Vlasov-Fokker-Planck simulations of electron gas in long scale length laser plasmas

Martin Mašek
e-mail: masekm@fzu.cz

Institute of Physics,
Academy of Science of the Czech Republic,
Na Slovance 2
182 21 Prague 8
Czech Republic

10^{th} Direct Drive and Fast Ignition Workshop, May 27-30, 2012, Prague, Czech Rep.

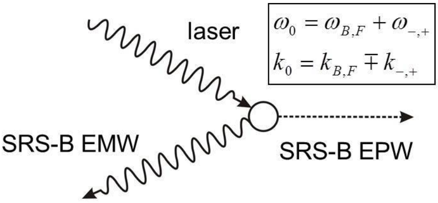
Fyzikální ústav
Akademie věd ČR, v. v. i.

Outline of the talk

1. Physics of outer corona
   - The stimulated Raman scattering
   - Damping of electron plasma waves
   - Electrons trapped in electron plasma waves
2. Numerical models of plasma corona
   - Self-consistent Vlasov-Maxwell model
   - Envelope model
3. Results of the numerical simulations
4. Conclusions

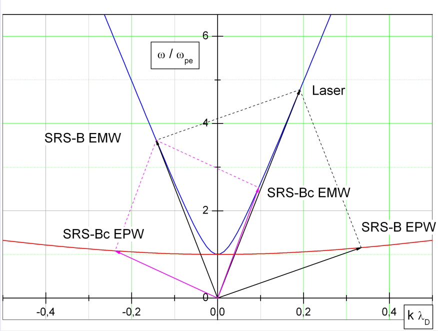
The stimulated Raman scattering

- Resonant decay of an incident laser beam into a forward-going electrostatic wave and into a scattered electromagnetic wave is the dominant 'wave-wave' interaction in well under-dense and long scale-length plasmas in target experiments.
- Strong growth of daughter waves causes generation of hot electrons travelling in the target direction.
- In the case of Raman backscattering part of laser energy is driven away from the target.



Raman backscattering

Raman backscattering and Raman cascade



- Raman backscattering matching conditions:

$$\omega_0 = \omega_B + \omega_-$$
$$k_0 = k_B + k_-$$

- Linear dispersion relations:

$$\omega_-^2 = \omega_{pe}^2 + c^2 k_-^2$$
$$\omega_B^2 = \omega_{pe}^2 + v_T^2 k_B^2$$

- Necessary condition for SRS:

$$\omega_{-,B} \geq \omega_{pe} \Rightarrow$$

$$\omega_0 \geq 2\omega_{pe} \Rightarrow n_e \leq \frac{n_c}{4}$$

- Cascading occurs only if:

$$\omega_0 \geq 3\omega_{pe} \Rightarrow n_e \leq \frac{n_c}{9}$$

Raman Scattering

Linear growth rates of Raman instability

Linear growth rate for Raman backscattering and forward scattering[a]:

$$\gamma_B = \frac{k v_{osc}}{4}\sqrt{\frac{\omega_{pe}^2}{\omega_e(\omega_0 - \omega_e)}} \qquad \gamma_F = \frac{k v_{osc}}{2\sqrt{2}}\sqrt{\frac{\omega_{pe}^2}{\omega_0^2 - \omega_{pe}^2}}$$

[a] Kruer, W. L., *The Physics of Laser Plasma Interactions*, 1988.

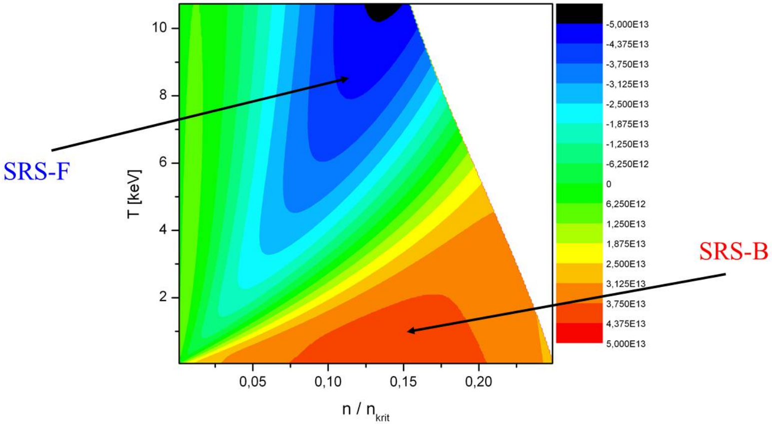
Wave number of SRS-B plasma wave

By solving the matching conditions together with the dispersion relations for participating waves, we get the relation for the wave number of electron plasma wave[a]

$$\frac{k_e}{k_L} = \frac{1 \pm \left(\frac{1-2\sqrt{N}}{1-N}\right)^{1/2}\left[1 - \frac{3}{2}\left(\frac{v_T}{c}\right)^2 \frac{(2-\sqrt{N})(1-\sqrt{N})}{1-2\sqrt{N}}\right]}{1 + 3\left(\frac{v_T}{c}\right)^2\left(\frac{1}{\sqrt{N}} - 1\right)}$$

[a] Karttunen, S. J., *Laser Part. Beams* **2** (1985), 157.

Existence regions of SRS-B and SRS-F



Temperature and density dependence of SRS growth rate for PALS laser at fundamental wavelength $\lambda = 1.3152\ \mu m$.

Damping of electron plasma waves

- Landau collisionless damping[a]:

$$\gamma_L = -\sqrt{\frac{\pi}{8}}\frac{\omega_{pe}^2\omega_e^2}{|k_e^3|v_T^3}\exp\left(-\frac{\omega_e^2}{2k_e^2 v_T^2}\right)$$

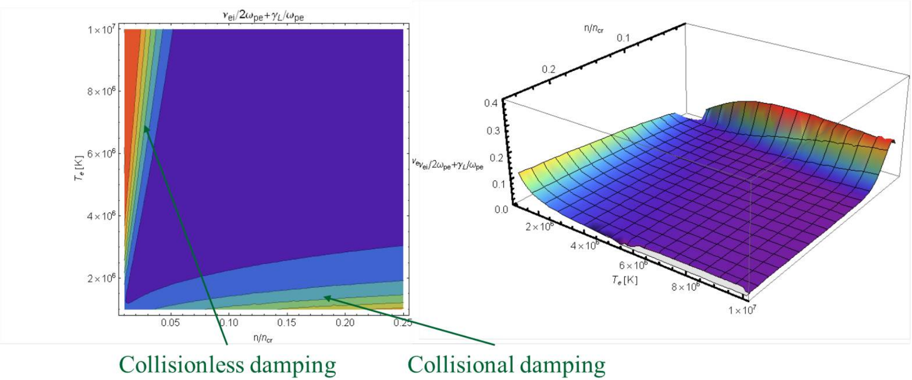
- Electron-ion collision frequency[b]:

$$\nu_{ei} = \frac{4(2\pi)^{1/2}Z^2 e^4 n_i \ln\Lambda}{3\sqrt{m_e}(K_B T_e)^{3/2}}$$

$$\nu_{ei} \approx 2,9\cdot 10^{-6}\frac{Z^2 n_i(cm^{-3})\ln\Lambda}{[T_e(eV)]^{3/2}}[s^{-1}]$$

[a] Kruer, W. L., *The Physics of Laser Plasma Interactions*, 1988.
[b] Eliezer, S., *The Interaction of High-Power Lasers with Plasmas*, 2002.

Total damping rate for SRS-B EPW



Temperature and density dependence of total damping rate for PALS laser at fundamental wavelength $\lambda = 1.3152\ \mu m$.

Charged particle motion in periodic potential

The particle motion is governed by the Newton equation:

$$m\ddot{x}'(t) = F(x', t),$$

$$F(x', t) = -eE(x', t).$$

Transformation to the wave frame:

$$x = x' - \frac{\omega}{k}t = x' - v_{ph}t.$$

By multiplication of the Newton equation by $\dot{x}$ we get:

$$\dot{x} = \sqrt{\frac{2e}{m_e}\varphi(x) + \frac{2\mathcal{E}}{m_e}},$$

$$\varphi(x) = -\int E(x)\,dx, \;\; \mathcal{E} = m_e v_0^2/2.$$

Electron plasma wave:

$$E(x) = E_0 \sin(kx), \varphi(x) = -\frac{E_0}{k}\cos(kx).$$

For free moving particles expression under root must be positive, so the condition for separatrix separating free and trapped particles:

$$v_{sep} - v_{ph} = \sqrt{\frac{2eE_0}{m_e k}\left(\cos(kx) + 1\right)}.$$

In the approximation of well trapped particles (particle moving close to the potential minimum):

$$\varphi(x) = -\frac{E_0}{k}\cos(kx) \simeq -\frac{E_0}{k} + \frac{1}{2}E_0 kx^2 + O[x^4],$$

$$E(x) = E_0 kx.$$

Finally, we get:

$$\omega_{Bf} = \sqrt{\frac{eE_0 k}{m_e}},$$

is the lowest estimation for electron bouncing frequency.

Trapped particle instability

- Trapped electrons oscilating around potential minima with frequency roughly ω_{Bf} act coherently like a beam.
- New satellite modes are generated in the way similar to the two-stream instabilty.
- The sidebands gradually dominates over the resonant wave mode.
- Stronger Landau damping of the waves with higher wave number causes a shift of broaden spectral line to the left in k-spectrum.

Linear dispersion relation[a]:

$$1 = \frac{f_T^2}{(\omega - kv_{ph})^2 - \omega_{Bf}^2} \left[\frac{1}{\varepsilon_L(k,\omega)} + \frac{1}{\varepsilon_L(k - 2k_e, \omega - 2\omega_e)} \right],$$

$$\varepsilon_L(k,\omega) = 1 - \frac{\omega_{pe}^2}{\omega^2 - 3k^2 v_T^2}.$$

[a] Kruer et al., *Phys. Rev. Lett.* **23** (1969), 838

Self-consistent Vlasov-Maxwell model

Assumptions of the model and the solution method

- Non-relativistic collisionless plasma with homogeneous neutralizing ion background
- 1D changes of physical quantities
- Velocity of electrons in the perpendicular direction in the Vlasov equation is replaced by the mean velocity of particles in the field of incident laser beam ($v_y = eA/m$)
- A Fourier-Hermite transform method is used for solution of the set of partial differential equations[a]
- Numerical stability is ensured by employing a simplified Fokker-Planck collision term in the Vlasov equation[b]

---

[a] Armstrong *et al.*, *In Methods in Computational Physics* **vol. 9**, Academic Press, London (1970), pp. 22-86

[b] Grant, F. C. and Feix, M. R., *Phys. Fluids* **10** (1967), 696 and 1356

Self-consistent Vlasov-Maxwell model

Set of equations for the kinetic model

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + \frac{e}{m}\left(\frac{\partial \varphi}{\partial x} - \frac{e}{m} A \frac{\partial A}{\partial x}\right)\frac{\partial f}{\partial v} = \nu_c \left(\frac{\partial (vf)}{\partial v} + \frac{\partial^2 f}{\partial v^2}\right),$$

$$\left[\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\omega_{pe}^2}{c^2}\frac{n_e}{n_0}\right] A \ = 0,$$

$$\frac{\partial^2 \varphi}{\partial x^2} = \frac{e}{m}(n_e - n_0),$$

$$\frac{n_e}{n_0} = \int_{-\infty}^{\infty} f \ dv.$$

- For more detailed information about the numerical method see our previous work[a]

[a] Mašek, M. and Rohlena, K., *Czech. J. Phys.* **55**, 8 (2005), 973.

Envelope model

- The starting set of equations consists of the motion equation for the electron fluid together with the full set of Maxwell's equations. These equations are coupled by the general form of the state equation.
- E denotes the longitudinal part of electric field and A is the transverse part of vector potential

(Weiland, J. and Wilhelmsson, H., *Coherent Non-Linear Interaction of Waves in Plasmas*, Pergamon Press, 1977)

Starting equations in 1D

$$\begin{aligned}\left[c^2\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial t^2}-\omega_{pe}^2\right]A &= -\omega_{pe}^2 A\frac{\partial E}{\partial x},\\ \left[v_T^2\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial t^2}-\omega_{pe}^2\right]E &= \omega_{pe}^2\frac{e}{m_e}A\frac{\partial A}{\partial x},\end{aligned}$$

Formalism of coupled wave modes

$$A(x,t) = \frac{1}{2}A_0(x,t)e^{i(k_0 x - \omega_0 t)} + \frac{1}{2}A_R(x,t)e^{i(k_R x - \omega_R t)} + c.c.$$
$$E(x,t) = \frac{1}{2}E_e(x,t)e^{i(k_e x - \omega_e t)} + c.c.$$

$A_0(x,t)$ and $A_R(x,t)$ denote the slowly varying amplitude of the impinging electromagnetic wave and of the scattered electromagnetic wave, respectively, and $E_e(x,t)$ is the amplitude of electron plasma wave. Since the fully self-consistent model is periodic, we assume a perfect wave numbers matching

$$k_0 = k_R + k_e,$$

while we allow for a mismatch in the matching condition for the frequencies:

$$\omega_0(k_0) + \Delta\omega = \omega_R(k_R) + \omega_e(k_e).$$

Three-wave envelope equations

Substituting slowly varying fields into the starting set of equations and using the standard cancelation due to the zeroth-order dispersion relation leaves the following three-waves envelope equations:

$$\begin{aligned}
\left[\frac{\partial}{\partial t} + v_{g0}\frac{\partial}{\partial x} + \nu_0\right] A_0 &= -\frac{e}{4m_e}\frac{k_e}{\omega_0} A_R E_e e^{i\Delta\omega t} + \nu_0 A_L \\
\left[\frac{\partial}{\partial t} + v_{gR}\frac{\partial}{\partial x} + \nu_R\right] A_R &= \frac{e}{4m_e}\frac{k_e}{\omega_R} A_0 E_e^* e^{-i\Delta\omega t} \\
\left[\frac{\partial}{\partial t} + v_{ge}\frac{\partial}{\partial x} + \nu_e\right] E_e &= \frac{e}{4m_e}\frac{k_e}{\omega_e} A_0 A_R^* e^{-i\Delta\omega t}
\end{aligned}$$

Group velocities

$$\begin{aligned}
v_{g0} &= c^2 k_0/\omega_0 \\
v_{gR} &= c^2 k_R/\omega_R \\
v_{ge} &= 3v_T^2 k_e/\omega_e
\end{aligned}$$

Collision damping rates

$$\nu_0 = \omega_{pe}^2 \nu_{ei}/2\omega_0^2,$$

$$\nu_R = \omega_{pe}^2 \nu_{ei}/2\omega_R^2,$$

$$\nu_e = \nu_{ei}/2 + \gamma_L.$$

Results of the numerical models

PALS laser facility parameters

$$\begin{aligned} \lambda &= 1.315\ \mu m \\ \tau &= 400\ ps \\ P &= 10^{20}\ \ W/m^2 \\ T_e &= 0.9\ keV \end{aligned}$$

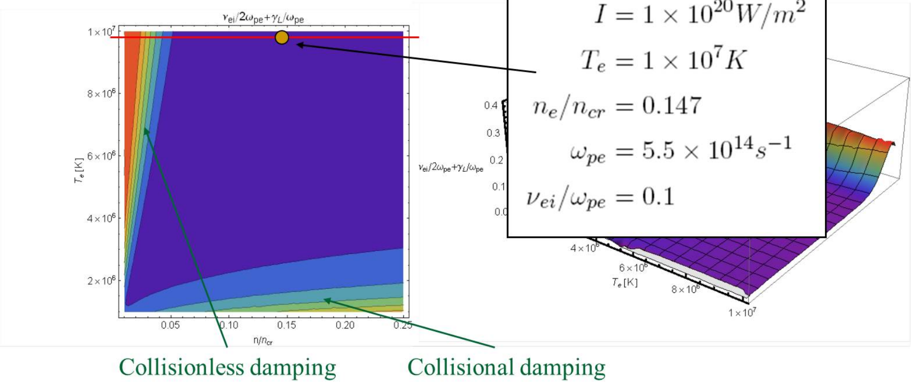
(a) $\omega_{pe} = 5.5 \times 10^{14}\ \ s^{-1}$

$$\begin{aligned} n_e/n_{crit} &= 0.147 \\ k_0\lambda_D &= 0.0987 \\ k_R\lambda_D &= 0.0498 \\ k_e\lambda_D &= 0.1485 \\ v_e^{ph}/v_T &= 6.95 \end{aligned}$$

(b) $\omega_{pe} = 3.0 \times 10^{14}\ \ s^{-1}$

$$\begin{aligned} n_e/n_{crit} &= 0.044 \\ k_0\lambda_D &= 0.192 \\ k_R\lambda_D &= 0.143 \\ k_e\lambda_D &= 0.335 \\ v_e^{ph}/v_T &= 3.45 \end{aligned}$$

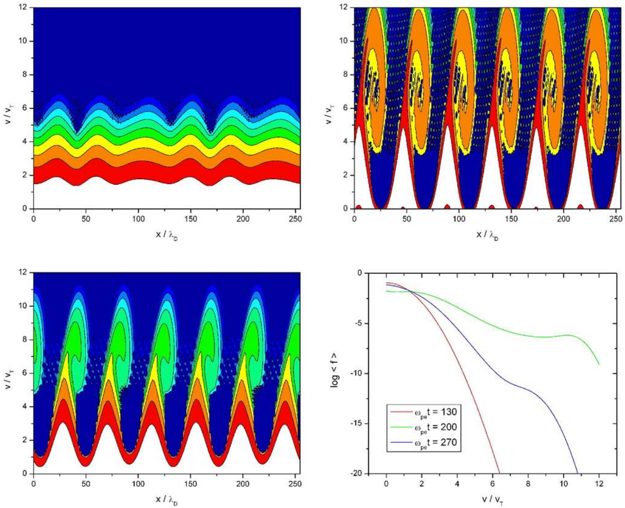
(a) Raman backscattering



$$I = 1\times 10^{20} W/m^2$$
$$T_e = 1\times 10^7 K$$
$$n_e/n_{cr} = 0.147$$
$$\omega_{pe} = 5.5\times 10^{14} s^{-1}$$
$$\nu_{ei}/\omega_{pe} = 0.1$$

The simplest case: weak SRS-F, no Raman cascading, simulation spectra set to avoid TPI

(a) Phase space evolution



Contour plot of phase space at $\omega_{pe}t = 130, 200, 270$.
$v_{ph}/v_T = 6.95$ and $v_{sep}/v_T = 5.57$.

(a) $\omega_{pe} = 5.5 \times 10^{14}\ s^{-1}$

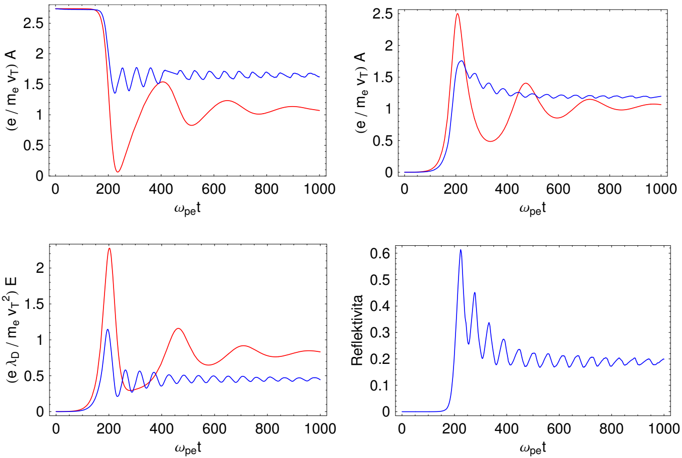
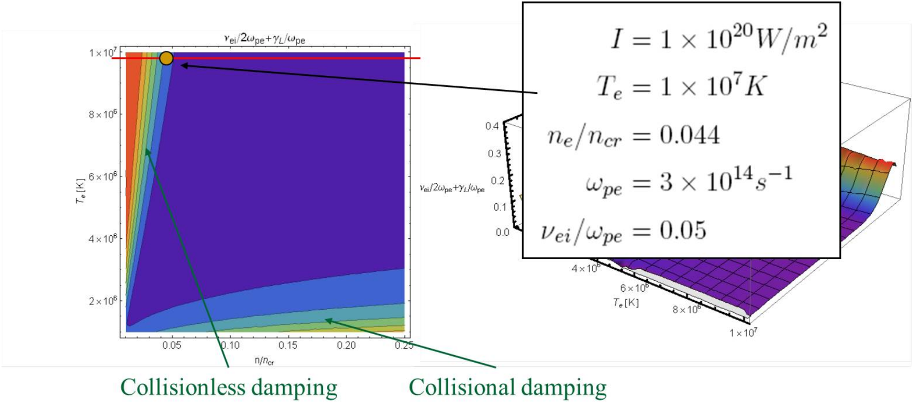


Figure: Comparison of the temporal evolutions of the participating wave modes obtained from the full kinetic simulation (blue) and from the envelope model (red) in a denser plasma ($n_e/n_{crit} = 0.147$).

Discussion (a)

- In a denser plasma ($n_{crit}/9 \leq n_e \leq n_{crit}/4$) no Raman cascading can occur and the Raman backscattering dominates the forward scattering.
- The phase velocity of the plasma wave is under the mentioned conditions relatively high ($v_{ph}/v_T = 6.95$) lying outside the body of electron distribution, so the influence of particle trapping is relatively weak and the results of the both models agree with a strong instability growth at the first stage of evolution.
- When the electrostatic wave reaches a sufficiently high amplitude (peak value from the Vlasov simulation is $E = 4.43 \times 10^{10}\ V/m$, which corresponds to the amplitude of separatrix $v_{sep}/v_T = 5.57$) to trap a large amount of electrons, the growth of the instability is saturated and the amplitudes of the participating waves reaches more or less equilibrium values.
- Small amplitude fluctuations correspond to the period of wobbling motion of trapped electrons in the wave
- Raman reflectivity $R = E_R^2/E_L^2 \approx 20\ \%$

(b) Raman cascade



Raman cascading is possible, presence of SRS-F, simulation spectra set to avoid TPI

(b) $\omega_{pe} = 3.0 \times 10^{14}\ s^{-1}$

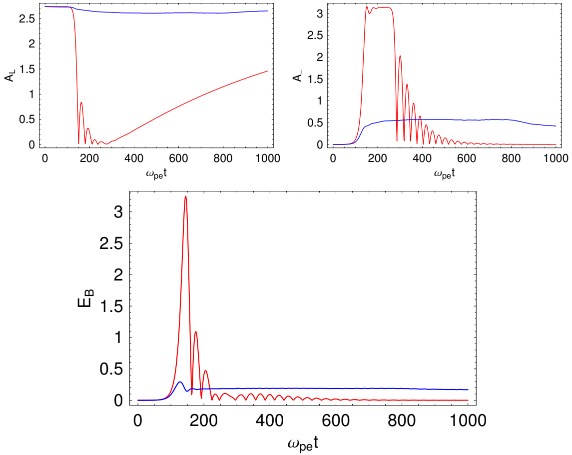
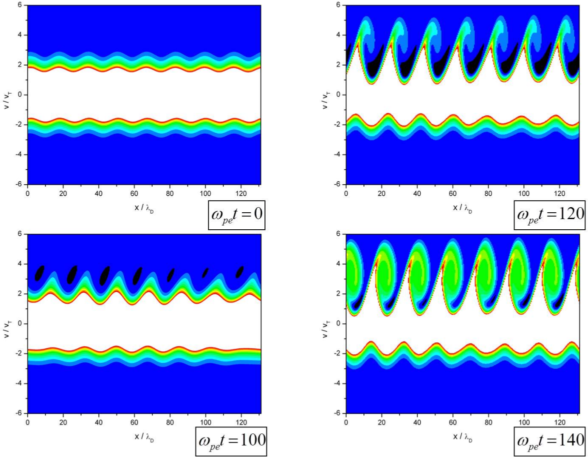


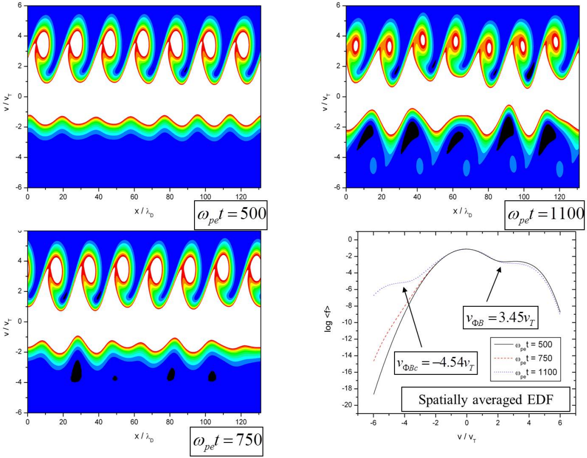
Figure: Comparison of the temporal evolutions of the participating wave modes obtained from the full kinetic simulation (blue) and from the envelope model (red) in a thinner plasma ($n_e/n_{crit} = 0.044$).

(b) Phase space evolution - Raman backscattering



Contour plot of phase space at $\omega_{pe}t = 0, 100, 120, 140$.
$v_{phB}/v_T = 3.45$ and $v_{sep}/v_T = 1.5$.

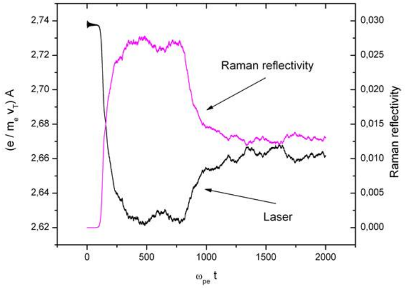
(b) Phase space evolution - SRS cascade

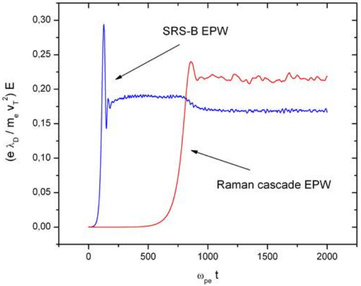


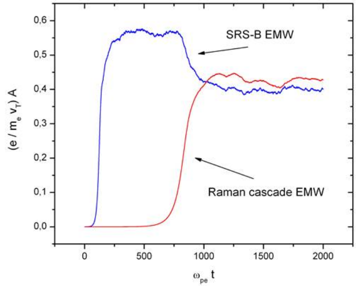
Contour plot of phase space at $\omega_{pe} t = 500, 750, 1100.$

$v_{phc}/v_T = -4.54.$

(b) Evolution of resonant wave mode amplitudes





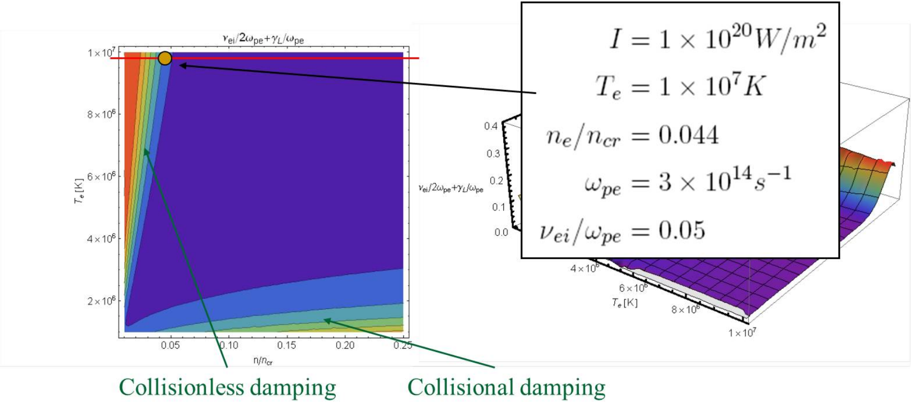


Due to the strong wave-particle interaction in a thinner plasma, Raman reflectivity is in order of a few percent and it is additionally reduced by the Raman cascading.

Discussion (b)

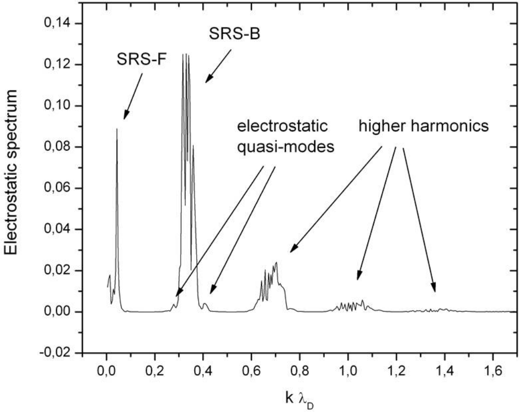
- In a thinner plasma ($n_e \leq n_{crit}/9$) the Raman backscattering is no longer a dominant process, since the forward scattering also appears. Moreover, the Raman cascading appears and there is also a possibility of electrostatic quasi-mode formation, through which the SRS-F plasma wave can interact with the electrons.
- SRS-B plasma wave has the phase velocity $v_{ph}/v_T = 3.45$, thus it is strongly interacting with the electrons from the very start of the growth.
- The large amount of laser energy is used for electron acceleration and a fast saturation of Raman backscattering occurs, which is still enhanced by the secondary decay of the back-scattered electromagnetic wave due to the Raman cascading.
- It results in a disagreement of results of the both methods, when the Raman reflectivity is about 1 % in the full model, while in the envelope model we observe almost a laser reflection.
- Later stage of the system evolution is affected by the Raman cascading, which is also taken into account in the envelope model.

(c) Denser Fourier k-spectrum



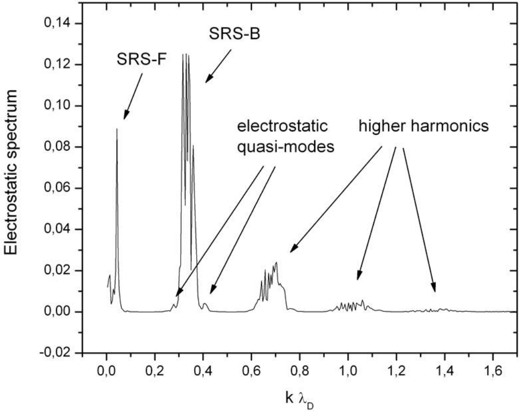
Raman cascading is possible, presence of SRS-F, TPI enabled, the same case as (b)

(c) Fully developed electrostatic spectrum at $\omega_{pe}t = 300$



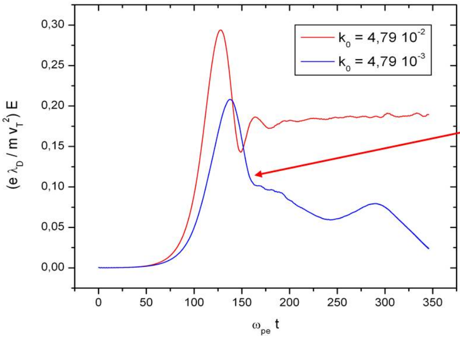
A fully developed electrostatic k-spectrum in the non-linear stage of SRS with a broadened and shifted peak of SRS-B due to the presence of trapped particle instability.

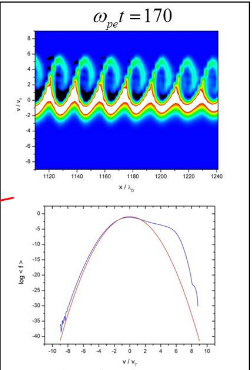
(c) Fully developed electrostatic spectrum at $\omega_{pe} t = 300$



A fully developed electrostatic k-spectrum in the non-linear stage of SRS with a broadened and shifted peak of SRS-B due to the presence of trapped particle instability.

(c) Temporal evolution of SRS-B plasma wave



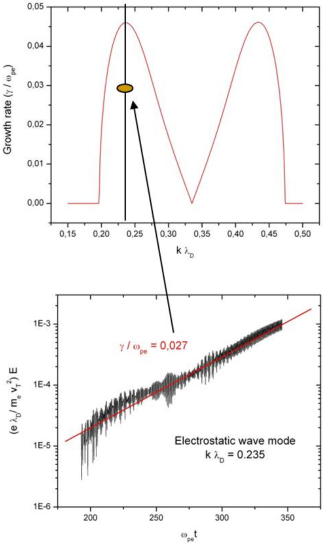


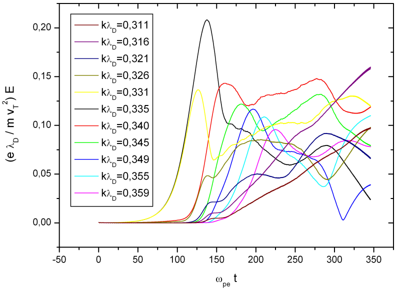
$$\omega_B = \sqrt{\frac{eEk}{m_e}} \Rightarrow \omega_B / \omega_{pe} = 0.18$$

$$f_T = 12.2\%$$

Comparison of the SRS-B plasma wave temporal evolution as a result of simulation with thinner Fourier spectrum (b) and with denser Fourier spectrum (c).

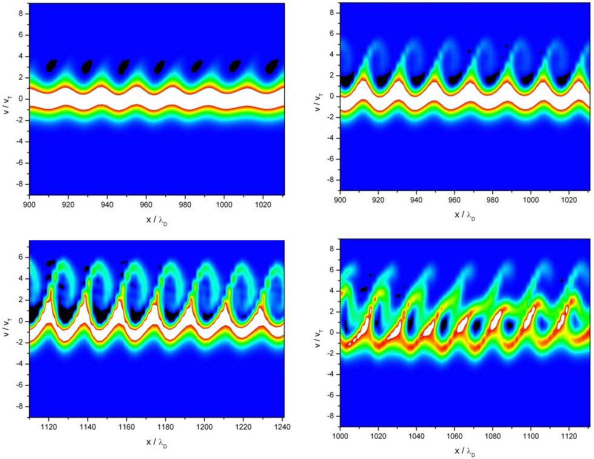
(c) Trapped particle instability growth rate

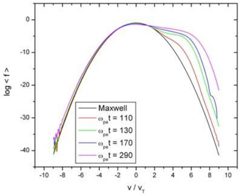




A certain disagreement between the results of the full model and of the linear theory is caused by the presence of collisions in the full model.

(c) Phase space evolution

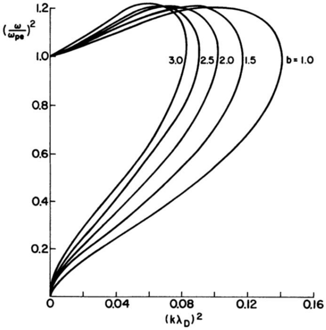




Contour plot of phase space at $\omega_{pe}t = 110, 130, 170, 290.$

(c) Phase space evolution

(c) Anomalous dispersion



Taken from V. Krapchev, A.K. Ram, *Phys. Rev. A* **22** (1980), 1229.

- More complex trapping electrostatic field formed as a consequence of the TPI frees the trapped electrons. The resonant wave mode is not then damped, sidebands disappeared and the EPW can trap electrons anew.
- This leads to a quasi-periodicity or an intermittency in the phase space.
- These perturbation are traveling in the opposite direction, which is caused by reversal of the group velocity as a consequence of existence of two sorts of electrons (free and trapped).

Discussion (c)

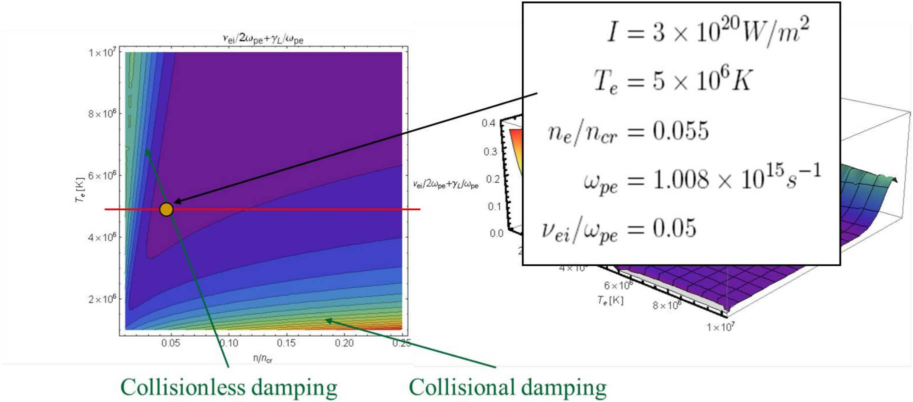
- The electrostatic daughter wave of the forward scattered Raman wave cannot interact with the plasma electrons directly owing to its high phase velocity, but it can combine with its partner generated by the Raman backscattering to form a non-resonant quasi-mode capable of electron trapping.
- The wobbling of the trapped electrons in the potential minima of the electrostatic backscattering daughter wave leads to a modulation of the electron density and to a generation of unstable sidebands of the main electrostatic daughter wave - trapped particle instability.
- In later times due to the Landau damping affecting mainly higher sidebands, the lower sideband takes over from the main wave, which is gradually outgrown.
- Intermittency, quasi-periodicity and reversal of the EPW group velocity.

Conclusions

- Comparing results of Vlasov-Maxwell and envelope model we are able to trace the influence of wave-particle interaction to the temporal evolution of electrostatic waves.
- In denser plasmas, where this interaction start to participate on system evolution later, the value of Raman reflectivity is relatively high.
- In thinner plasmas, where wave-particle interaction is very strong, the Raman reflectivity is in order of a few percents.
- This value is significantly reduced by the Raman cascading.
- Intermittency, quasi-periodicity and reversal of the EPW group velocity as a consequence of particle trapping.
- Our other studies showed higher Raman reflectivity in more collisional plasmas ($R \approx 20$ % and more).

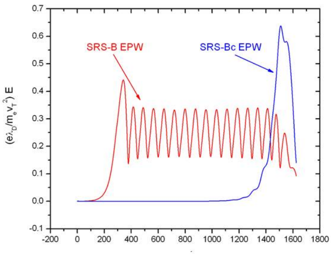
Thank you for your attention!

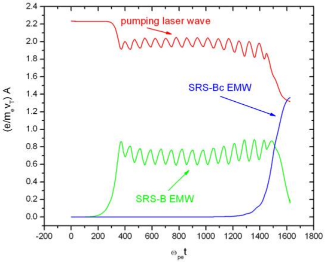
(d) Increased collisionality - 3ω plasma

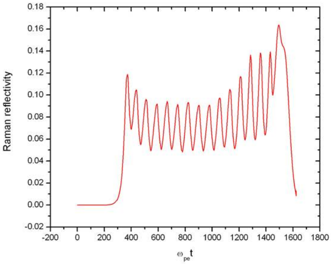


Raman cascading is possible, evidence of SRS-F, TPI enabled, the same case as (b)

(d) Increased collisionality - 3ω plasma







$$E_B = 2.2 \times 10^{10} V/m$$
$$v_{ph_B}/v_T = 5.098$$
$$v_{sep_B}/v_T = 2.905$$
$$\omega_{Bf}/\omega_{pe} = 0.303$$

$$E_{Rc} = 3.2 \times 10^{10} V/m$$
$$v_{ph_{Rc}}/v_T = 7.148$$
$$v_{sep_{Rc}}/v_T = 4.207$$

Discussion (d)

- As expected a strongly increased collisionality deeper inside the corona suppresses the kinetic effects in the corona as obvious from the detailed phase space evolution, the collisionality not in all the cases can substitute the suppressed Landau damping.
- With the suppression of Landau damping and of the associated higher order kinetic effects (such as a generation of hot electrons) the Raman reflectivity goes up. Although, the Raman instability is, in addition, damped by the electron-ion collisions its value in the denser 3ω plasma may reach 20 % or more.
- The trapped electron wobbling frequency is no longer significant for the fluctuations of the scattered wave amplitudes. It is rather the mutual energy interplay between the driving and the scattered modes, which controls the scattered amplitude oscillations around the saturation values, which is also reflected in a modulation of the phase space evolution.
- The most efficient suppression of the primary local Raman reflectivity is due to the cascading process. By the secondary scattering of backward going scattered electromagnetic wave the flow of energy is very efficiently returned to the forward direction and the back-scatter is significantly