Plasma expansion into a vacuum and ion acceleration

Patrick Mora

Centre de Physique Théorique
Ecole Polytechnique, Palaiseau

Prague, september 24, 2003

Motivations

- Plasma expansion into a vacuum: an old problem relevant to recent experiments on ion acceleration
- Importance of a correct theoretical/numerical prediction of the ion energy spectrum and of its cut-off at high energy (in relation with the structure of the ion front)
- Among the few works which did address the structure of the ion front and the ion energy spectrum, no clear picture comes out and contradictory results are given.

Previous description of the ion front

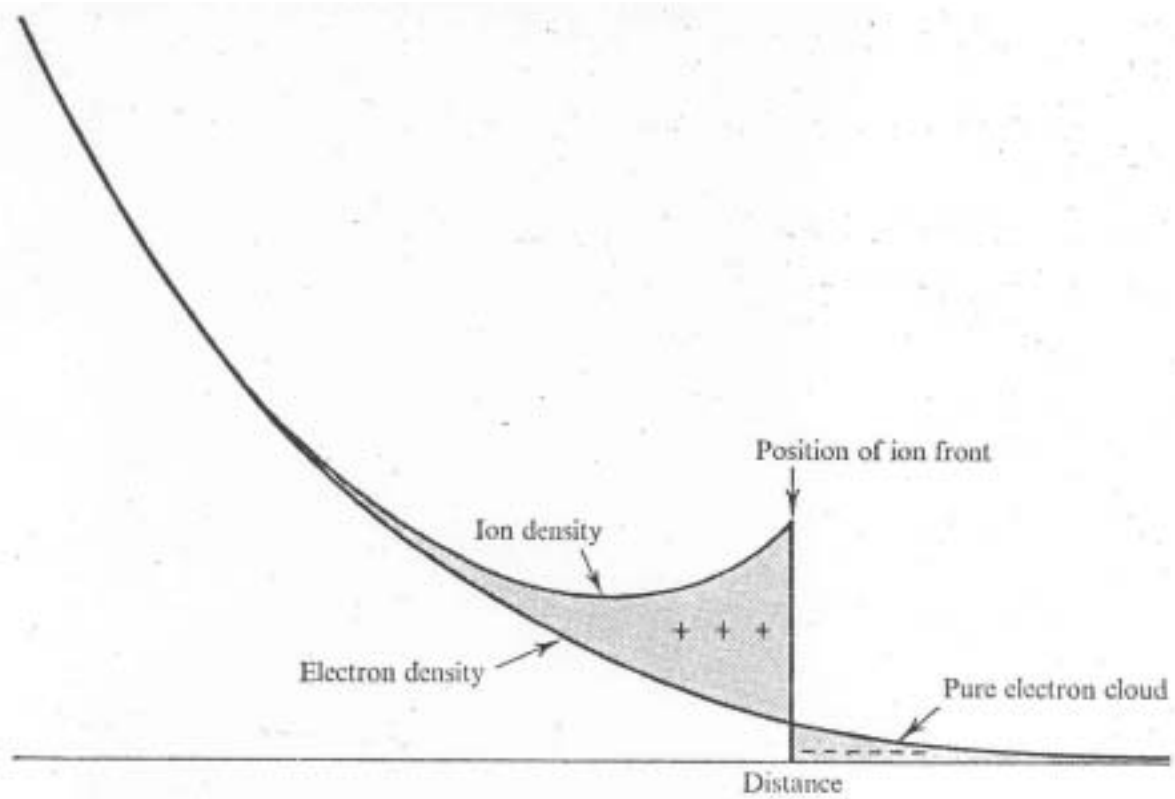


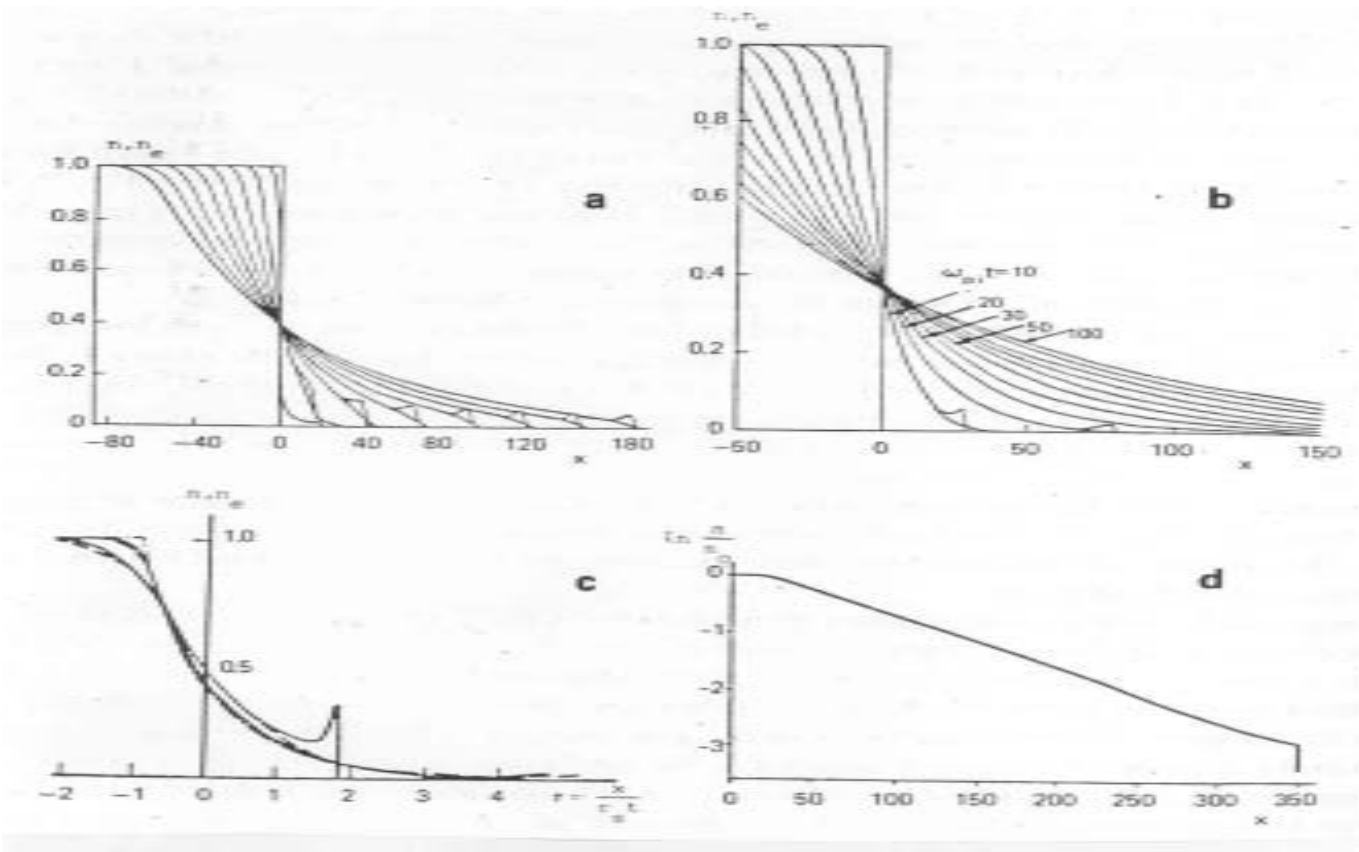
FIGURE 2. Variation of ion and electron densities at the front.

J. E. Crow, *et al.*, Plasma Physics **14**, 65 (1975)

Previous numerical results

Widner *et al.*, Phys. Fluids **14**, 795 (1971)

Crow *et al.*, Plasma Physics **14**, 65 (1975)



Gurevich *et al.*, Sov. Phys JETP **53**, 937 (1981)

True *et al.*, Phys. Fluids **24**, 1885 (1981)

Theoretical model: initial condition

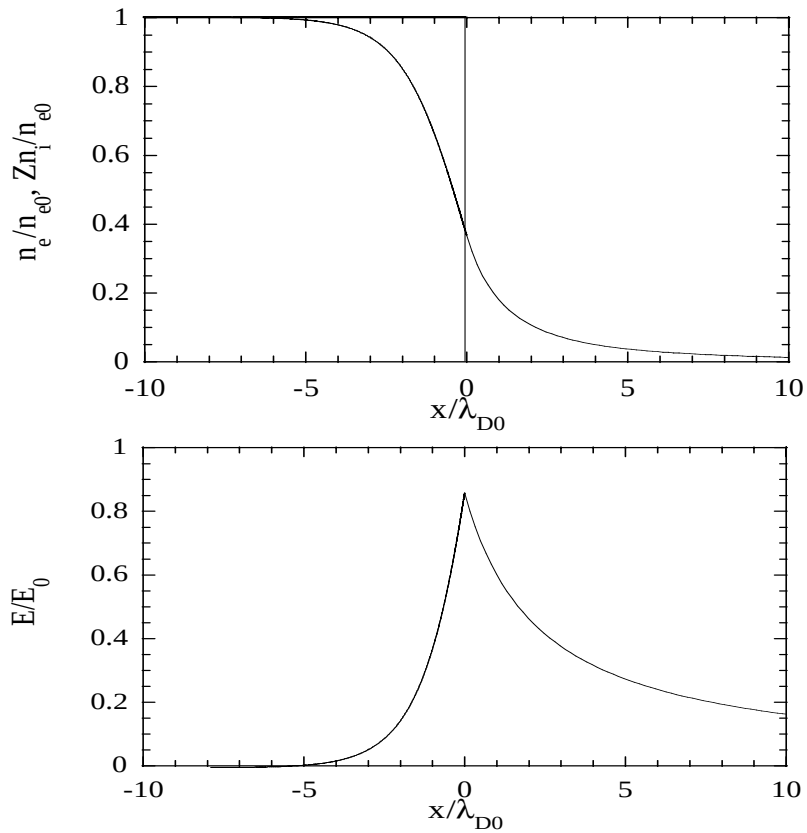
At time t=0 a plasma occupies the half space x<0 and begins to expand into a vacuum.

$$n_e = n_{e0} \exp(e\Phi / k_B T_e)$$

$$\varepsilon_0 \frac{\partial^2 \Phi}{\partial x^2} = e(n_e - Zn_i)$$

$$E_{front} = \sqrt{\frac{2}{e}} E_0$$

$$E_0 = \left(\frac{n_{e0} k_B T_e}{\varepsilon_0} \right)^{1/2} = \frac{k_B T_e}{e \lambda_{D0}}$$



Equations of the model for t>0

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right) n_i = -n_i \frac{\partial v}{\partial x}$$

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right) v = -\frac{Ze}{m_i}\frac{\partial \Phi}{\partial x}$$

$$n_e = n_{e0} \exp(e\Phi / k_B T_e)$$

$$\varepsilon_0 \frac{\partial^2 \Phi}{\partial x^2} = e\left(n_e - Zn_i\right)$$

Self similar solution for t>0 and $x+c_s t>0$

$$\boxed{v = c_s + x/t}$$

$$\boxed{n_e = Zn_i = n_{e0}\exp(-x/c_s t - 1)}$$

$$\boxed{E_{ss} = \frac{k_B T_e}{e c_s t} = \frac{E_0}{\omega_{pi} t}}$$

Validity : $\boxed{c_s t > \lambda_D}$ or $\begin{cases} \boxed{\omega_{pi} t > 1} \\ \boxed{2\ln\left(\omega_{pi} t\right) > 1 + x/c_s t} \end{cases}$

Ion front position and velocity

If one assumes that the ion front coincides with the point where the self-similar solution breaks down, i. e., where $c_s t \approx \lambda_D$, one gets

$$\boxed{x_{front} / c_s t \approx 2 \ln\left(\omega_{pi} t\right) - 1}$$

$$\boxed{v_{front} \approx 2 c_s \ln\left(\omega_{pi} t\right)}$$

Note that this implies

$$\boxed{E_{front} \approx 2 E_{ss} = 2 E_0 / \omega_{pi} t}$$

Numerical solution

Lagrangian code solving fluid eqs. + Poisson eq., using « standard » methods:

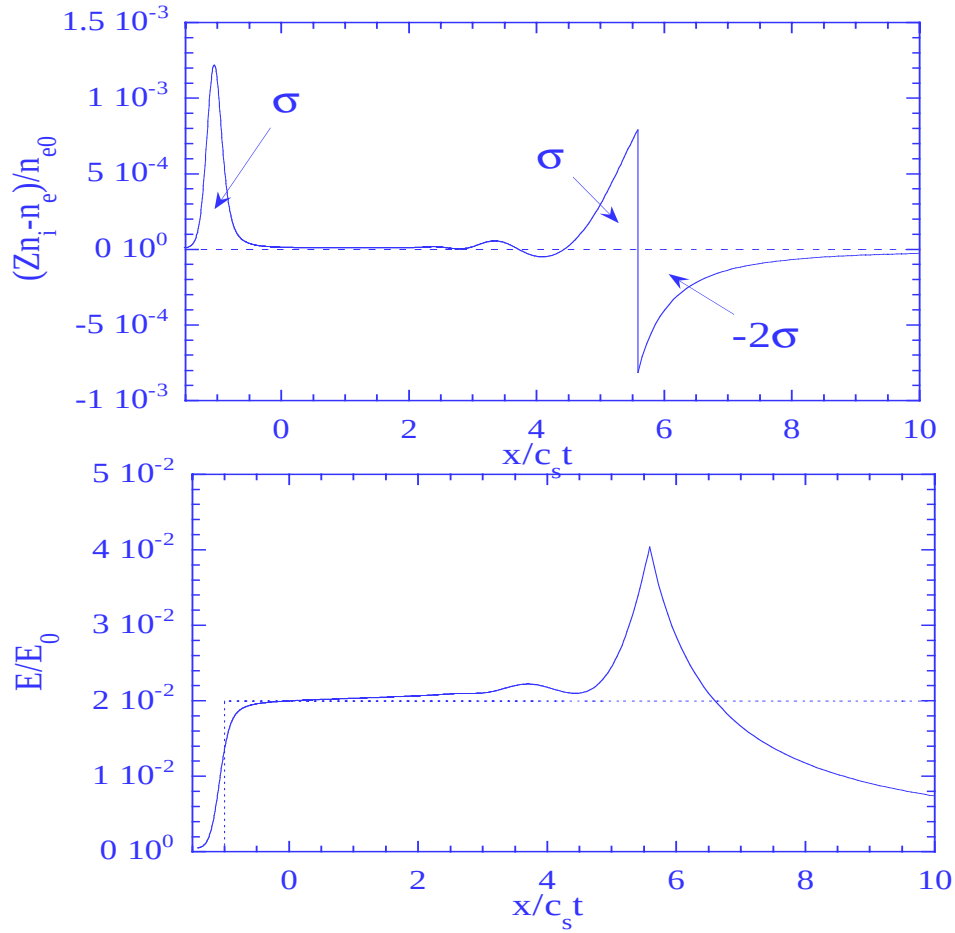
- leap-frog for ion motion,
- nonlinear Poisson Eq. is linearized with respect to small variations from the previous time step solution with a fast converging iterative method:

$$\exp\left(\frac{e\Phi}{k_B T_e}\right) \approx \exp\left(\frac{e\Phi_{old}}{k_B T_e}\right) \times \left(1 - \frac{e\Phi_{old}}{k_B T_e} + \frac{e\Phi}{k_B T_e}\right)$$

Boundary condition :

$$E_{front} = \sqrt{2}\,\frac{k_B T_e}{e\lambda_D}$$

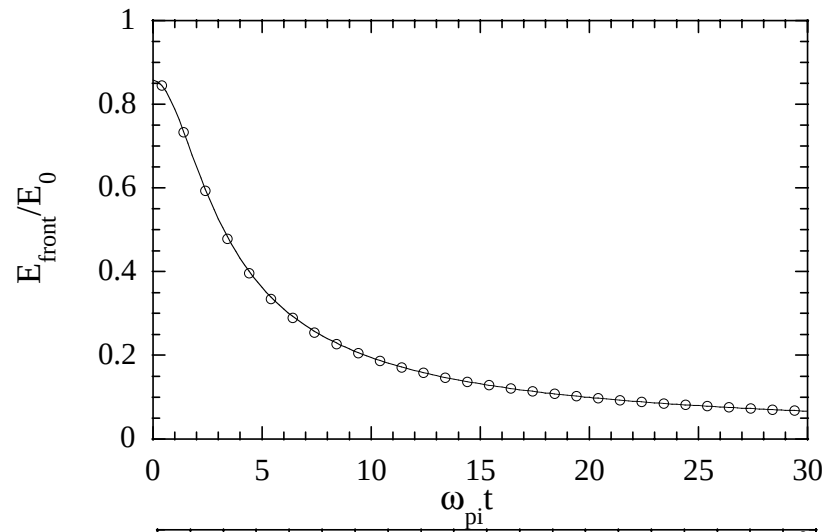
Charge density and electric field at $\omega_{pi}t=50$



$$\sigma = \varepsilon_0 E_{ss}$$

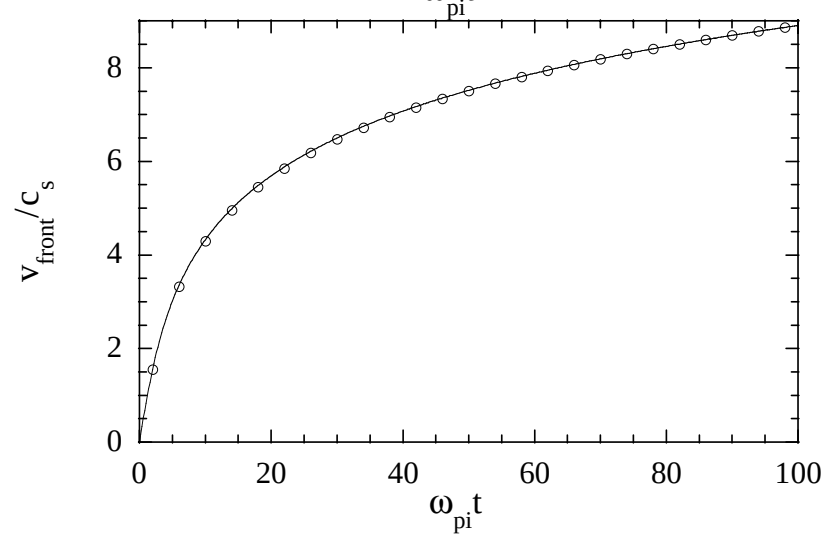
Time evolution of E_{front} and v_{front}

$$E_{front} \approx \frac{2E_0}{\sqrt{2e + \omega_{pi}^2 t^2}}$$



$$v_{front} \approx 2c_s \ln\left(\tau + \sqrt{\tau^2 + 1}\right)$$

where $\tau = \omega_{pi} t / \sqrt{2e}$



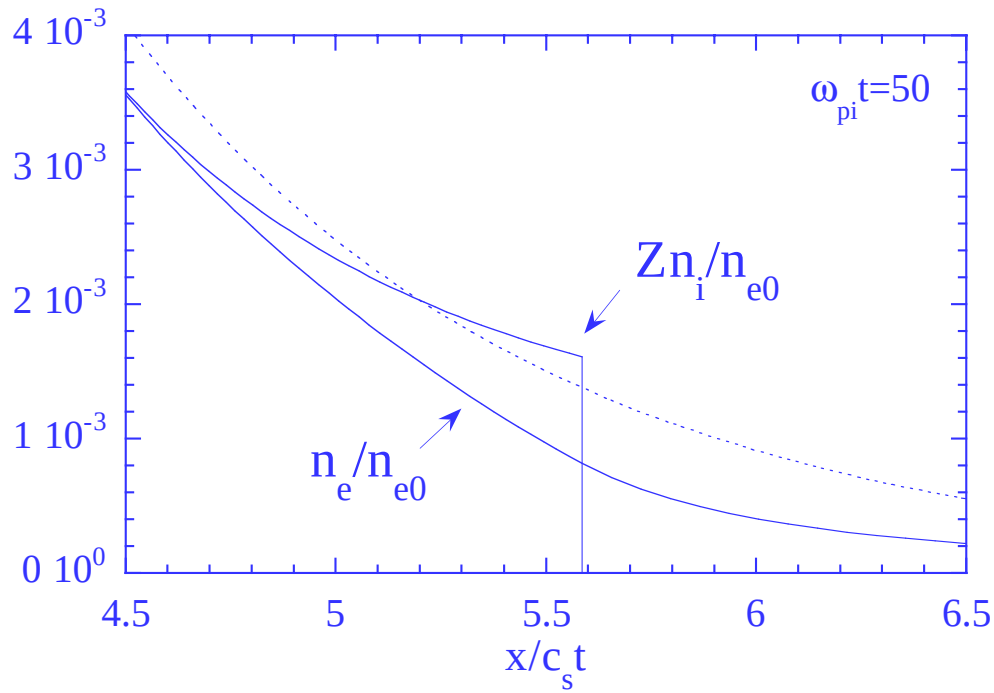
Structure of the ion front

$$n_{e,\,front} \approx \frac{2n_{e0}}{\omega_{pi}^2 t^2}$$

$$n_{i,\,front} \approx \frac{4n_{i0}}{\omega_{pi}^2 t^2}$$

$$\left(\frac{\partial}{\partial x}\ln n_e\right)_{front} \approx -\frac{2}{c_s t}$$

$$\left(\frac{\partial}{\partial x}\ln n_i\right)_{front} \approx -\frac{1}{2c_s t}$$



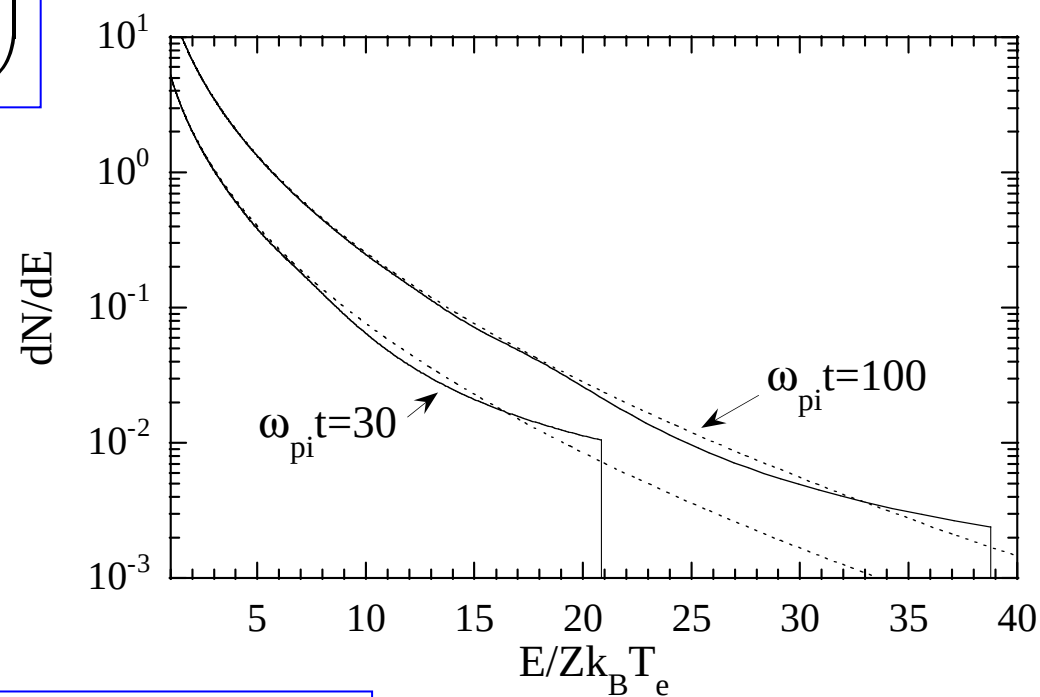
No ion bump !

Energy spectrum

$$\frac{dN}{dE} = \frac{n_{i0} c_s t}{\sqrt{2EE_0}} \exp\left(-\sqrt{\frac{2E}{E_0}}\right)$$

where

$$E_0 = Zk_B T_e$$



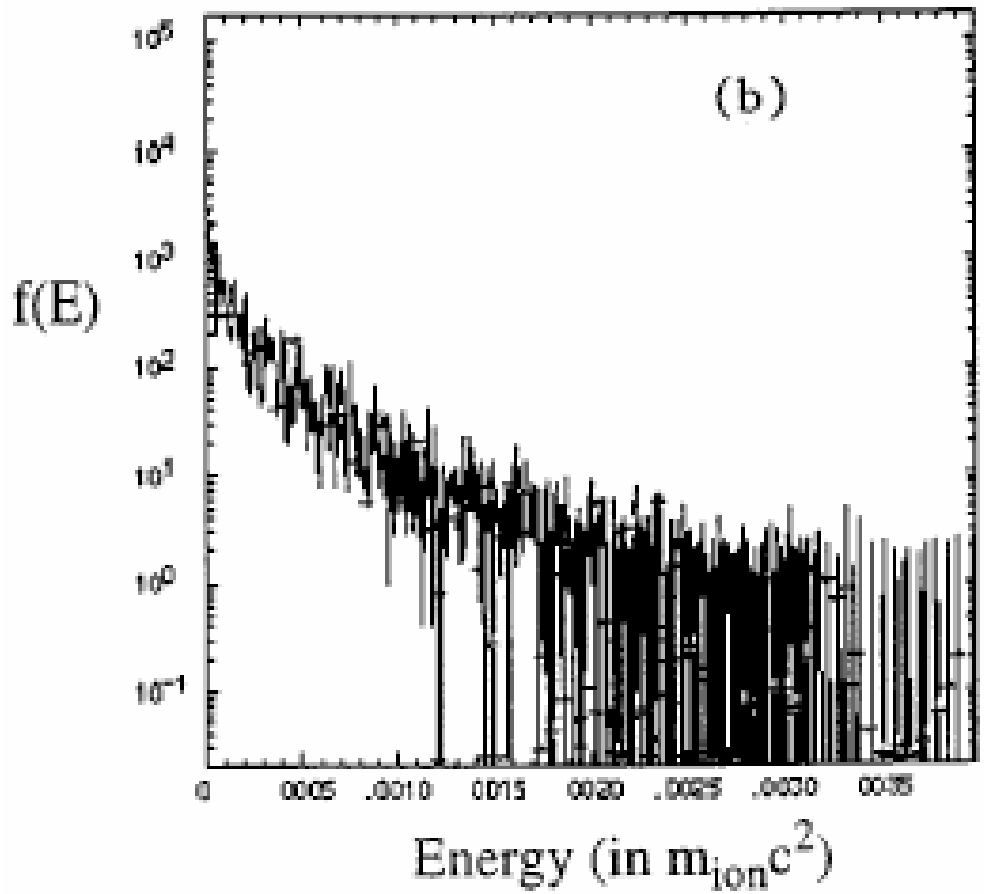
$$E_{max} \approx 2E_0 [\ln(2\tau)]^2$$

$$E_{max} \approx \frac{1}{2} Zk_B T_e \left[2\ln(\omega_{pi} t) + \ln 2 - 1\right]^2$$

Energy spectrum (PIC code)

$$\omega_{pi} t = 74$$

$$T_e = 500\,keV$$

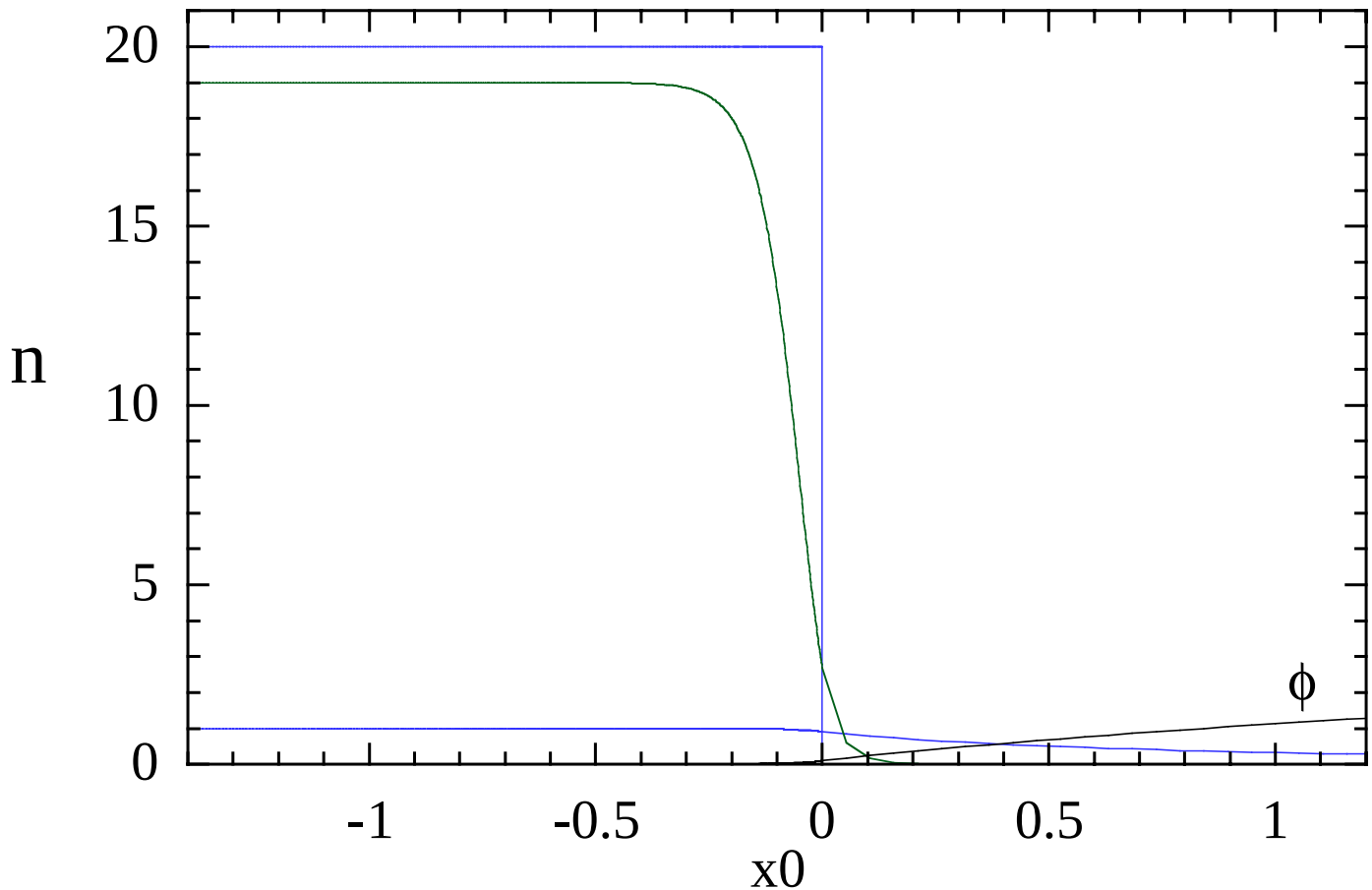


First conclusions and remarks

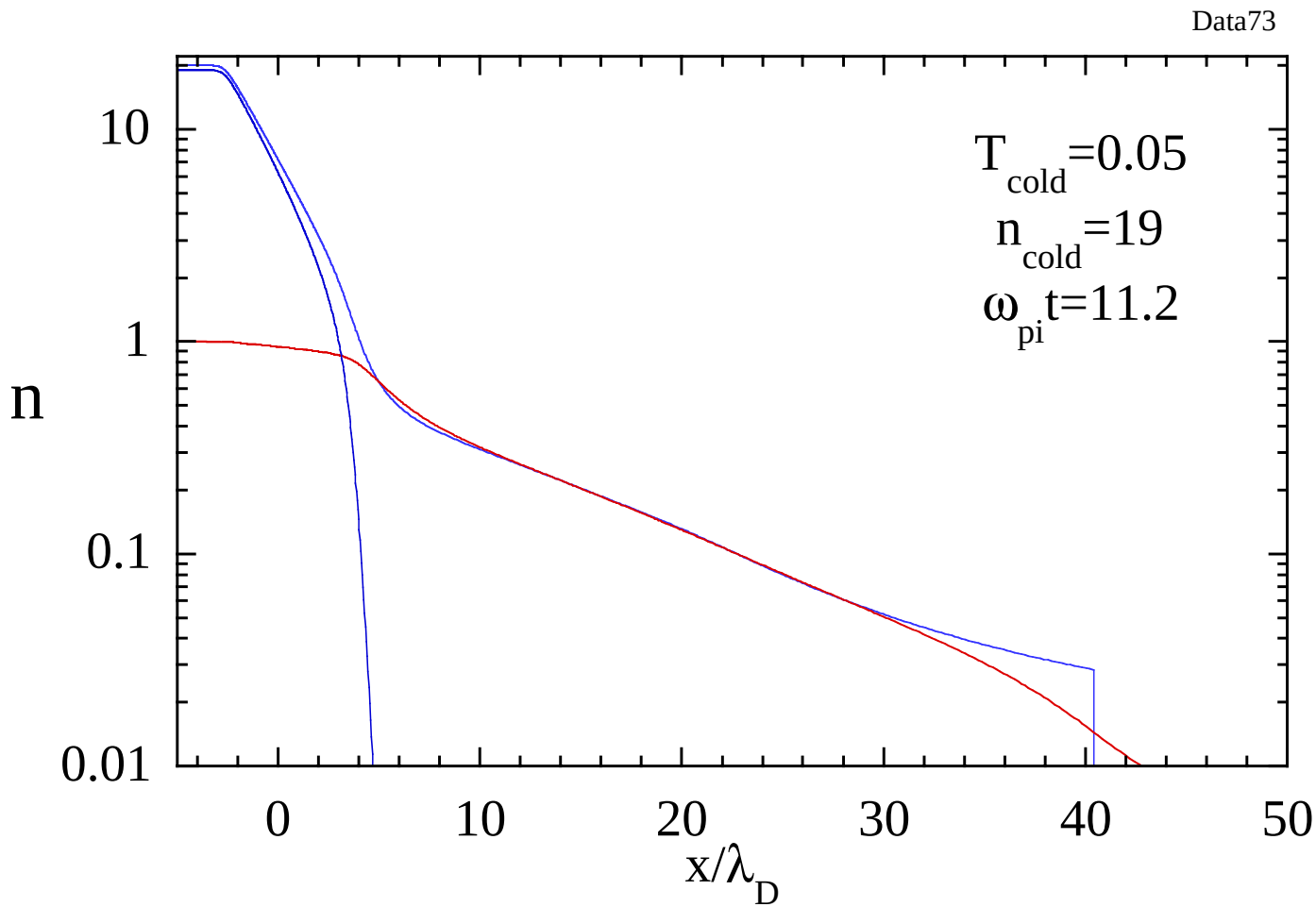
- Controversy about the existence of an ion bump solved

- In the interpretation of a real experiment additional effects have to be taken into account: 2 temperatures, time dependence (energy conservation), etc.

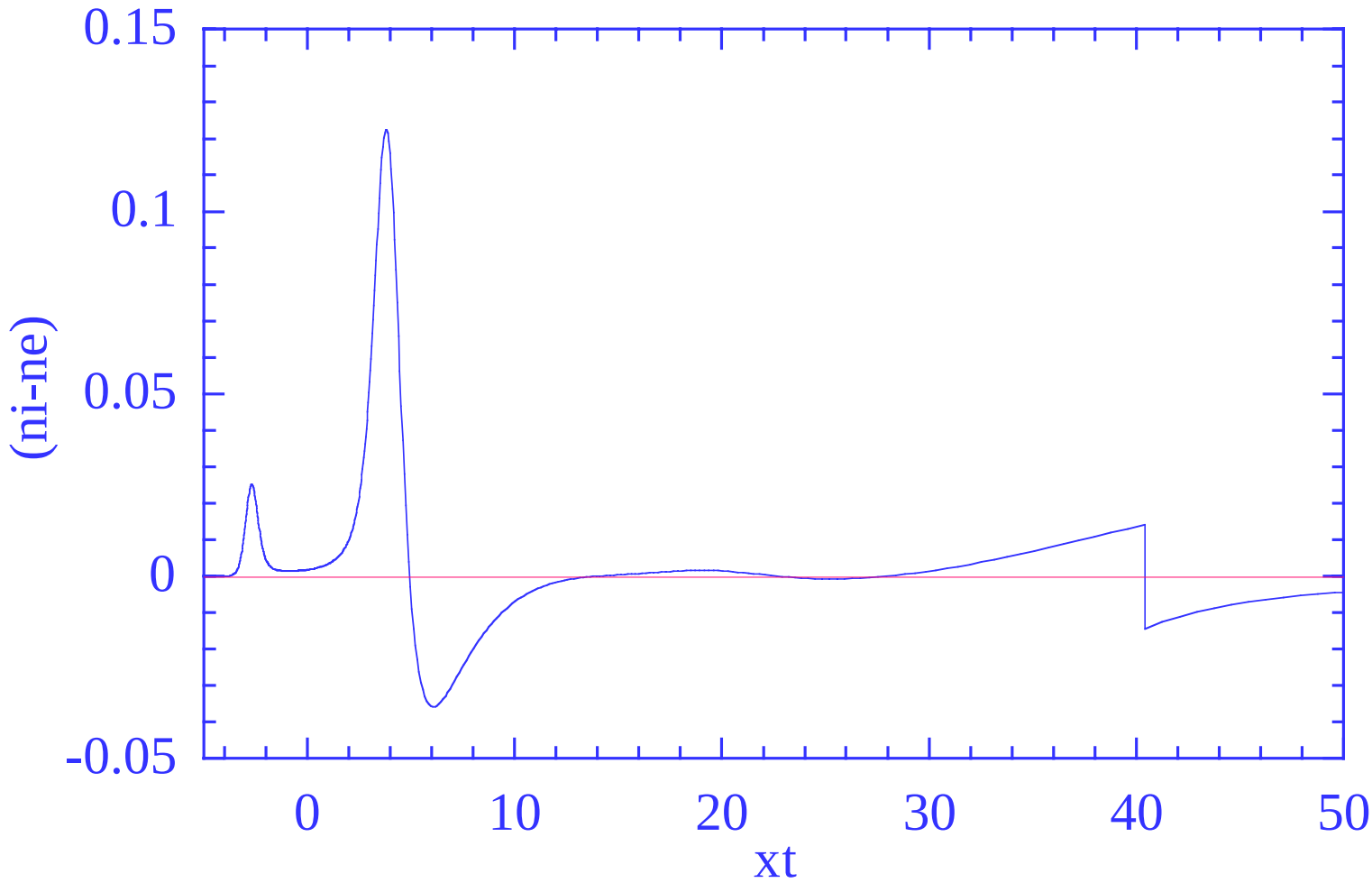
Two temperatures: initial conditions



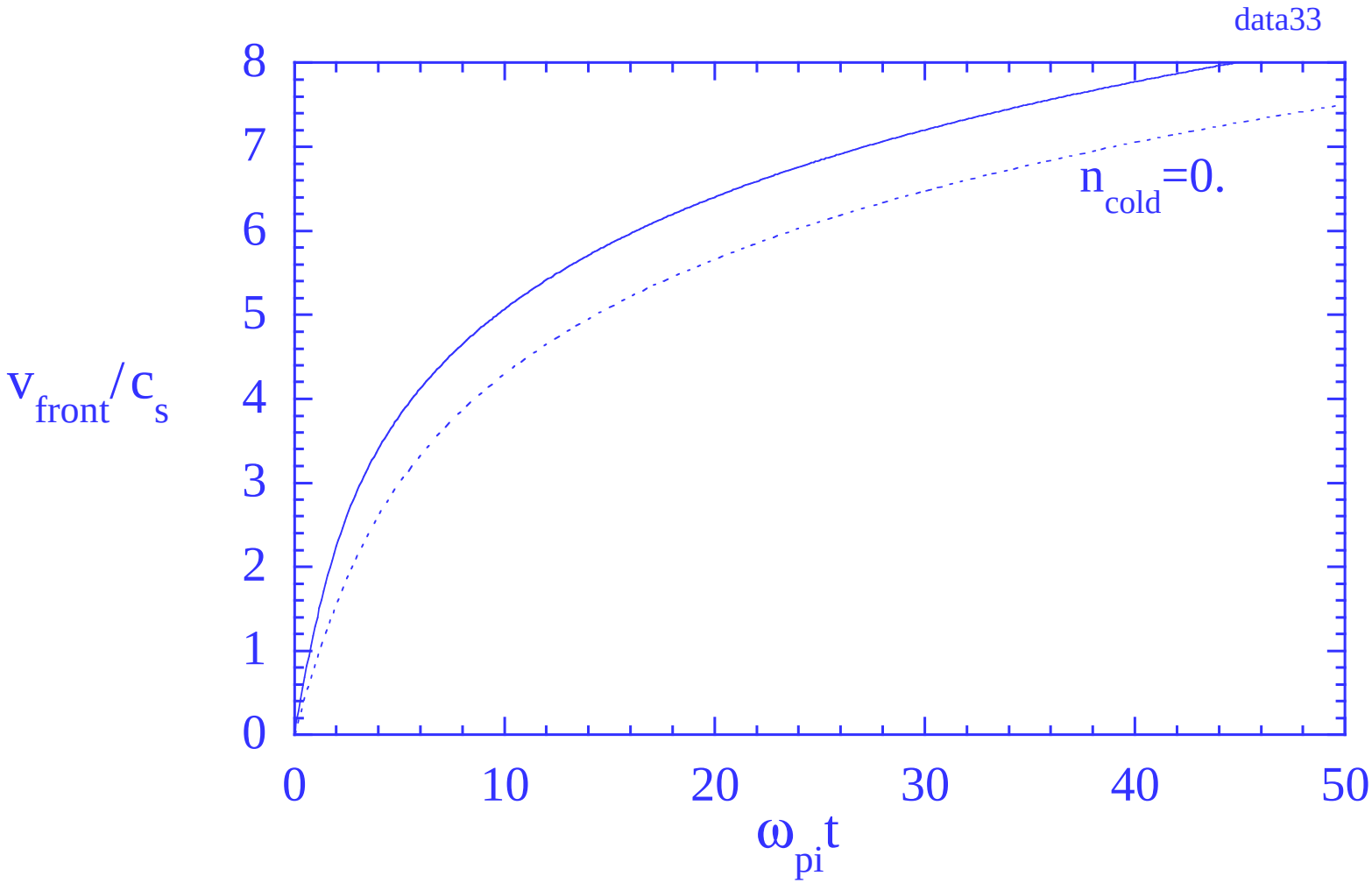
Expansion in the 2 temperatures case:



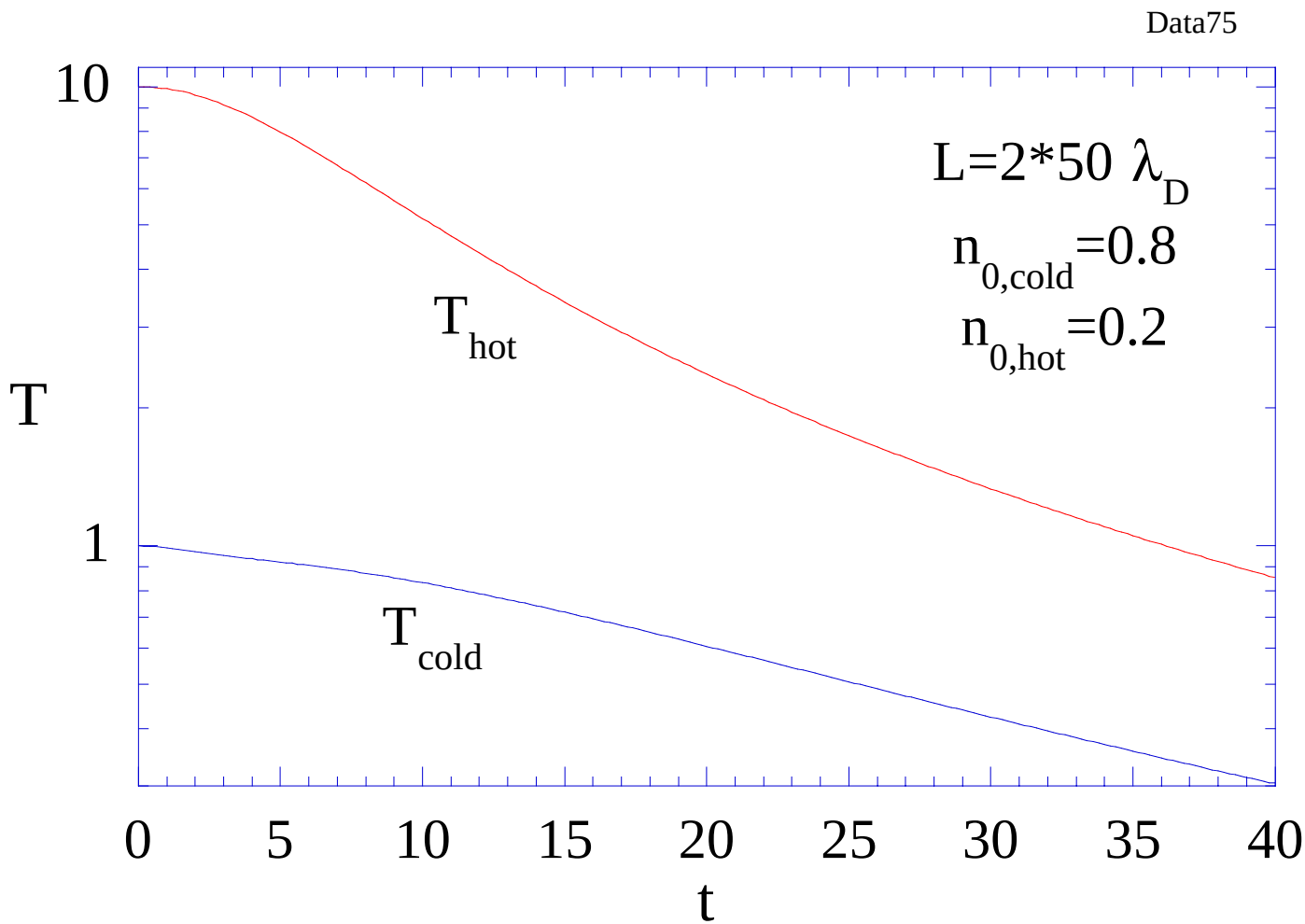
Charge separation in the 2 temperatures case



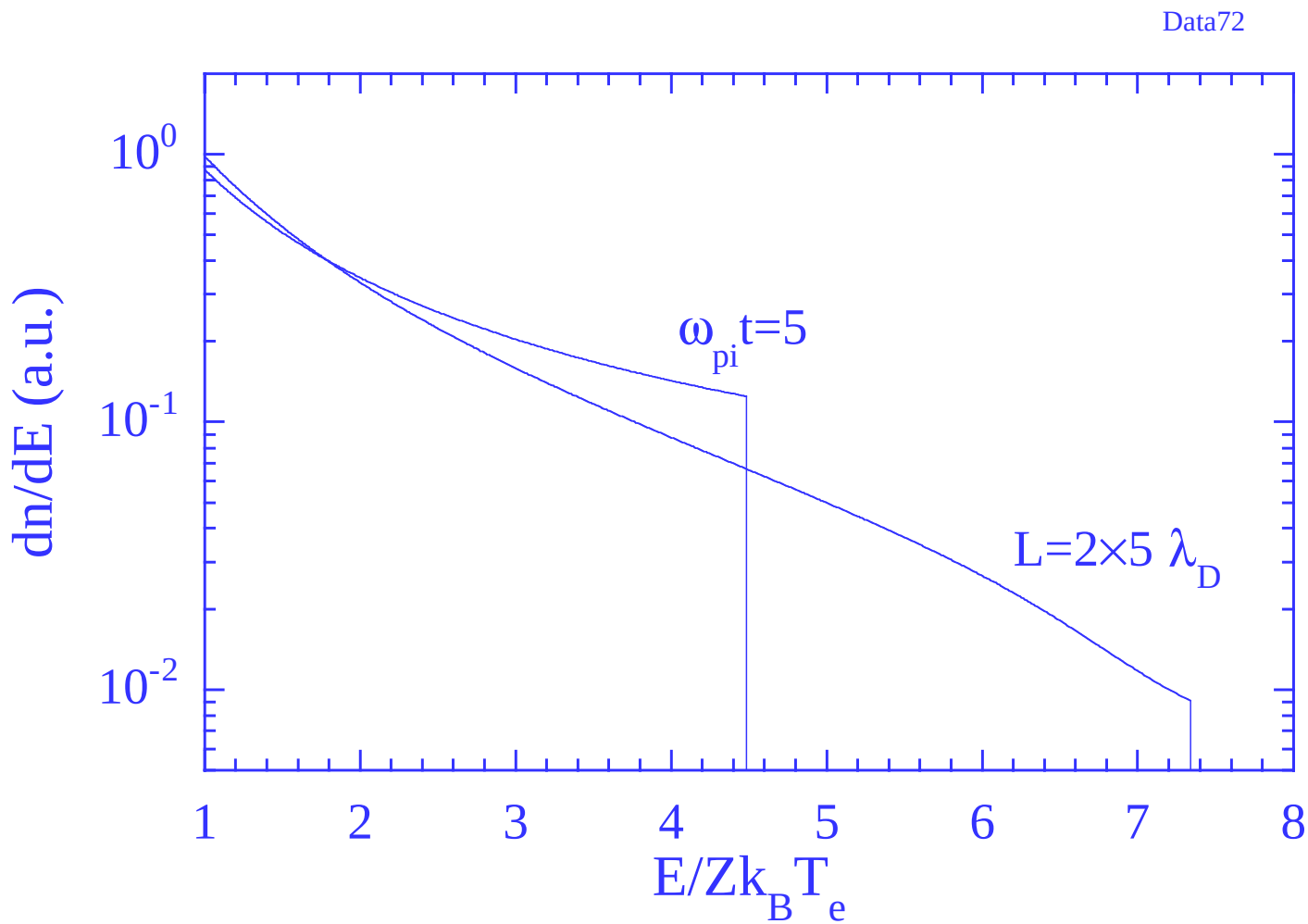
Comparaison with the one-temperature case



Thin foil: time variation of the temperature



Comparaison with the constant temperature case



Conclusion

- Clear picture of the isothermal expansion model

- In the 2 temperatures case, the hot component dominates the expansion: the cold component slightly modifies the fast ions spectrum.

- In the thin foil case, the electron cooling limits the ion energy. The final ion spectrum might differ significantly from the predictions of the constant temperature case frozen at a finite time.