Using Mathematica System in Mechanics Problems.

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Mechanics as a science used computer-aided symbolic calculations since their appearance and indirectly helped to develop computer algebra in its modern form. It is possible to think that mutual interest of these disciplines is partially explained by the fact that Newton's dynamics for long period of its development offered a huge diversity of problems, methods and algorithms. The fields of authors interest presented in this paper are qualitative theory of ordinary differential equations, stability of motion and dynamics of solid body. Using the methods of qualitative analysis requires constructing and possible transformation of motion equations of complex mechanical systems in the Lagrange, Hamilton, Euler-Lagrange, Routh and other forms. It is labour-intensive but well algorithmizable job.

In many problems of gyroscopia, space dynamics and robotics the representation of motion equations in symbolic form allows to avoid unreasonably great numerical experiments, stimulates effective usage and development of qualitative methods. In these areas computer algebra systems usually offer the significant help both on modelling stage (construction of kinetic energy and force function of mechanical system, construction and transformation of motion equations) and during qualitative analysis of obtained equations. This help is significant even for objects of moderate dimension.

Authors of the report together with the colleagues have developed and use software package based on computer algebra system Mathematica. This package implements modelling of motion of solid bodies systems with holonomic links in Lagrange's formalism; construction of kinetic energy and force functions for certain force fields, construction of differential equations in the Lagrange, Hamilton and Routh forms; qualitative analysis of the constructed equations; construction and analysis of stationary solutions stability by second Lyapunov's method (including the first approximation methods that use the Routh-Hurwitz method and its generalizations); finding of first integrals and investigation of system spectrum, bifurcation sets of stationary manifolds, etc. with the help of these integrals. The algorithms of mechanical systems modelling and qualitative analysis of their motion equations are published in [1,2]

Let us discuss several applications of the above mentioned software package.

1. Modelling of solid body (satellite) with a point of mass m_2 attached to main body with visco-elastic spring, moving in Newton's central force field (Fig.1).

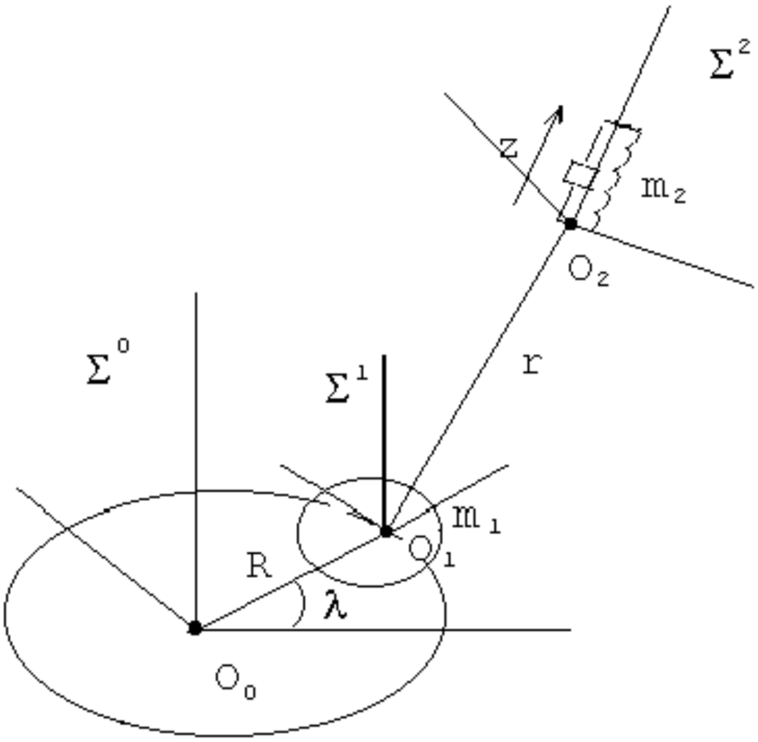

Fig.1.

Motion of holonomical mechanical system with n degrees of freedom in potential force field is described by Lagrange's function

$$L = T + U.$$

$T = T(q, \dot{q})$ is kinetic energy of the system; $U(q)$ is force function of potential force; $q_1, \ldots, q_n$ are generalized coordinates that describe a position of mechanical system in configuration space; $\dot{q}_1, \ldots, \dot{q}_n$ are derivatives of the coordinates by t.

To calculate Lagrange's function of the mechanical system in Fig.1, we must supply the description of the system geometry to program, as it shown below. We use three coordinate systems for this description: inertial Σ^0, orbital Σ^1 and Σ^2 connected to the point m_2.

Program input.

The description of motion of the coordinate system Σ^1 relative to Σ^0:
mass = 0,

$r^0_{O_1} = \{R\cos\lambda, R\sin\lambda, 0\}$,
$r^1_{C_1} = \{0,0,0\}$,
$v^0_{O_1} = \{-Rw_0\sin\lambda, Rw_0\cos\lambda, 0\}$,
$J^{O_1} = \{\{0,0,0\},\{0,0,0\},\{0,0,0\}\}$,
the numbers of rotation axes $= \{3,0,0\}$,
the rotation angles $= \{\lambda,0,0\}$,
the list of coordinates $= \{\lambda\}$.

The description of the satellite motion in the coordinate system Σ^1:
mass $= m_1$,
$r^1_{O_1} = \{0,0,0\}$,
$r^1_{C_1} = \{0,0,0\}$,
$v^1_{O_1} = \{0,0,0\}$,
$J^{O_1} = \{\{A,0,0\},\{0,B,0\},\{0,0,C\}\}$,
the numbers of rotation axes $= \{1,2,1\}$,
the rotation angles $= \{\psi,\theta,\varphi\}$,
the list of coordinates $= \{\psi,\theta,\varphi\}$.

The description of the coordinate system Σ^2 motion relative to Σ^1:
mass $= 0$,
$r^1_{O_2} = \{0,0,r\}$,
$r^2_{C_2} = \{0,0,0\}$,
$v^1_{O_2} = \{0,0,0\}$,
$J^{O_2} = \{\{0,0,0\},\{0,0,0\},\{0,0,0\}\}$,
the numbers of rotation axes $= \{0,0,0\}$,
the rotation angles $= \{0,0,0\}$,
the list of coordinates $= \{\}$.

The description of motion of the point m_2 in the coordinate system Σ^2:
mass $= m_2$,
$r^2_{O_2} = \{0,0,z\}$,
$r^2_{C_2} = \{0,0,0\}$,
$v^2_{O_2} = \{0,0,\dot z\}$,
$J^{O_2} = \{\{0,0,0\},\{0,0,0\},\{0,0,0\}\}$,
the numbers of rotation axes $= \{0,0,0\}$,
the rotation angles $= \{0,0,0\}$,
the list of coordinates $= \{z\}$,
Rayleigh's function $= \frac{1}{2}k_1\dot z^2$,
the potential energy of the spring $= -\frac{1}{2}k_2 z^2$.

Here $r^j_{O_i}$ is the radius-vector of attachment point O_i of i-th and j-th bodies in coordinate system Σ^j; $r^i_{C_i}$ is the radius-vector of mass center of i-th body in coordinate system Σ^i; $v^j_{O_i}$ is the relative speed of point O_i in projections on axes of Σ^j; J^{O_i} is the inertia tenzor of solid body; O_1 is the inertia center of the satellite; $w_0 = \dot{\lambda}$ is the angle velocity of mass center O_1; m_1 is the mass of the satellite; A, B, C are the main central inertia moments of the satellite; $R = |O_0\ O_1|$; $r = |O_1\ O_2|$; z is the extension of the spring; k_1, k_2 are the viscosity and the elasticity coefficients of the spring.

Program output.

Lagrange's function:

$$(-(m_2Rw_0(r+z)\cos\varphi\cos\psi) + m_2Rw_0(r+z)\cos\theta\sin\varphi\sin\psi - w_0(A+m_2r^2+$$

$$+2m_2rz+m_2z^2)\cos\psi\sin\theta)\dot{\varphi} + \frac{A+m_2r^2+2m_2rz+m_2z^2}{2}\dot{\varphi}^2+$$

$$+(-(m_2Rw_0(r+z)\cos\varphi\cos\psi\cos\theta)+m_2Rw_0(r+z)\sin\varphi\sin\psi - w_0(A+m_2r^2+$$

$$+2m_2rz+m_2z^2)\cos\psi\cos\theta\sin\theta + Cw_0\cos^2\varphi\cos\psi\cos\theta\sin\theta-$$

$$-Cw_0\cos\varphi\sin\varphi\sin\psi\sin\theta+w_0(B+m_2r^2+2m_2rz+m_2z^2)\sin\varphi(\cos\psi\cos\theta\sin\varphi+$$

$$+\cos\varphi\sin\psi)\sin\theta)\dot{\psi} + (A+m_2r^2+2m_2rz+m_2z^2)\cos\theta\dot{\varphi}\dot{\psi}+$$

$$+\frac{A\cos^2\theta + C\cos^2\varphi\sin^2\theta + B\sin^2\varphi\sin^2\theta + m2(r+z)^2(\cos^2\theta+\sin^2\varphi\sin^2\theta)}{2}\dot{\psi}^2+$$

$$+(-(Cw_0\cos\varphi\cos\psi\cos\theta\sin\varphi) + Cw_0\sin^2\varphi\sin\psi + w_0(B+m_2r^2+2m_2rz+$$

$$+m_2z^2)\cos\varphi(\cos\psi\cos\theta\sin\varphi+\cos\varphi\sin\psi)+m_2Rw_0(r+z)\cos\varphi\sin\psi\sin\theta)\dot{\theta}+$$

$$+(B-C+m_2r^2+2m_2rz+m_2z^2)\cos\varphi\sin\varphi\sin\theta\dot{\psi}\dot{\theta}+$$

$$+\frac{(B+m_2r^2+2m_2rz+m_2z^2)\cos^2\varphi + C\sin^2\varphi}{2}\dot{\theta}^2-$$

$$-m_2Rw_0(\cos\psi\sin\varphi+\cos\varphi\cos\theta\sin\psi)\dot{z} + \frac{m2}{2}\dot{z}^2+$$

$$\frac{A\nu+B\nu+C\nu+2m_1\nu R^2+2m_2\nu R^2+m_1R^5w_0^2+m_2R^5w_0^2-k_2R^3z^2}{2R^3} - \frac{3}{2}Aw_0^2\cos^2\theta-$$

$$-\frac{m_2w_0^2(r+z)^2(\cos^2\varphi\cos^2\theta+\sin^2\varphi)}{2}+(B-C)w_0^2\cos\varphi\cos\psi\cos\theta\sin\varphi\sin\psi+$$

$$+\frac{Bw_0^2(cos^2\psi\cos^2\theta\sin^2\varphi+cos^2\varphi\sin^2\psi)}{2}+\frac{Cw_0^2(\cos^2\varphi\cos^2\psi\cos^2\theta+\sin^2\varphi\sin^2\psi)}{2}+$$

$$+\frac{m_2(-\nu+R^3w_0^2)(r+z)\cos\varphi\sin\theta}{R^2} - \frac{3}{2}Cw_0^2\cos^2\varphi\sin^2\theta + \frac{Aw_0^2\cos^2\psi\sin^2\theta}{2}-$$

$$-\frac{3}{2}Bw_0^2\sin^2\varphi\sin^2\theta + m_2w_0^2(r+z)^2\cos\varphi(\cos\psi\cos\theta\sin\varphi\sin\psi+\cos\varphi\sin^2\theta)+$$

$$+\frac{m_2 w_0^2 (r+z)^2 (\cos^2\psi\cos^2\theta\sin^2\varphi + \cos^2\varphi\sin^2\psi + \cos^2\psi\sin^2\theta)}{2}.$$

Rayleigh's function: $\frac{1}{2}k_1\dot{z}^2$
The list of generalized coordinates: $\{\psi, \theta, \varphi, z\}$
Elapsed time: 0 h. 0 min. 43:1 sec.

2. Modelling of RLC-circuits.

Authors tried to apply the methods of modelling and analysis of mechanical systems to electrical ones. The results of computer-aided modelling and analysis of linear circuits are presented in [3]. Here nonlinear circuits are discussed.

According to [5,6], the state of nonlinear RLC-circuit can be described by the equations system:

$$L_k(i_k)\frac{di_k}{dt} = \frac{\partial P}{\partial i_k}, \quad (k = 1, \ldots, r)$$

$$C_l(v_l)\frac{dv_l}{dt} = -\frac{\partial P}{\partial v_l}, \quad (l = r+1, \ldots, s)$$

where $L_k(i_k)$ is the inductance; $C_l(v_l)$ is the capacitance; $i = (i_1, \ldots, i_r)$ denotes the vector of currents; $v = (v_{r+1}, \ldots, v_{r+s})$ denotes the vector of voltages; *P(i,v)* is the function called the mixed potential that is defined like

$$P(i,v) = \int\limits_{\Gamma} \sum_{\mu > r+s} v_\mu di_\mu + \sum_{l=r+1}^{r+s} i_l v_l |_\Gamma.$$

Γ is a curve in the space of currents and voltages of circuit where Kirchhoff's laws are correct.

The function $P(i,v)$ can be used not just for the construction of circuit equations, but also for qualitative analysis of them. In latter case it could be treated as an analogue for Lyapunov's function.

In [5] it is discussed the construction of the mixed potential for the class of *complete* circuits. The *complete* means that a circuit is described by the *complete* set of variables. The complete set of variables is chosen independently without leading to a violation of Kirchhoff's laws and determines current or voltage in any branch of circuit.

The set of currents in inductors and voltages across capacitors is complete. If a circuit is not *complete* then it is transformed to the *complete* one by adding inductors in series and capacitors in parallel. The original circuit is considered as a limiting case of the new one.

Let us describe the procedure of constructing the mixed potential for complete circuits. It consists of the following steps.

1. Choose in the graph a maximal tree τ with the set of links α.

2. Choose in τ a subtree $\dot{\tau}$ with the set of links $\dot{\alpha}$. $\dot{\alpha}$ is all branches which connect two nodes of $\dot{\tau}$ and form a loop with branches of $\dot{\tau}$ only.

According to [5], the currents $i_1, \ldots, i_r$ in $\alpha - \dot{\alpha}$ and the voltages $v_{r+1}, \ldots, v_{r+s}$ in $\dot{\tau}$ form the *complete* set of variables. This is correct also for $r = 0, s > 0$; $r > 0, s = 0$; $rs > 0$.

3. Calculate the mixed potential by the formula:

$$P(i^*, v^*) = F(i^*) - G(v^*) + (i^*, \gamma v^*), \tag{1}$$

$$F(i^*) = \sum_{\mu > r+s, \Omega_i} \int_\Gamma v_\mu di_\mu,\ G(v^*) = \sum_{\mu > r+s, \Omega_v} \int_\Gamma i_\mu dv_\mu,\ (i^*, \gamma v^*) = \sum_{\rho=1}^{r} i_\rho \sum_{\Lambda_\rho \cap \dot{\tau}} \pm v_\sigma, \tag{2}$$

where $i^* = (i_1, \ldots, i_r)$; $v^* = (v_{r+1}, \ldots, v_{r+s})$; $\gamma = (\gamma_{\rho\sigma})$ is matrix $r \times s$; $\gamma_{\rho\sigma} =$ +1,-1,0;

Ω_i is the set of branches where the currents can be determined from $i_1, \ldots, i_r$ by Kirchhoff's node law; Ω_v is the set of branches where the voltages can be determined from $v_{r+1}, \ldots, v_{r+s}$ by Kirchhoff's loop law; Λ_ρ $(\rho \in \alpha - \dot{\alpha})$ is a loop; i_ρ is a loop current; v_σ is the voltage of branches of $\dot{\tau}$ in Λ_ρ.

To illustrate the procedure of constructing the mixed potential we will consider the circuit in Fig.2 and its graph in Fig.3.

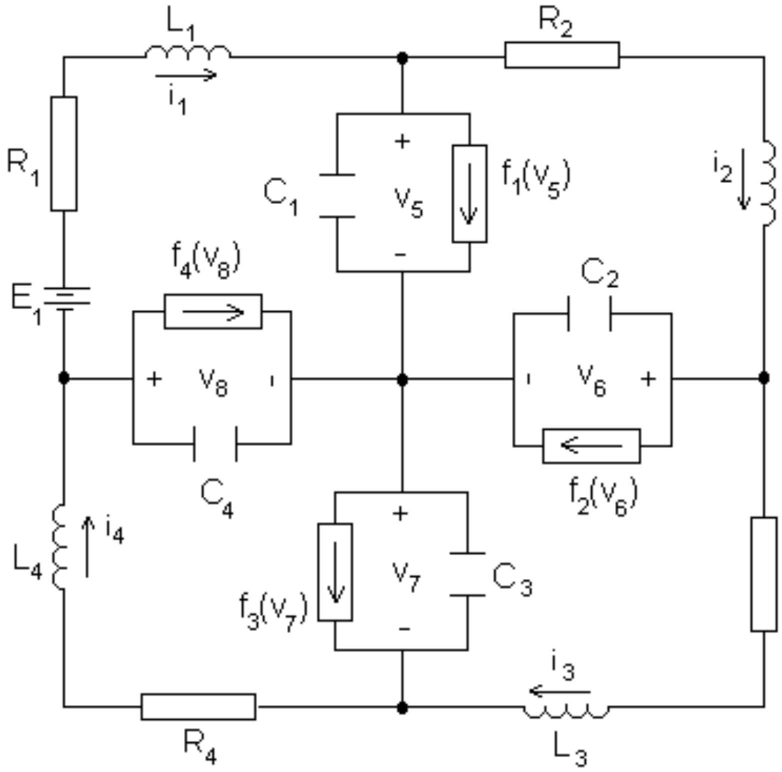
L1
R2
i1
R1
C1
+
v5
-
f1(v5)
i2
f4(v8)
C2
E1
+
v8
-
v6
+
C4
f2(v6)
L4
i4
+
v7
-
C3
f3(v7)
i3
R4
L3

Fig.2.

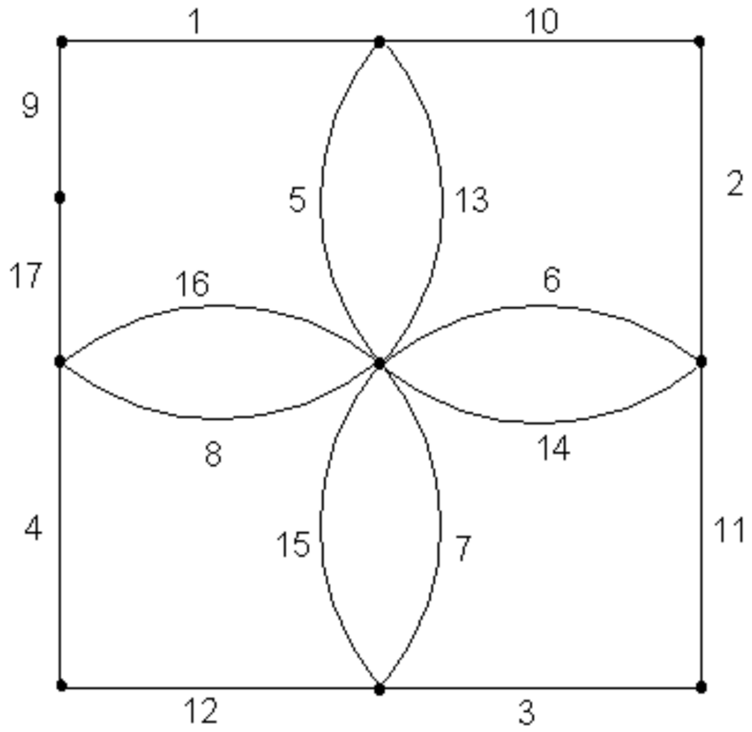

Fig.3.

The graph of the circuit of Figure 2

Here the nonlinear elements are resistors $f_1(v_5),\ f_2(v_6),\ f_3(v_7),\ f_4(v_8)$.

The maximal tree τ consists of branches $\{5,6,7,8,9,10,11,12,17\}$, $\alpha = \{1,2,3,4,13,14,15,16\}$, $\grave{\tau} = \{5,6,7,8\}$, $\grave{\alpha} = \{13,14,15,16\}$, $\alpha - \grave{\alpha} = \{1,2,3,4\}$. So the currents i_1, i_2, i_3, i_4 in the inductors L_1, L_2, L_3, L_4 and the voltages v_5, v_6, v_7, v_8 across the capacitors C_1, C_2, C_3, C_4 are the complete set of variables.

Now we can construct the mixed potential by the formula (1).

$\Omega_i = \{1,2,3,4,9,10,11,12,17\}$. According to (2), $F(i_1, i_2, i_3, i_4)$ is calculated only for the resistive branches $\{9,10,11,12,17\}$.

$$F(i_1, i_2, i_3, i_4) = E_1 i_1 - \frac{1}{2}R_1 i_1^2 - \frac{1}{2}R_2 i_2^2 - \frac{1}{2}R_3 i_3^2 - \frac{1}{2}R_4 i_4^2.$$

$\Omega_v = \{5,6,7,8,13,14,15,16\}$. We choose the branches $\{13,14,15,16\}$ with the resistors.

$$G(v_5, v_6, v_7, v_8) = -\int_0^{v_5} f_1(v)dv - \int_0^{v_6} f_2(v)dv - \int_0^{v_7} f_3(v)dv - \int_0^{v_8} f_4(v)dv.$$

To calculate $(i^*, \gamma v^*)$ we construct the loops $\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4$ and consider their intersections with $\dot{\tau}$.

$\Lambda_1 = \{1, 5, 8, 17, 9\}$, $\Lambda_2 = \{2, 6, 5, 10\}$, $\Lambda_3 = \{3, 7, 6, 11\}$, $\Lambda_4 = \{4, 8, 7, 12\}$. $\Lambda_1 \cap \dot{\tau} = \{5, 8\}$, $\Lambda_2 \cap \dot{\tau} = \{5, 6\}$, $\Lambda_3 \cap \dot{\tau} = \{6, 7\}$, $\Lambda_4 \cap \dot{\tau} = \{7, 8\}$.

According to (2), $(i^*, \gamma v^*) = i_1(v_5 - v_8) + i_2(v_6 - v_5) + i_3(-v_6 - v_7) + i_4(v_7 + v_8)$ and the mixed potential

$$P(i^*, v^*) = E_1 i_1 - \frac{1}{2}R_1 i_1^2 - \frac{1}{2}R_2 i_2^2 - \frac{1}{2}R_3 i_3^2 - \frac{1}{2}R_4 i_4^2 + \int\limits_0^{v_5} f_1(v)dv + \int\limits_0^{v_6} f_2(v)dv +$$

$$+ \int\limits_0^{v_7} f_3(v)dv + \int\limits_0^{v_8} f_4(v)dv + i_1(-v_5 + v_8) + i_2(v_5 - v_6) + i_3(v_6 + v_7) + i_4(-v_7 - v_8).$$

Ability to apply the analytical methods of mechanical systems to electrical ones allows to formulate a problem of modelling of mechanical systems with circuits. This problem can be applied, for example, to analysis of transient states of mechanical systems after losing stability near the boundaries of stationary motions family. The number of these problems could be increased. It is important that parameters of circuits can be measured easier than parameters of mechanical ones.

3. Finally, we present the results of investigation of one generic mechanical system with first integrals. Let us discuss the system of absolutely solid bodies, connected by spherical and cylindrical hinges to single carrying body, that has a fixed point. The positions of carried bodies we will define as generalized coordinates $q_1, q_2, ..., q_n$. Let us suppose that forces acting on the system are potential and could be determined by the force function $U(q_1, q_2, ..., q_n)$. The full mechanical energy of such system looks like

$$2H = \sum_{\alpha=1}^{3}\sum_{\beta=1}^{3} J_{\alpha\beta}(q)\omega_\alpha\omega_\beta + 2\sum_{k=1}^{n}\sum_{\alpha=1}^{3} e_{k\alpha}(q)\omega_\alpha\dot{q}_k + \sum_{j=1}^{n}\sum_{i=1}^{n} c_{ij}(q)\dot{q}_i\dot{q}_j - 2U(q_1, ...q_n).$$

Here ω_α $(\alpha = 1, 2, 3)$ are the projections of angle velocity of carrying body to coordinate axes connected to this body; $q_i, \dot{q}_i$ $(i = 1, \ldots, n)$ are the coordinates and the velocities of carried bodies relative to carrying one and to each other. There is another possible interpretation of given characteristic function [4].

The differential equations of described system motion have the following form:

$$\sum_{j=1}^{n} c_{ij}\ddot{q}_j + \sum_{\alpha=1}^{3} e_{i\alpha}\dot{\omega}_\alpha - \sum_{k=1}^{n}\sum_{\alpha=1}^{3}(\frac{\partial e_{k\alpha}}{\partial q_i} - \frac{\partial e_{i\alpha}}{\partial q_k})\dot{q}_k\omega_\alpha + \sum_{j=1}^{n}\sum_{k=1}^{n}(\frac{\partial c_{ij}}{\partial q_k} - \frac{1}{2}\frac{\partial c_{kj}}{\partial q_k})\dot{q}_k\dot{q}_j -$$

$$-\frac{1}{2}\sum_{\alpha=1}^{3}\sum_{\beta=1}^{3}\frac{\partial J_{\alpha\beta}}{\partial q_i}\omega_\alpha\omega_\beta-\frac{\partial U}{\partial q_i}=0 \qquad (i=1,\dots,n),$$

$$\sum_{\beta=1}^{3}J_{1\beta}\dot{\omega}_\beta+\sum_{k=1}^{n}e_{k1}\ddot{q}_k+\sum_{k=1}^{n}(e_{k3}\omega_2-e_{k2}\omega_3+\sum_{\beta=1}^{3}\frac{\partial J_{1\beta}}{\partial q_k}\omega_\beta)\dot{q}_k+$$

$$+\sum_{\beta=1}^{3}(J_{3\beta}\omega_2-J_{2\beta}\omega_3)\omega_\beta+\sum_{j=1}^{n}\sum_{k=1}^{n}\frac{\partial e_{k1}}{\partial q_j}\dot{q}_j\dot{q}_k=0,$$

$$\dot{\gamma}_1=\gamma_2\omega_3-\gamma_3\omega_2.$$

Another four equations (not shown here) can be obtained from last two by circular substitution of indices 1,2,3. For this differential equations system it is possible to obtain three first integrals in addition to the integral of the full mechanical energy $H=h=const$. They are:
the integral of kinetic moment projection to *vertical* axis:

$$V_1=\sum_{\beta=1}^{3}[\sum_{\alpha=1}^{3}J_{\beta\alpha}\omega_\alpha+\sum_{k=1}^{n}e_{k\beta}\dot{q}_k]\gamma_\beta=m;$$

the integral of square of modulus of kinetic moment:

$$V_2=\sum_{\beta=1}^{3}[\sum_{\alpha=1}^{3}J_{\beta\alpha}\omega_\alpha+\sum_{k=1}^{n}e_{k\beta}\dot{q}_k]^2=k^2;$$

and the cosine integral:

$$V_3=\sum_{k=1}^{3}{\gamma_k}^2=1.$$

For the entire class of problems that are described by the above differential equations we can make qualitative analysis of phase space by selecting a set of invariant manifolds that provide stationary values for different first integrals from algebra of first integrals with basis formed by the integrals H, V_1, V_2, V_3. Also we can study stability and bifurcations of selected invariant manifolds of stationary motions using Lyapunov and Poincaré methods. For example, let us discuss bundle of the first integrals

$$K=\frac{1}{2}V_2-\lambda_1V_1+\frac{1}{2}\lambda_1^2V_3,\ \lambda_1=const.$$

The stationary conditions of function K can be written in the form:

$$\frac{\partial K}{\partial\omega_i}=\sum_{j=1}^{3}\varphi_jJ_{ij}=0,$$

$$
\begin{aligned}
\frac{\partial K}{\partial \gamma_i} &= -\lambda_1 \varphi_i = 0 \qquad\qquad (i = 1, 2, 3),\\
\frac{\partial K}{\partial \dot{q}_m} &= \sum_{i=1}^{3} \varphi_i e_{mi} = 0, \qquad\qquad (A)\\
\frac{\partial K}{\partial q_m} &= \sum_{i=1}^{3} \varphi_i (\sum_{\alpha=1}^{3} \frac{\partial J_{i\alpha}}{\partial q_m} \omega_\alpha + \sum_{k=1}^{n} \frac{\partial e_{ki}}{\partial q_m} \dot{q}_k) = 0 \qquad (m = 1, \ldots, n).
\end{aligned}
$$

Where

$$
\varphi_i = \sum_{\alpha=1}^{3} J_{i\alpha} \omega_\alpha + \sum_{k=1}^{n} e_{ki} \dot{q}_k - \lambda_1 \gamma_i \quad (i = 1, \ldots, 3).
$$

According to the theorem of [3] it is possible to state that the elements of manifolds family

$$
\varphi_1 = 0, \quad \varphi_2 = 0, \quad \varphi_3 = 0
$$

with parameter λ_1 that provide extremal value to the first integrals from bundle K with any fixed λ_1 form the family of invariant manifolds for the original differential equations system, because the rank of system (A) matrix is equal to 3. This is the consequence of submatrix elements properties:

$$
\begin{pmatrix} J_{11}(q) & J_{12}(q) & J_{13}(q) \\ J_{21}(q) & J_{22}(q) & J_{23}(q) \\ J_{31}(q) & J_{32}(q) & J_{33}(q) \end{pmatrix},
$$

that are interpreted as the inertia moments of mechanical system.

Using this approach we can obtain stability conditions for the elements of found family of invariant manifolds of stationary motions (IMSM) by second Lyapunov's method. Non-writing the equations of disturbed motion in the neighbourhood of our IMSM, let us consider the first integral K as Lyapunov's function. Let us write it in the terms of deviation from the manifold in question, i.e (in our case) in variables φ_i. The direct calculation gives:

$$
K = \frac{1}{2} ({\varphi_1}^2 + {\varphi_2}^2 + {\varphi_3}^2).
$$

Because K is the sign-definite function of variables $\varphi_1, \varphi_2, \varphi_3$, using well known Zubov's theorem on stability of sets we conclude, that all elements of discussed IMSM family are stable with all values of $\lambda_1 \neq 0$.

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