsymbol{\nabla} \cdot \boldsymbol{q}_{\perp} = 0,$  $\frac{\mathrm{d}n}{\mathrm{d}t} + n\mathcal{C}(\phi) - \mathcal{C}(nT) = \Lambda(n)$  $\frac{\mathrm{d}T}{\mathrm{d}t} + \frac{2T}{3}\mathcal{C}(\phi) - \frac{7T}{3}\mathcal{C}(T) - \frac{2T^2}{3n}\mathcal{C}(n) = \Lambda(T)$  $\frac{\mathrm{d}\Omega}{\mathrm{d}t} - \mathcal{C}(nT) = \Lambda(\Omega)$  $\Omega = \boldsymbol{\nabla} \times v_E = B^{-2} \boldsymbol{\nabla} \times (\boldsymbol{B} \times \boldsymbol{\nabla} \phi) = \nabla^2_{\perp} \phi.$ 

### Simple 2D model - without temperature

#### 2D model without temperature:

- Turbulence can be evolved even without temperature.
- Considering **constant temperature** simplification.
- Poisson equation is unchanged.
- Numerical solution:
  - Poisson equation-Poisson solver
  - Operator d/dt
  - Curvature operator C(.)
  - $\circ$  Diffusion operator A

$$\frac{dn}{dt} + nC(\phi) - C(n) = \Lambda(n)$$

$$\frac{d\Omega}{dt} - C(n) = \Lambda(\Omega)$$

 $\Delta \phi = \Omega$ 

#### Simple 2D model - implementation

$$\frac{dn}{dt} + nC(\phi) - C(n) = \Lambda(n)$$

Total time derivative -> time change + convection

Curvature operator - derivative in y direction

Diffusion term -> diffusion in x and y + parallel decay

$$\Lambda(f) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} - \frac{1}{\tau_{\parallel}}f$$

1. derivative in x:

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+\Delta h,y) - f(x-\Delta h,y)}{\Delta h}$$

2. derivative in y:

$$\frac{\partial^2 f(x,y)}{\partial x^2} \approx \frac{f(x+\Delta h,y) - 2f(x,y) + f(x-\Delta h,y)}{\Delta h^2}$$

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#### Simple 2D model - Arakawa scheme

- Convection term using Arakawa
   scheme for numerical stability.
- Arakawa conserves:

- Mean vorticity
- Mean-square vorticity
- Kinetic energy

 $\partial \zeta / \partial t = (\partial \zeta / \partial x) (\partial \psi / \partial y) - (\partial \zeta / \partial y) (\partial \psi / \partial x) \equiv J(\zeta, \psi)$ 

$$\begin{aligned} \mathcal{J}_{i,j}(\zeta, \psi) &= -\frac{1}{12d^2} \left[ (\psi_{i,j-1} + \psi_{i+1,j-1} - \psi_{i,j+1} - \psi_{i+1,j+1}) (\zeta_{i+1,j} - \zeta_{i,j}) \right. \\ &+ (\psi_{i-1,j-1} + \psi_{i,j-1} - \psi_{i-1,j+1} - \psi_{i,j+1}) (\zeta_{i,j} - \zeta_{i-1,j}) \right. \\ &+ (\psi_{i+1,j} + \psi_{i+1,j+1} - \psi_{i-1,j} - \psi_{i-1,j+1}) (\zeta_{i,j+1} - \zeta_{i,j}) \\ &+ (\psi_{i+1,j-1} + \psi_{i+1,j} - \psi_{i-1,j-1} - \psi_{i-1,j}) (\zeta_{i,j} - \zeta_{i,j-1}) \end{aligned}$$

$$+ (\psi_{i+1,j} - \psi_{i,j+1})(\zeta_{i+1,j+1} - \zeta_{i,j}) + (\psi_{i,j-1} - \psi_{i-1,j})(\zeta_{i,j} - \zeta_{i-1,j-1}) + (\psi_{i,j+1} - \psi_{i-1,j})(\zeta_{i-1,j+1} - \zeta_{i,j}) + (\psi_{i+1,j} - \psi_{i,j-1})(\zeta_{i,j} - \zeta_{i+1,j-1})],$$
(45)

#### Simple 2D model - Poisson Solver

Solve Poisson equation in 2D using periodical BCs in y:

• The Poisson equation in 2D:

• Fourier transform in y direction:

• Discretization - solving for phi:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \Omega$$

$$\frac{\partial^2 \hat{\phi}}{\partial x^2} - k^2 \hat{\phi} = \hat{\Omega}$$

$$\frac{\hat{\phi}_{i+1,j} - 2\hat{\phi}_{i,j} + \hat{\phi}_{i-1,j}}{\Delta x^2} - k^2 \hat{\phi}_{i,j} = \hat{\Omega}_{i,j}$$

#### Simple 2D model - Poisson Solver

Solve Poisson equation in 2D using general BCs:

• The Poisson equation in 2D:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \Omega$$

• Fourier transform cannot be used (no periodic BCs) -> finite difference matrix solver [1]:

$$\frac{\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{\Delta x^2} + \frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{\Delta y^2} = \Omega_{i,j}$$

### Simple 2D model - Boundary conditions

Left:

• Temperature and density set to 1 (normalization).

Right:

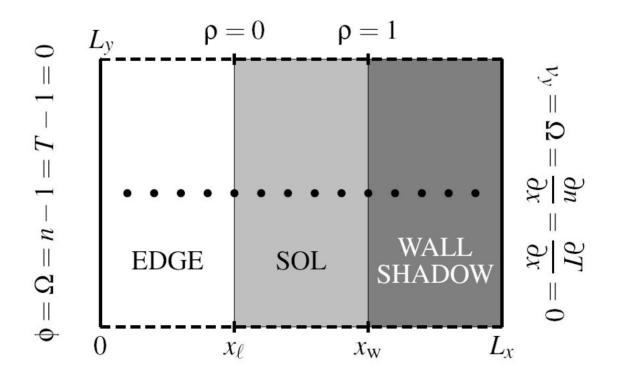
- Neumann for temperature, density and potential.
- Dirichlet for vorticity.

Top and bottom:

• Periodic boundary conditions for all the fields.

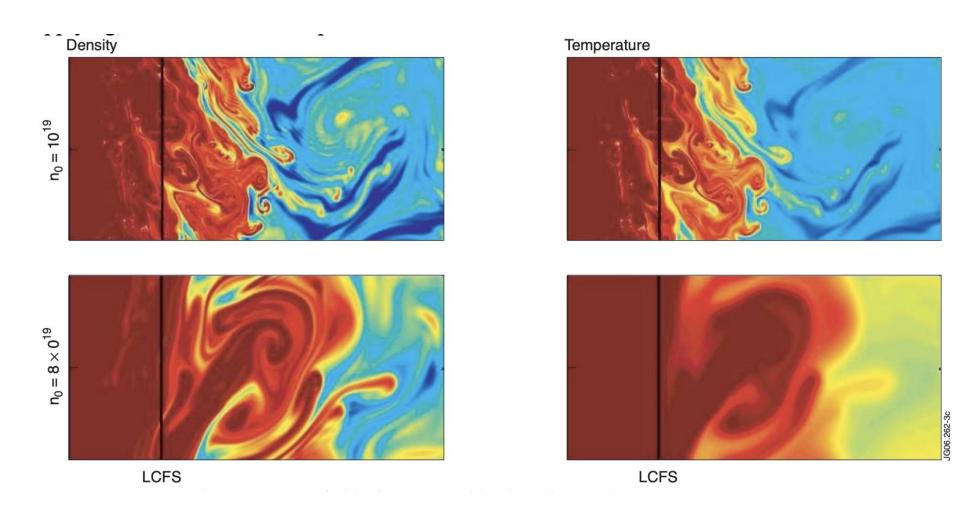
Parallel transport:

- Exponential decay in SOL and WALL SHADOW.
- Represents region of open / closed mg. field lines.



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#### Simple 2D model - results



21st IAEA Fusion Energy Conference

#### Simple 2D model - limits

#### Advantages

- Reduced computational resources.
- Faster simulation time
- Easier interpretation of the results.
- Easier implementation (simple equations).
- Some processes can be reasonably approximated by 2D model.
- Validation of more complex 3D codes.

#### Disadvantages

- Limited accuracy (neglects 3rd dimension).
- Oversimplification (some processes cannot be described in 2D).
- Not possible to perform full-size simulation.
- Cannot describe the complex tokamak geometry.



#### Towards complex 3D fluid model

#### Towards Braginskii equations I

$$\begin{aligned} \frac{\partial f}{\partial t} + \nabla \cdot (vf) + \nabla \cdot \left(\frac{F}{m}f\right) &= C \\ & & \\ \frac{\partial n_{e}}{\partial t} &= -\nabla \cdot (n_{e}\mathbf{v}_{e}) + n_{n}\nu_{iz} - n_{i}\nu_{rec} + S_{n} \end{aligned}$$

$$\begin{aligned} \frac{\partial n_{i}}{\partial t} &= -\nabla \cdot (n_{i}\mathbf{v}_{i}) + n_{n}\nu_{iz} - n_{i}\nu_{rec} + S_{n} \end{aligned}$$

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$$\begin{aligned} \frac{\partial n_{i}}{\partial t} &= -\nabla \cdot (n_{i}\mathbf{v}_{i}) + n_{n}\nu_{iz} - \nabla \cdot \mathbf{v}_{i} - \mathbf{v}_{i} + \frac{\partial n_{i}}{\partial n_{i}} S_{n} \end{aligned}$$

$$\begin{aligned} \frac{\partial n_{i}}{\partial t} &= -\nabla \cdot (n_{i}\mathbf{v}_{i}) + n_{i}\nu_{iz} - \nabla \cdot \mathbf{v}_{i} - \nabla \cdot$$

#### Towards Braginskii equations II

• Assumptions on drift reduced limit, electrostatic limit, etc...

$$\begin{split} \frac{\partial n}{\partial t} + \nabla \cdot \left( \mathbf{V}_{\mathbf{E} \times \mathbf{B}} + \mathbf{V}_{\mathrm{dia,e}} + \mathbf{V}_{\parallel e} \right) &= 0 \\ \frac{nc}{B\omega_{\mathrm{i}}} \frac{d}{dt} \left( -\nabla_{\perp}^{2} \phi - \frac{1}{en} \nabla_{\perp}^{2} p_{\mathrm{i}} \right) + \frac{1}{3m_{\mathrm{i}}\omega_{\mathrm{i}}} \mathbf{b} \times \kappa \cdot \nabla G_{\mathrm{i}} + \nabla_{\parallel} \frac{j_{\parallel}}{e} + \\ \nabla \cdot n \left( \mathbf{V}_{\mathrm{dia,i}} - \mathbf{V}_{\mathrm{dia,e}} \right) &= 0 \\ m_{\mathrm{e}} \frac{dV_{\parallel e}}{dt} &= -\frac{1}{n} \nabla_{\parallel} p_{\mathrm{e}} - \frac{2}{3} \nabla_{\parallel} G_{\mathrm{e}} + e \nabla_{\parallel} \phi - \frac{e}{c} \frac{\psi}{\partial t} + e \frac{j_{\parallel}}{\sigma_{\parallel}} - 0.71 \nabla_{\parallel} T_{\mathrm{e}} \\ m_{\mathrm{i}} \frac{dV_{\parallel i}}{dt} &= -\frac{1}{n} \nabla (p_{\mathrm{i}} + p_{\mathrm{e}}) - p_{\mathrm{i}} \nabla \times \frac{\mathbf{b}}{\omega_{\mathrm{i}}} \cdot \nabla V_{\parallel \mathrm{i}} - \frac{2}{3} \nabla_{\parallel} G_{\mathrm{i}} \\ \frac{3}{2} n \frac{dT_{\mathrm{i}}}{dt} + \frac{3}{2} n \mathbf{V}_{\mathrm{dia,e}} \cdot \nabla T_{\mathrm{e}} + p_{\mathrm{e}} \nabla \cdot \left( \mathbf{V}_{\perp \mathrm{e}} + \mathbf{V}_{\parallel \mathrm{e}} \right) - \frac{5}{2} \frac{c}{e} \nabla \cdot p_{\mathrm{e}} \left( \frac{\mathbf{b}}{B} \times \nabla T_{\mathrm{e}} \right) - \\ 0.71 T_{\mathrm{e}} \nabla_{\parallel} j_{\parallel} - \nabla \cdot \left( \chi_{\parallel \mathrm{e}} \nabla_{\parallel \mathrm{T}_{\mathrm{e}}} \right) = 0 \\ \frac{3}{2} n \frac{dT_{\mathrm{i}}}{dt} + T_{\mathrm{i}} \left[ m \cdot \left( \mathbf{V}_{\mathrm{EB}} + \mathbf{V}_{\parallel \mathrm{e}} \right) + \nabla \cdot \left( n \mathbf{V}_{\mathrm{dia,e}} \right) \right] + \frac{5}{2} \frac{c}{e} p_{\mathrm{i}} \left( \nabla \frac{\mathbf{b}}{B} \right) \cdot \nabla T_{\mathrm{i}} = 0 \end{split}$$

### Towards Braginskii equations III

$$\begin{split} \frac{\partial n}{\partial t} &= -\frac{1}{B} [\phi, n] + \frac{2}{eB} \Big[ C(p_e) - nC(\phi) \Big] - \nabla_{\parallel} (nv_{\parallel e}) + D_n \nabla_{\perp}^2 n + s_n + v_{iz} n_n - v_{rec} n, \quad (1) \\ \frac{\partial \Omega}{\partial t} &= -\frac{1}{B} \nabla \cdot [\phi, \omega] - \nabla \cdot (v_{\parallel i} \nabla_{\parallel} \omega) + \frac{B\Omega_{ci}}{e} \nabla_{\parallel} j_{\parallel} + \frac{2\Omega_{ci}}{e} C(p_e + p_i) \\ &+ \frac{\Omega_{ci}}{3e} C(G_i) + D_\Omega \nabla_{\perp}^2 \Omega - \frac{n_n}{n} v_{cx} \Omega, \quad (2) \\ \frac{\partial U_{\parallel e}}{\partial t} &= -\frac{1}{B} [\phi, v_{\parallel e}] - v_{\parallel e} \nabla_{\parallel} v_{\parallel e} + \frac{e}{m_e} \Big( \frac{j_{\parallel}}{\sigma_{\parallel}} + \nabla_{\parallel} \phi - \frac{1}{en} \nabla_{\parallel} p_e - \frac{0.71}{e} \nabla_{\parallel} T_e - \frac{2}{3en} \nabla_{\parallel} G_e \Big) \\ &+ D_{v_{\parallel e}} \nabla_{\perp}^2 v_{\parallel e} + \frac{n_n}{n} (v_{cn} + 2v_{iz}) (v_{\parallel n} - v_{\parallel e}), \quad (3) \\ \frac{\partial v_{\parallel i}}{\partial t} &= -\frac{1}{B} [\phi, v_{\parallel i}] - v_{\parallel i} \nabla_{\parallel} v_{\parallel i} - \frac{1}{m_i n} \nabla_{\parallel} (p_e + p_i) - \frac{2}{3m_i n} \nabla_{\parallel} G_i \\ &+ D_{v_{\parallel i}} \nabla_{\perp}^2 v_{\parallel i} + \frac{n_n}{n} (v_{iz} + v_{cx}) (v_{\parallel n} - v_{\parallel i}), \quad (4) \\ \frac{\partial T_e}{\partial t} &= -\frac{1}{B} [\phi, T_e] - v_{\parallel e} \nabla_{\parallel} T_e + \frac{2}{3} T_e \Big[ 0.71 \frac{\nabla_{\parallel} j_{\parallel}}{en} - \nabla_{\parallel} v_{\parallel e} \Big] + \frac{4}{3} \frac{T_e}{eB} \Big[ \frac{7}{2} C(T_e) + \frac{T_e}{n} C(n) - eC(\phi) \Big] \\ &+ \nabla_{\parallel} (\chi_{\parallel e} \nabla_{\parallel} T_e) + D_{T_e} \nabla_{\perp}^2 T_e + s_{T_e} - \frac{n_n}{n} v_{cn} m_e^2 \frac{2}{3} v_{\parallel e} (v_{\parallel n} - v_{\parallel e}) \\ &- 2 \frac{m_e}{m_i} \frac{1}{\tau_e} (T_e - T_i) + \frac{n_n}{n} v_{iz} \Big[ -\frac{2}{3} E_{iz} - T_e + m_e v_{\parallel e} \Big( v_{\parallel e} - \frac{4}{3} v_{\parallel n} \Big) \Big], \quad (5) \\ &+ \frac{2}{3} T_i \Big[ (v_{\parallel i} - v_{\parallel e}) \frac{\nabla_{\parallel} n}{n} - \nabla_{\parallel} v_{\parallel e} \Big] + \nabla_{\parallel} (\chi_{\parallel} \nabla_{\parallel} n) + D_{T_e} \nabla_{\perp}^2 T_i + s_{T_i} \\ &+ 2 \frac{m_e}{m_i} \frac{1}{\tau_e} (T_e - T_i) + \frac{n_n}{n} (v_{iz} + v_{cx}) \Big[ T_n - T_i + \frac{1}{3} (v_{\parallel} n - v_{\parallel i})^2 \Big], \quad (6) \end{split}$$

convection
$$[\phi, f] = \mathbf{b} \cdot (\nabla \phi \times \nabla f)$$
curvature $C(f) = \frac{B}{2} (\nabla \times \frac{\mathbf{b}}{B}) \cdot \nabla f$ parallel gadient $\nabla_{\parallel} f = \mathbf{b} \cdot \nabla f + \frac{1}{B} [\Psi, f]$ perp. Laplace $\nabla_{\perp}^2 f = \nabla \cdot \left[ (\mathbf{b} \times \nabla f) \times \mathbf{b} \right]$ Gyroviscous  
terms $G_i = -\eta_{0i} \left[ 2 \nabla_{\parallel} v_{\parallel i} + \frac{1}{B} C(\phi) + \frac{1}{enB} C(p_i) \right]$ 



### **Operators used in fluid codes**

Poisson bracket

Curvature operator

$$[\phi, f] = \frac{B_{\varphi}}{B} (\partial_Z \phi \, \partial_R f - \partial_R \phi \, \partial_Z f)$$

$$C(f) = \frac{B_{\varphi}}{B_0} \partial_Z f$$

Perpendicular laplacian

Parallel laplacian

$$\nabla_{\perp}^2 f = \partial_{RR}^2 f + \partial_{ZZ}^2 f$$

$$\nabla_{\parallel} f = \partial_Z \Psi \, \partial_R f - \partial_R \Psi \, \partial_Z f + \frac{B_{\varphi}}{B_0} \partial_{\varphi} f$$

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### **Derivatives in 3D model**

$$(\partial_Z f)_{i,j,k} = \frac{1}{\Delta Z} \left( \frac{1}{12} f_{i,j,k-2} - \frac{2}{3} f_{i,j,k-1} + \frac{2}{3} f_{i,j,k+1} - \frac{1}{12} f_{i,j,k+2} \right)$$
$$(\partial_{ZZ} f)_{i,j,k} = \frac{1}{\Delta Z^2} \left( -\frac{1}{12} f_{i,j,k-2} + \frac{4}{3} f_{i,j,k-1} - \frac{5}{2} f_{i,j,k} + \frac{4}{3} f_{i,j,k+1} - \frac{1}{12} f_{i,j,k+2} \right)$$

• Poisson equation:

$$abla \cdot \left( n 
abla_{\perp} \phi 
ight) = \Omega - rac{
abla_{\perp}^2 p_i}{e}$$

• Ampere equation:

$$\left(\nabla_{\perp}^{2} - \frac{e^{2}\mu_{0}}{m_{e}}n\right)v_{\parallel e} = \nabla_{\perp}^{2}U_{\parallel e} - \frac{e^{2}\mu_{0}}{m_{e}}nv_{\parallel i} + \frac{e^{2}\mu_{0}}{m_{e}}\overline{j}_{\parallel}$$

-



#### **Complex 3D codes**

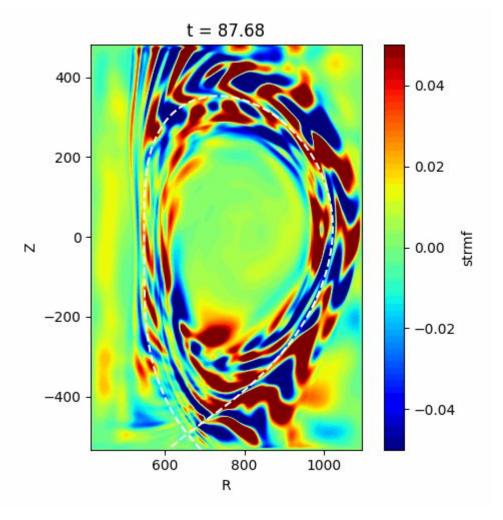
#### 3D, flux-driven turbulence codes, based on drift-reduced Braginskii model:

#### • GBS

- Non-aligned grid, includes plasma core, kinetic neutrals, electromagnetic effects, ion dynamics.
- GRILLIX
  - Cylindrical grid, includes plasma core, electron-ion heat exchange, drift corrections at the magnetic presheath.
  - Evolves parallel component of the electromagnetic vector potential A<sub>1</sub>.
- TOKAM3X
  - Electron-ion heat exchange, drift corrections at the magnetic presheath.
- BOUT++
  - Framework for writing plasma simulations.
  - Any set of equations can be inserted and solved.
  - Can perform fluid or kinetic simulations.

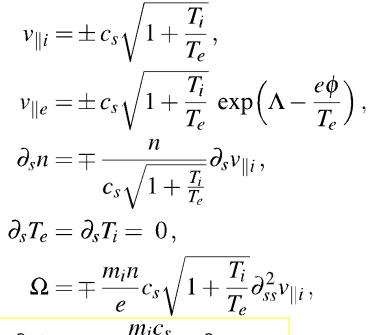
#### **GBS - FLUID TURBULENCE SIMULATION**

- Global Braginskii Solver first principle, 3D, flux-driven, global, turbulence code for plasma edge simulations based on Braginskii equations.
- Full plasma volume, Divertor geometry, electromagnetic effects, kinetic neutrals, ion temperature dynamics, self-consistent turbulence evolution.
- High computational requirements (~2000 cores, ~5-10 M CPU hours / simulation), however still lower compared to full kinetic models.



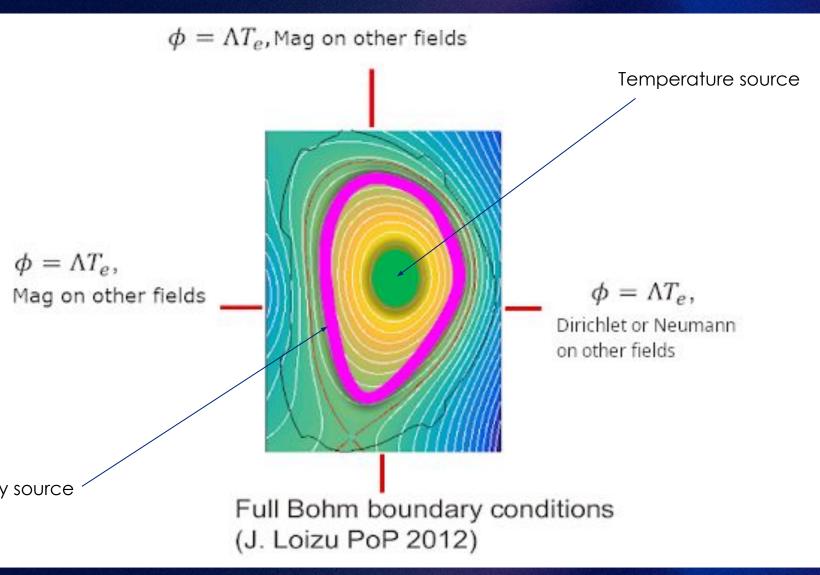
#### **GBS - SIMULATION DOMAIN**

Set of Magnetic boundary conditions (Bohm Chodura boundary conditions)

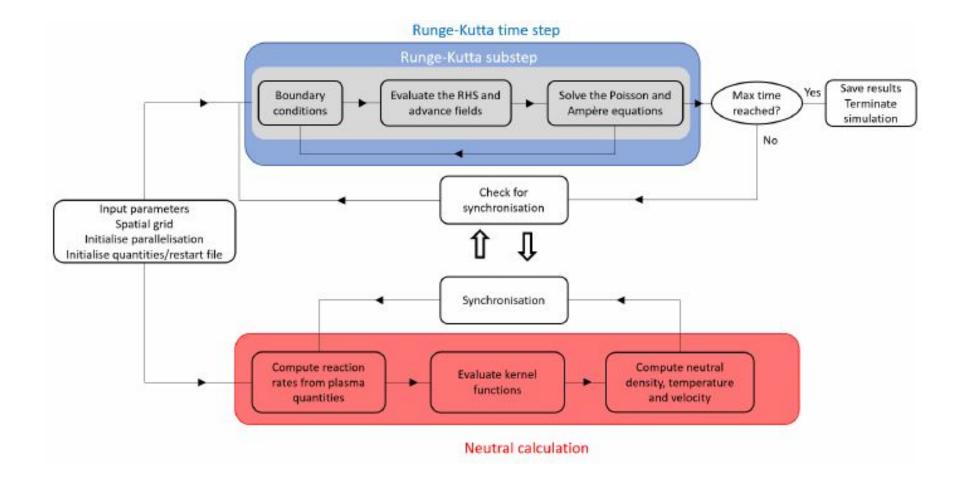


$$\partial_s \phi = \mp rac{m_i c_s}{e \sqrt{1+rac{T_i}{T_e}}} \partial_s 
u_{\parallel i} \, ,$$

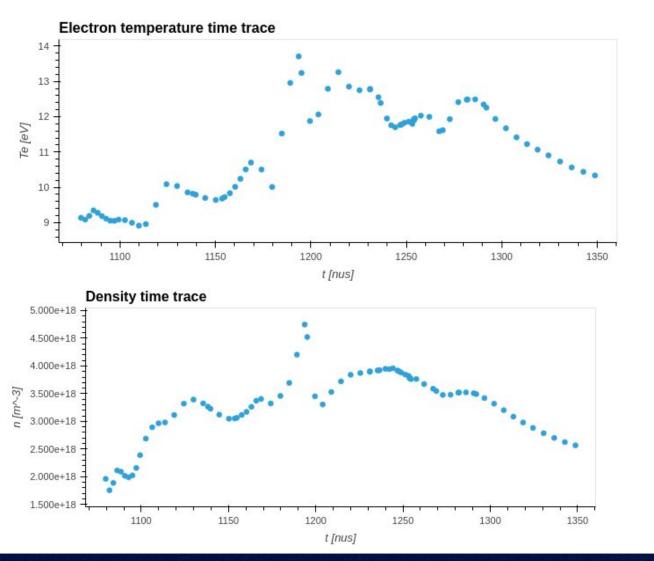
Density source

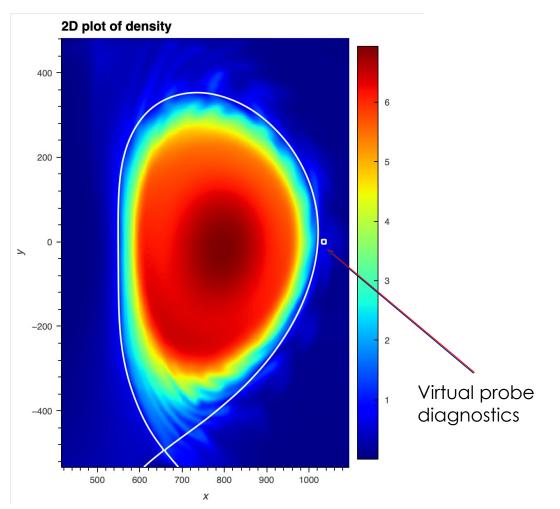


#### **GBS - SIMULATION DOMAIN**



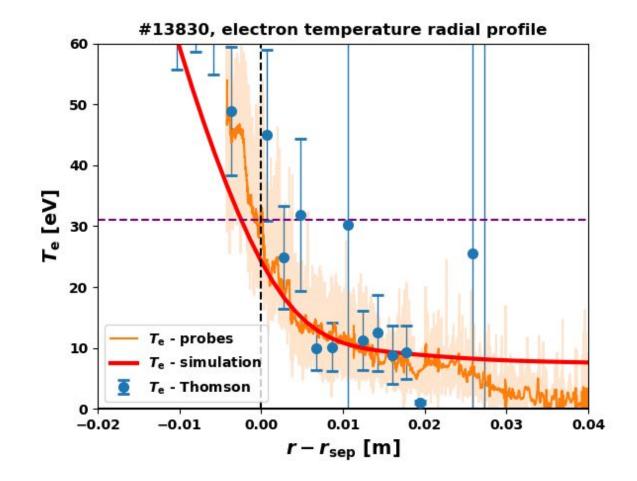
#### SIMULATION RESULTS EXAMPLES





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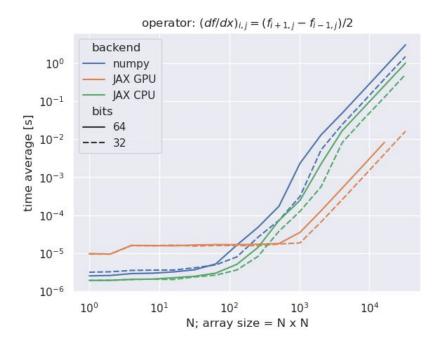
#### SIMULATION RESULTS EXAMPLES



#### Example of derivative on GPU

#### Boosting simulation using GPU

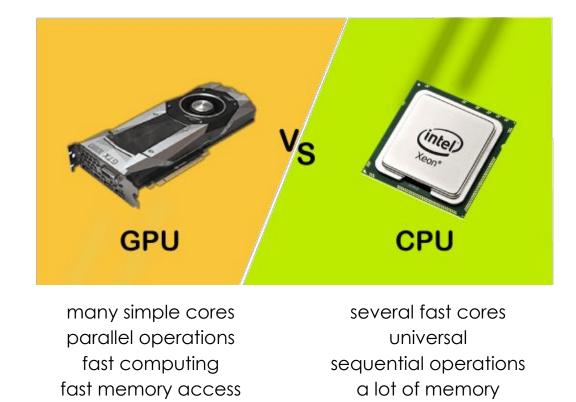
- Python allows simple transformations of the code to GPU.
- All the fluid terms can be transformed.
- Central difference:  $\frac{\partial f(t, x, y)}{\partial x} \approx \frac{f(t, x + \Delta h, y) f(t, x \Delta h, y)}{2\Delta h}$
- Compare of speeds using numpy vs NUMBA vs JAX CPU vs JAX GPU.
- Huge boost on GPU.
- Problem limited memory on GPUs.



#### Using GPUs in Python

#### GPUs using Python

- **JAX** module for calculations on GPU.
- Very easy to use and to convert code into JAX.
- Numpy-like syntax very easy to be used.
- The code is parallelized along GPU automatically.
- JAX insluces **JIT** (just-in-time) compiler boosting the code.
- Uses machine learning for boosting the code even more.





- Turbulence plays a key role in particle and heat transport.
- Understanding and controlling turbulence in plasma edge can lead to better confinement.
- Simulations can provide interpretation or prediction on turbulent transport.
- Fluid models offer higher speeds while still encompassing important physics.
  - Much faster compared to kinetic / gyrokinetic codes.
  - Does not include kinetic effects (Maxwellian distribution is assumed).
- Simple 2D model can be written very easily, providing still good results.
- Complex 3D models are more complicated, but almost the only way to perform full-size simulations.
- Consider using GPUs in your future works.



- 1. M.A. Zaman Electronics **2022**, 11(15), 2365
- 2. M. Giacomin et al J. Comput. Phys. 463 (2022) 111294 (The GBS code for the self-consistent simulation of plasma turbulence and kinetic neutral dynamics in the tokamak boundary)
- 3. M. Giacomin *et al* 2021 *Nucl. Fusion* **61** 076002 (Theory-based scaling laws of near and far scrape-off layer widths in single-null L-mode discharges)



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