A hybrid PIC-MHD model for high-density laser-plasma simulations

(using CALDER)

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Introduction

- For long simulation times, PIC simulations suffer from numerical heating
- Numerical heating can be mitigated by ensuring that the timestep $\Delta t \lesssim \omega_p^{-1}$
- As $\omega_p \propto {n_e}^{1/2}$, then high density plasmas require restrictively small timesteps
- Recently Cohen *et Al.*[1,2] has proposed a PIC-MHD hybrid model to break this restriction
- This work aims to incorporate the PIC-MHD hybrid model into the parallel, relativistic, collisional code CALDER[3]

[1] B.J. Cohen et Al. *Journal of Computational Physics*, **229** (2010) pp. 4591-4612.
[2] F. Fiuza et Al. *Plasma Phys. Control. Fusion,* **53** (2011) 074004.
[3] E. Lefebvre et Al. *Nuclear Fusion*, **43** (2003) 629.

PIC-MHD hybrid

- Highly collisional[1]:
  - Dense, Cold
  - Waves are heavily damped
- Resistive MHD theory (Ohm's law):

$$\mathbf{E} = \eta \mathbf{J}_c - \frac{\nabla p_e}{e n_e} - \mathbf{v}_c \times \mathbf{B} + \cdots$$

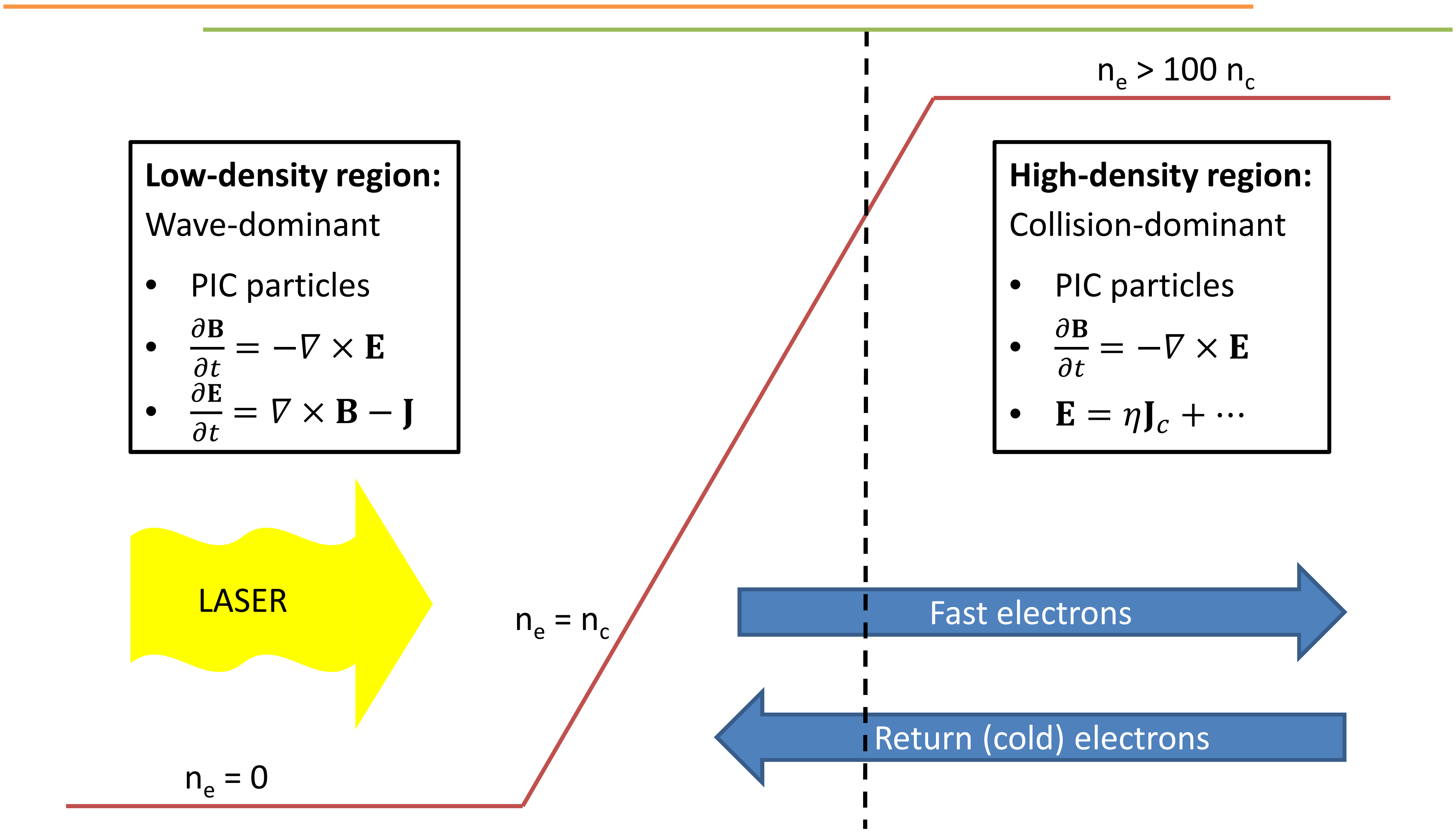
- Spitzer theory for resistivity[2]:

$$\eta \propto \frac{Z \ln \Lambda_{ei}}{T^{3/2}}$$

[1] K. Nanbu, *Phys. Rev. E.* **55** (1997) p. 4642.

[2] A. Decoster, et Al. Modeling of collisions (Gauthier-Villars, 1998)

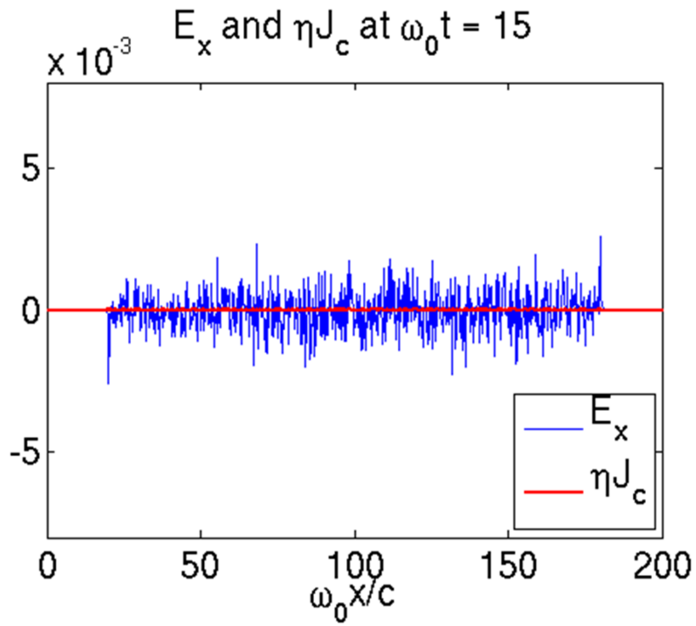
Schematic



$n_e > 100\ n_c$
Low-density region:
Wave-dominant
• PIC particles
• $\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$
• $\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B} - \mathbf{J}$
High-density region:
Collision-dominant
• PIC particles
• $\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$
• $\mathbf{E} = \eta \mathbf{J}_c + \cdots$
LASER
$n_e = n_c$
Fast electrons
Return (cold) electrons
$n_e = 0$

Nanbu and Spitzer

- Nanbu's collision model has been implemented in CALDER[1]
- Spitzer's model of resistivity is to be used in the high-density regions
- For a smooth transition, these two models must be self-consistent



[1] F. Perez, *Thèse du doctorat*, École Polytechnique (2010).

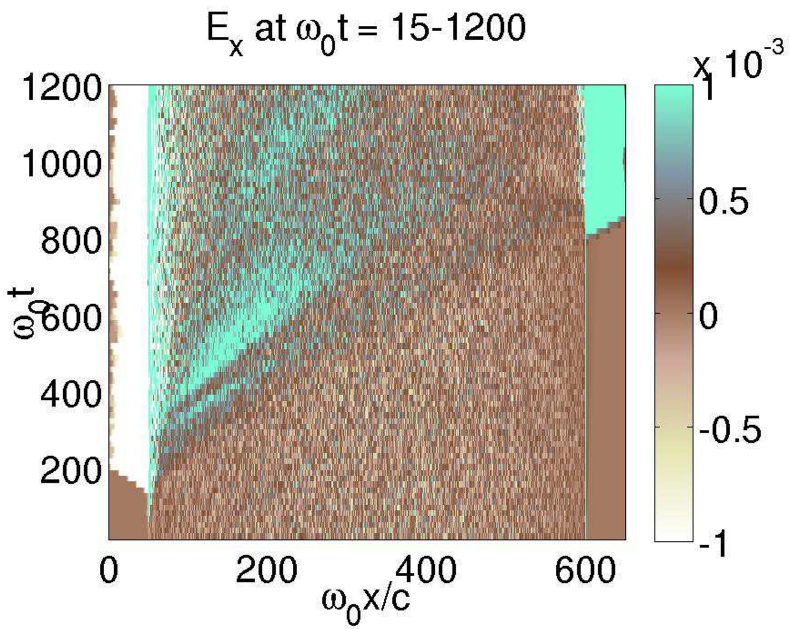
Stability condition

Ampère's law was the cause of the stability criterion $\Delta t \lesssim \omega_p^{-1}$.
The new stability condition depends on how $\mathbf{J}_c$ is determined

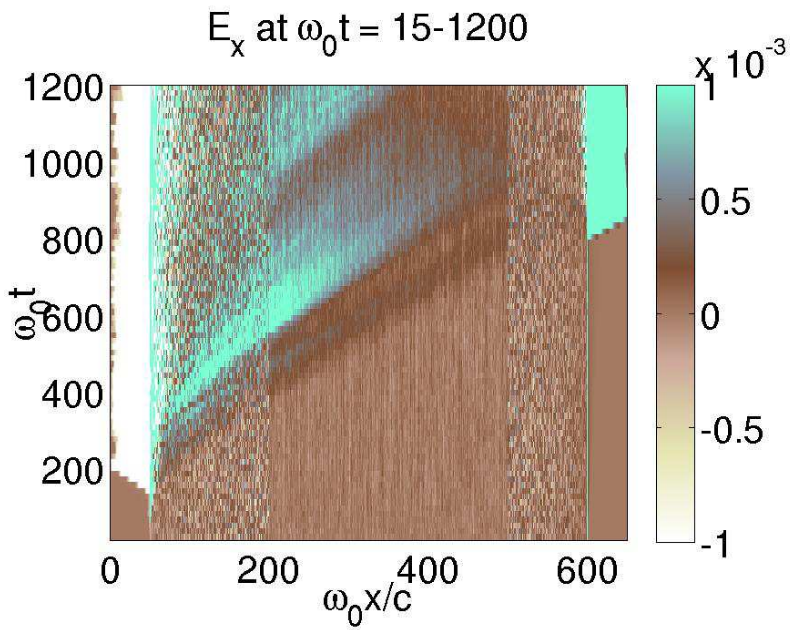
$\mathbf{J}_c$	Evaluated directly from cold electrons	→	N/A
$\mathbf{J}_c = \frac{1}{\mu_0} \nabla \times \mathbf{B} - \mathbf{J}_f$		→	Unconditionally Unstable
$\mathbf{J}_c = \frac{1}{\mu_0} \nabla \times \mathbf{B} - \mathbf{J}_f$	Smoothing of E and B fields	→	$4\eta\epsilon_0 c^2 \,\Delta t < \Delta x^2$
$\mathbf{J}_c = \frac{1}{\mu_0} \nabla \times \mathbf{B} - \mathbf{J}_f - \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$		→	$c\sqrt{\nu}\,\Delta t < \Delta x$

1D simulations

Purely PIC

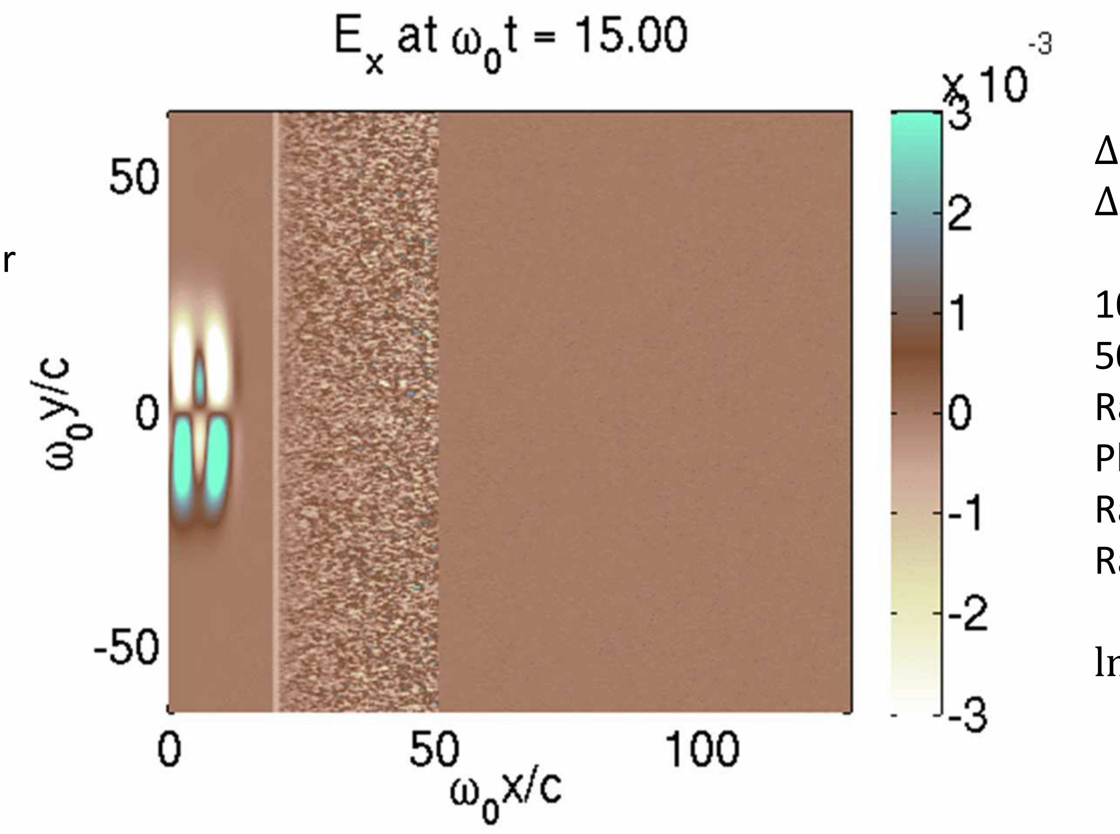


Hybrid



Electric Field

3.15×10^{19} W.cm^{-2}
laser rises linearly over
20 cycles of 1.06 μm



Δx = 0.05
Δt = 0.03

100 particles per cell
50 eV Al^{3+} plasma
Ramp profile, with
Plateau at 200 n_c
Ramp start 20
Ramp end 60

$\ln \Lambda$ = 2

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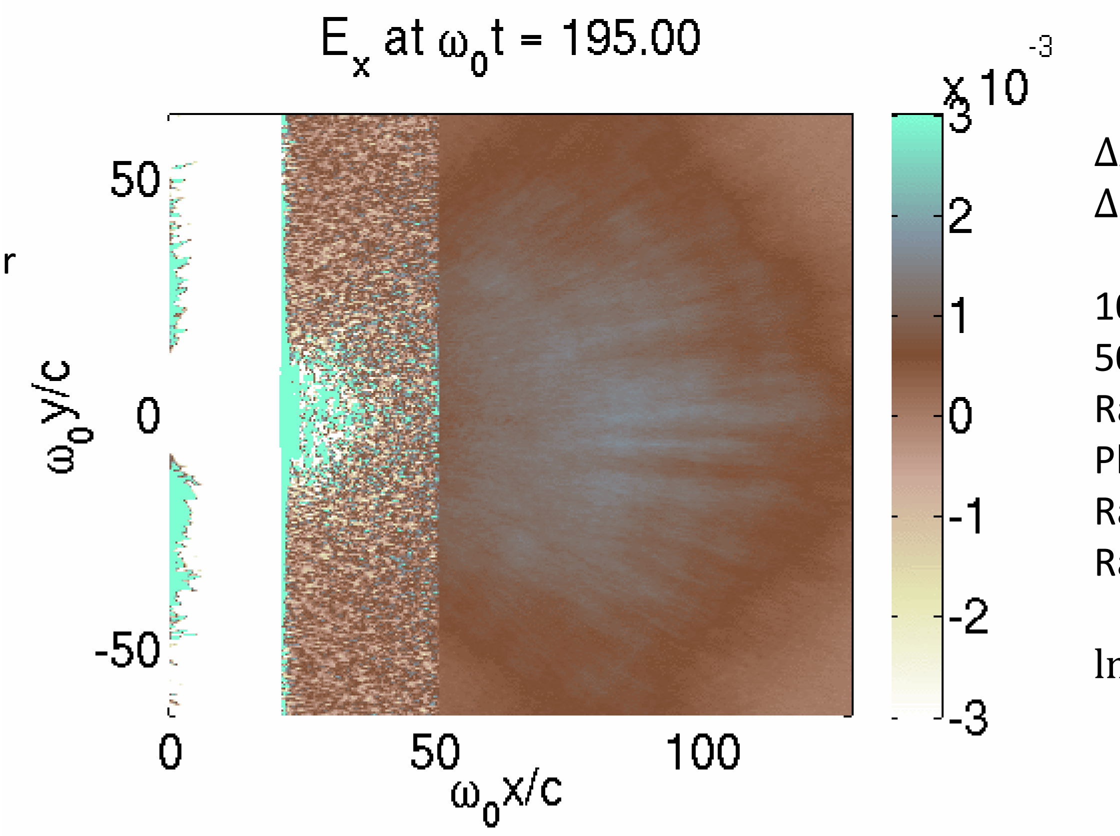

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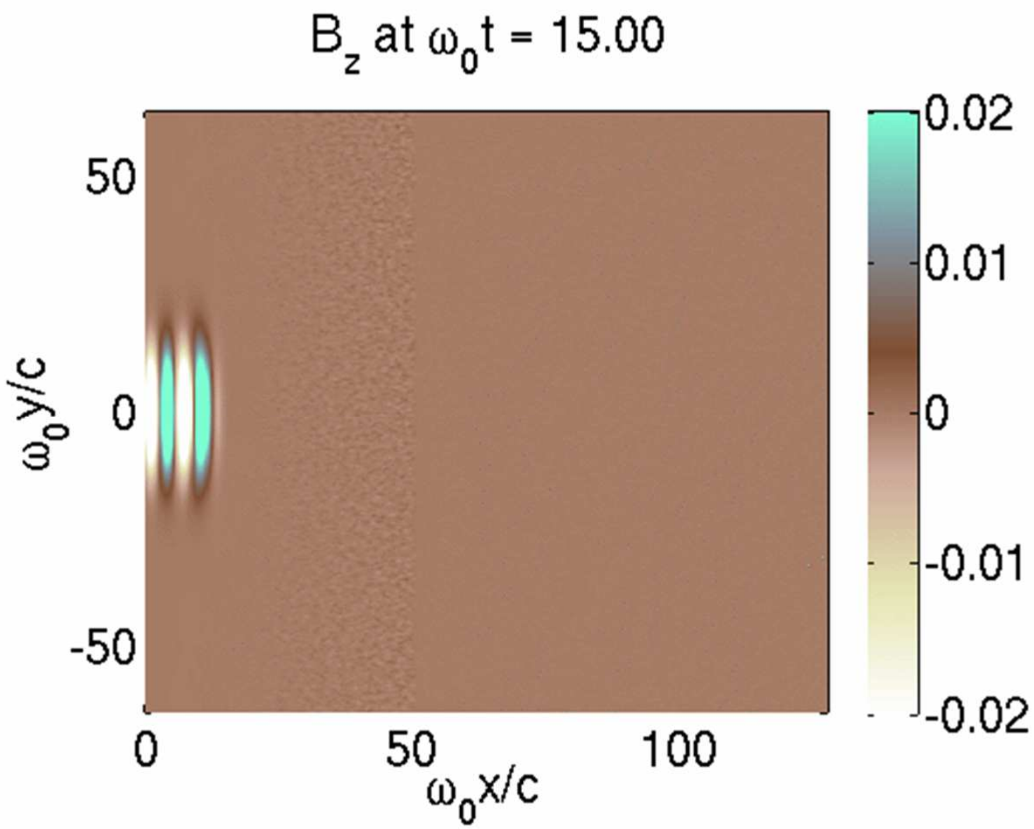

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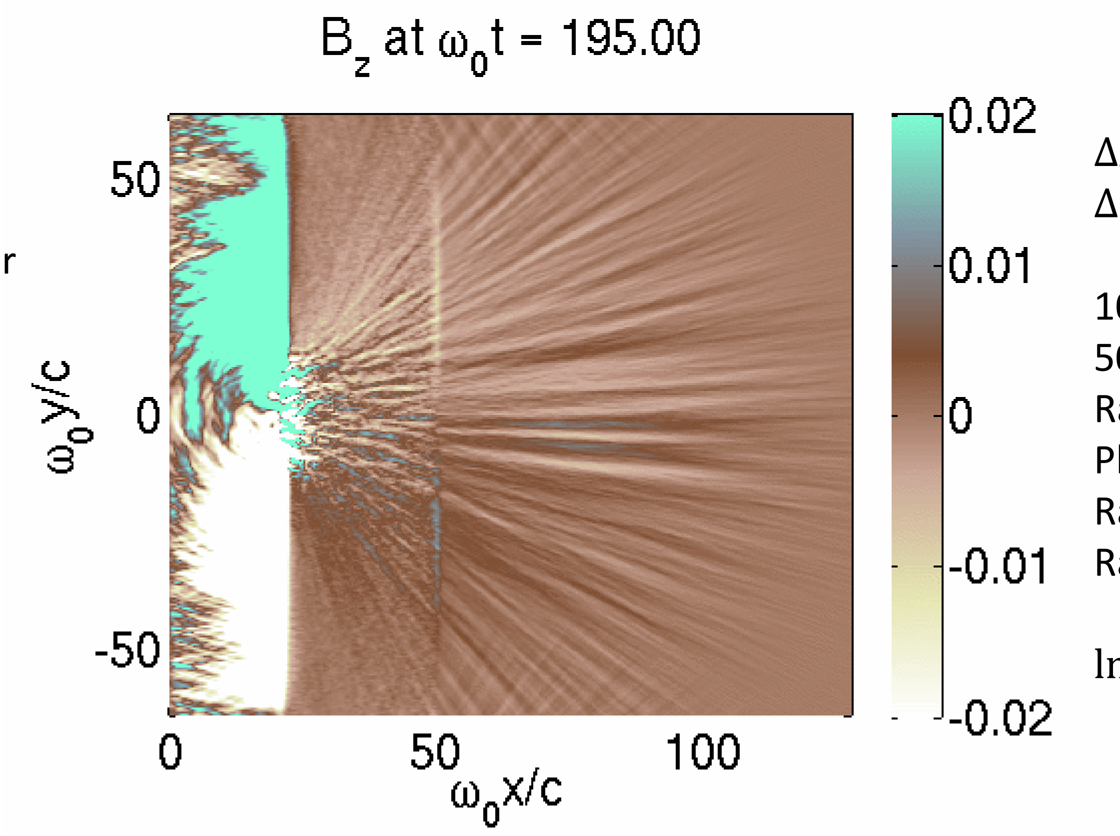

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Electron Temperature

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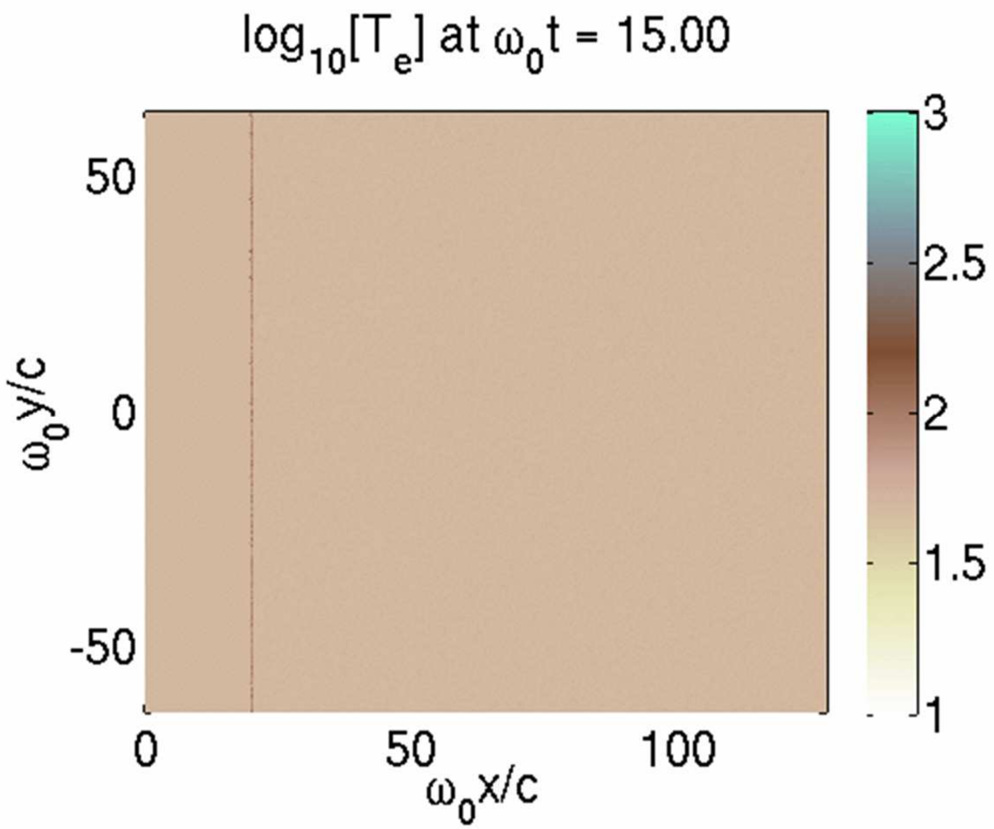

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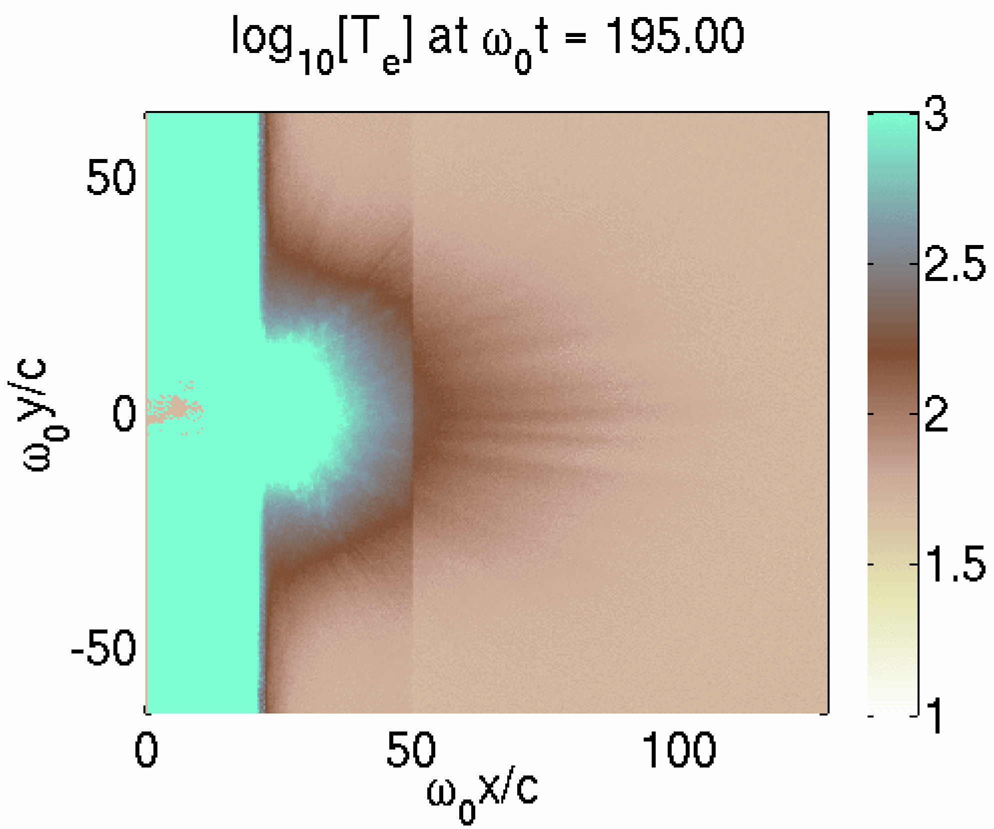

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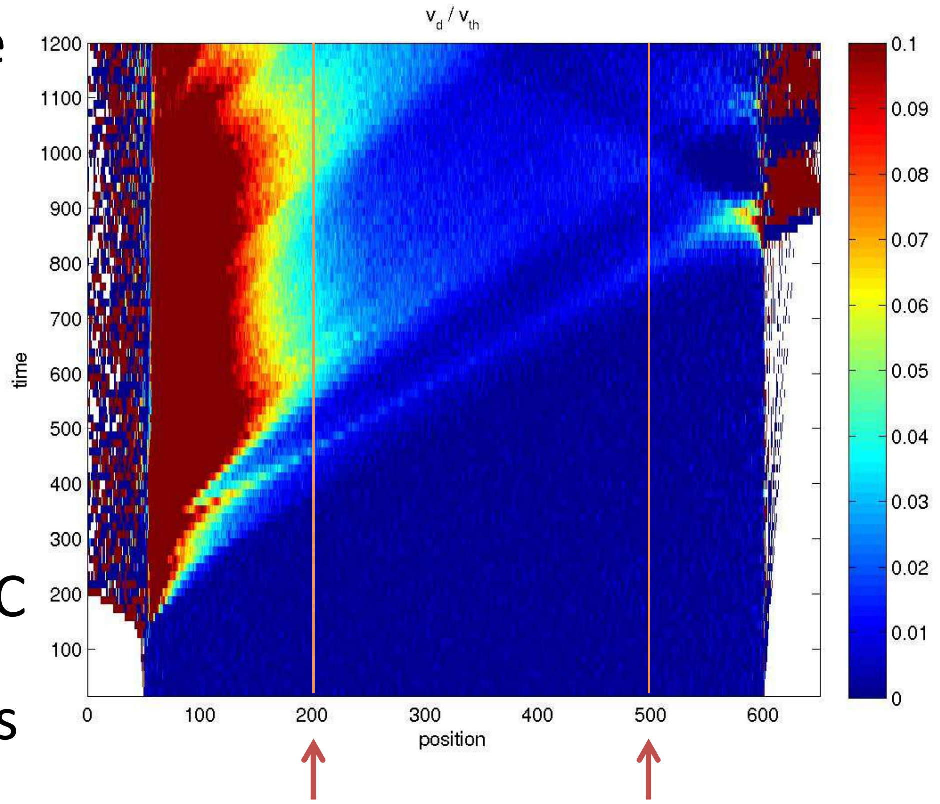
Weak-field limit

- The Spitzer model for resistivity is only applicable in the weak-field limit
- We have lowered the laser intensity, and raised the plasma density to enforce this
- Right is v_d/v_{th} of a fully PIC model. For the Spitzer model to be applicable, this needs to be small

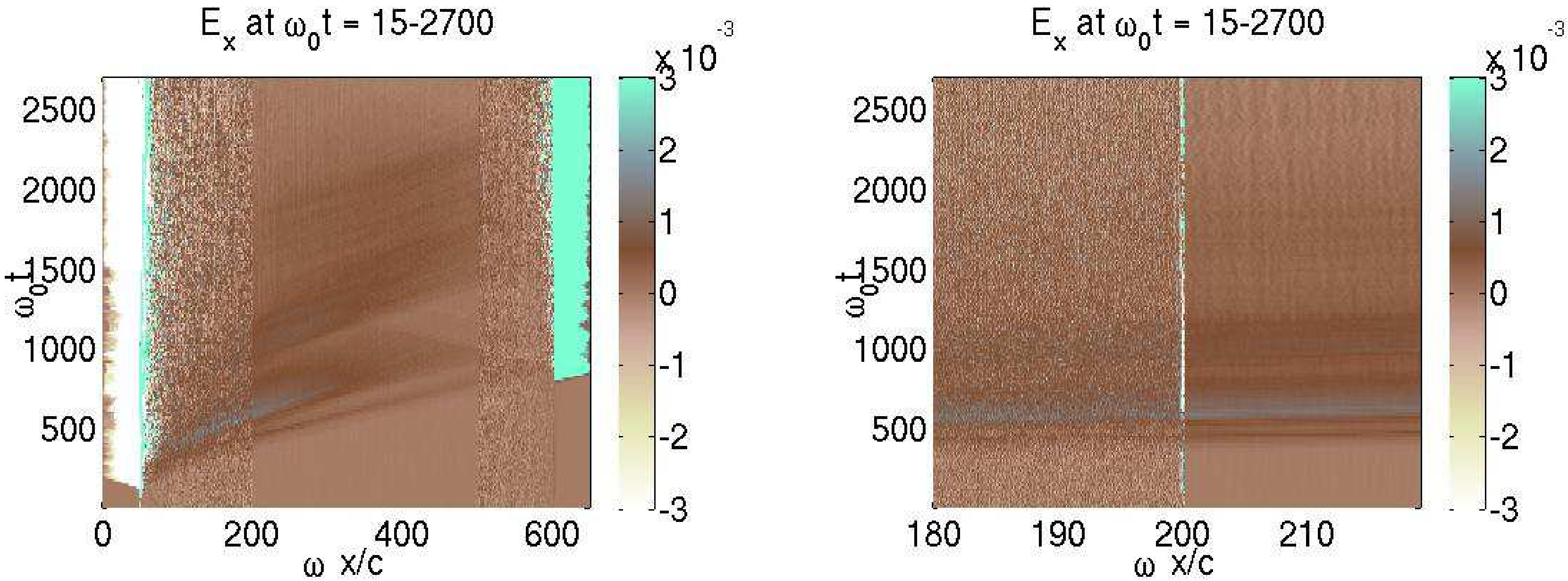


Laser : a = 1

Plasma : n_0 = 400 n_c

1D simulations

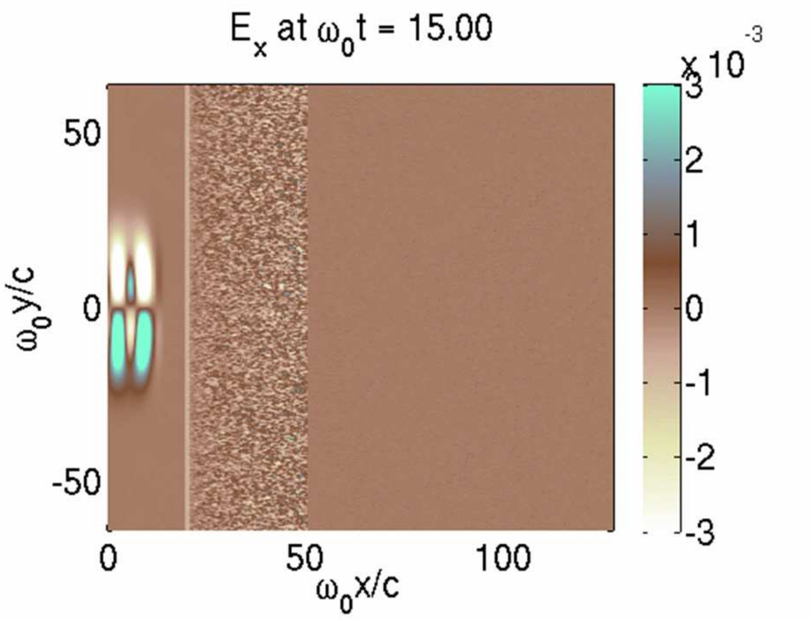
Despite this, 1D simulations still exhibit a problem in a tiny region on the boundary



Abrupt transition

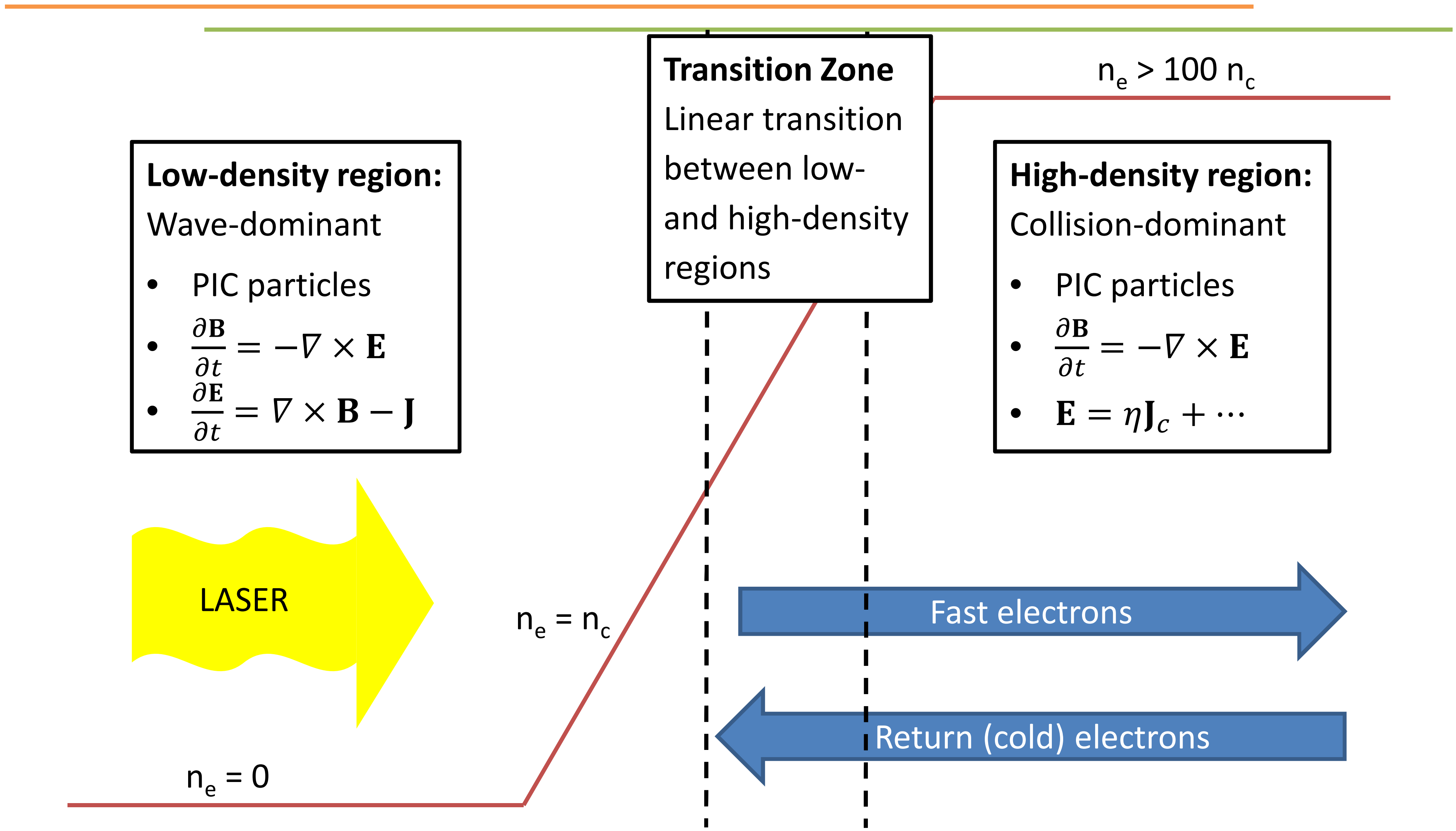
- Causes?
  - Fast-electron wave excitation
  - Numerical heating in *low-density* regions, but not present in *high-density* regions
  - Mis-calculation of the resistivity
  - Abrupt transition between the regions

- Filtering of the electric field
  - Spatial
  - Temporal (Friedman[1])



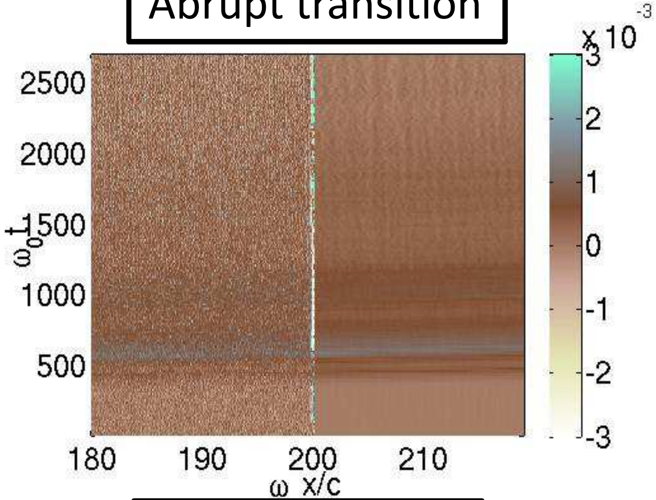
[1] A. Friedman, *Journal of Computational Physics* **90** (1990) pp. 292-312.

Transition Zone

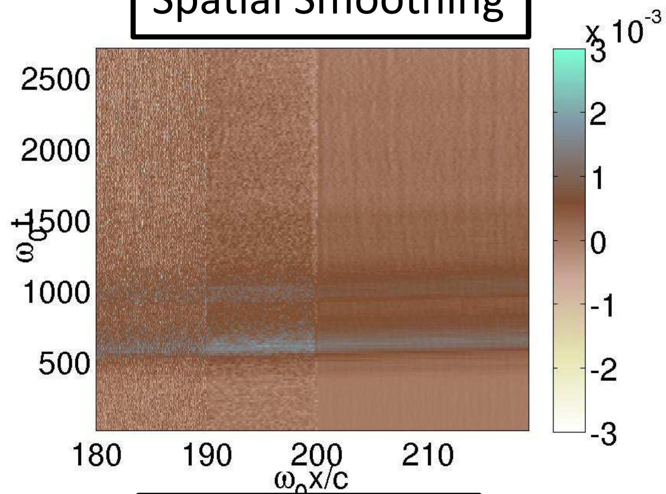


Electric fields (1D, zoomed in)

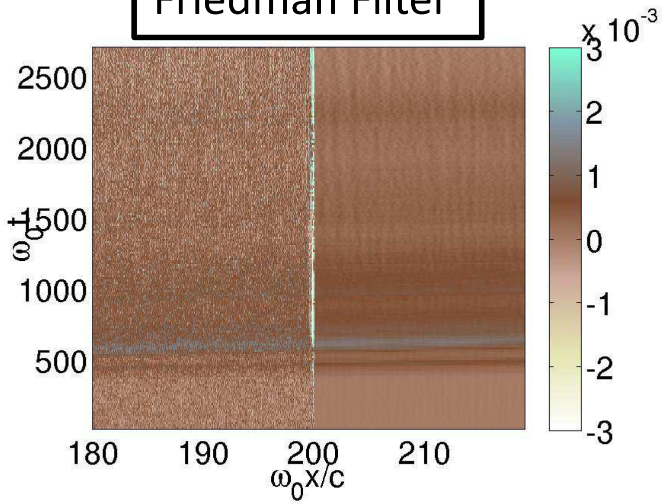
Abrupt transition



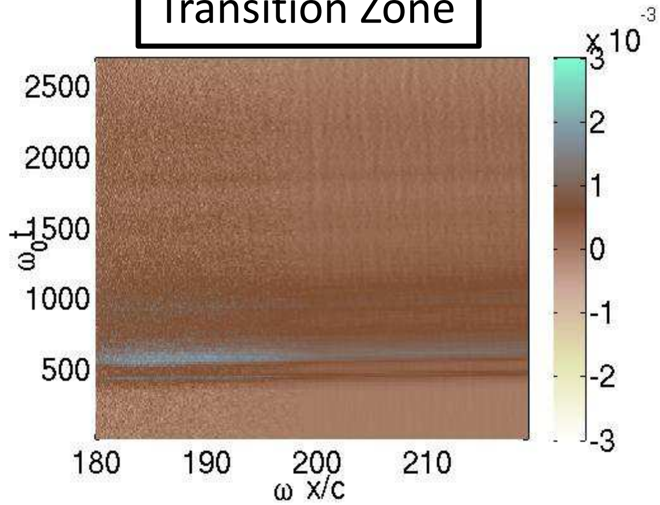
Spatial Smoothing



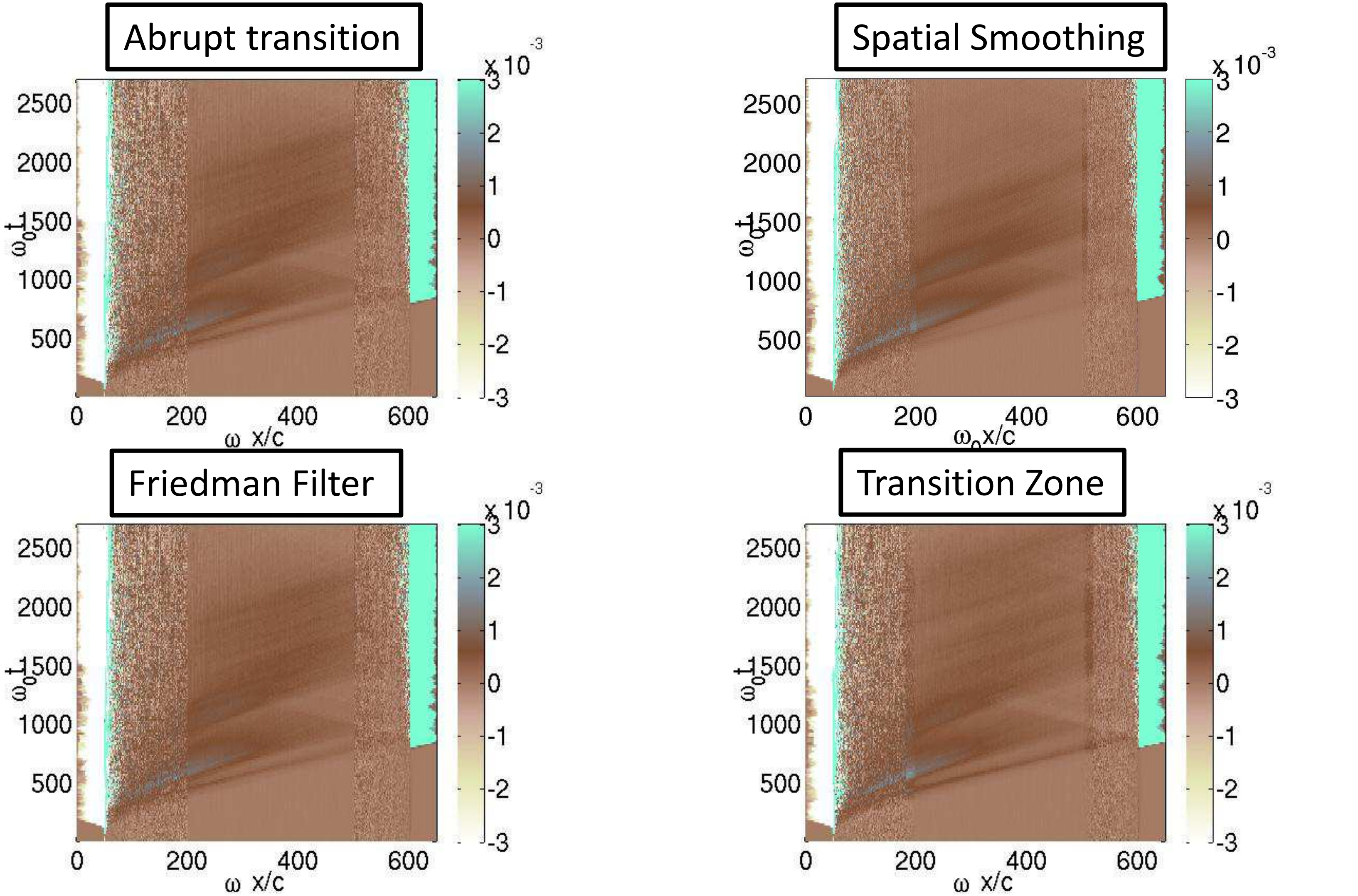
Friedman Filter



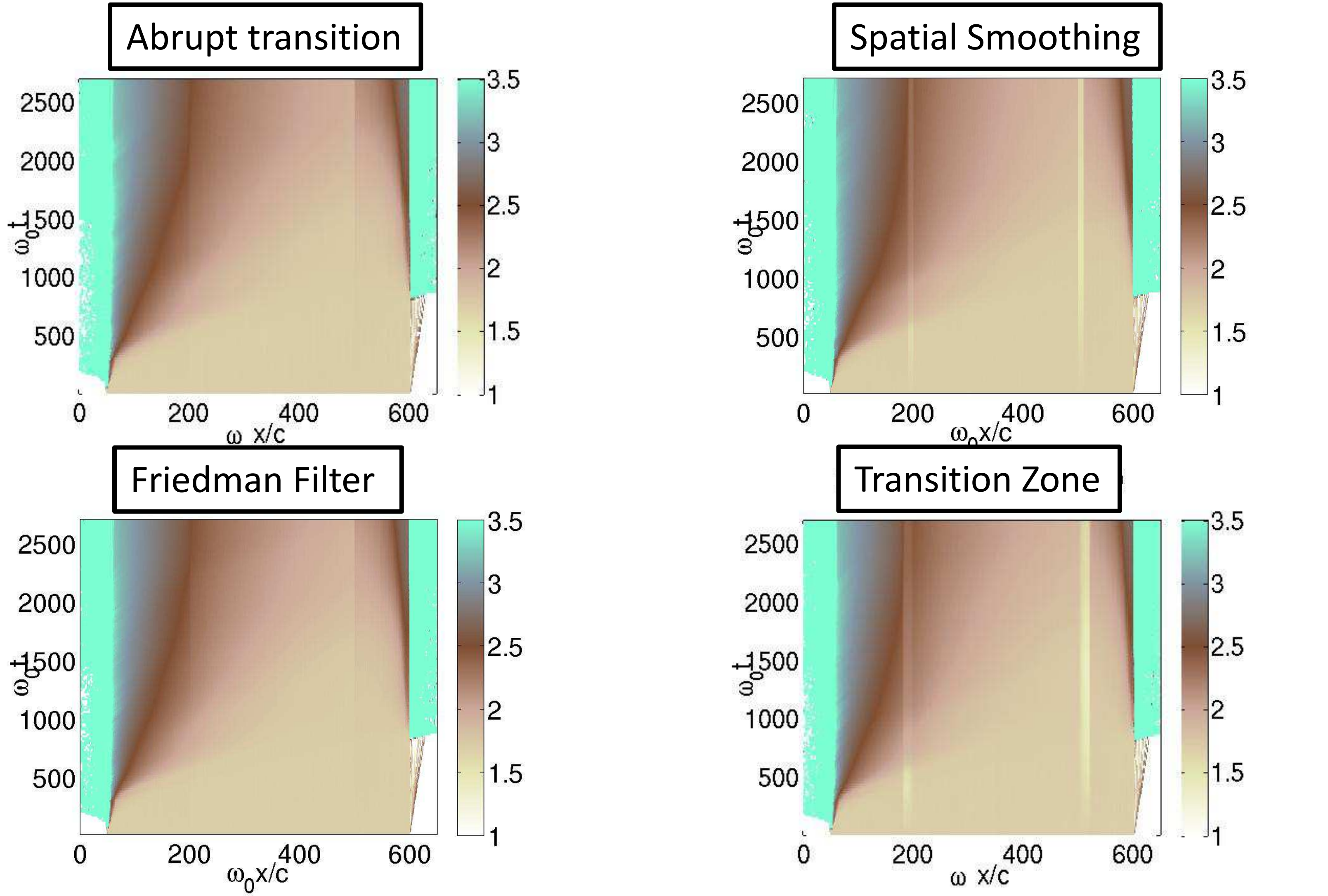
Transition Zone



Electric fields (1D, zoomed out)



Temperature (1D)



Conclusion

- For numerical stability, the displacement current $\partial E/\partial t$ must be included in the determination of the cold electron current
- The boundary between the two regions must be chosen carefully to ensure that v_d/v_{th} remains small
- A transition zone and/or spatial filtering of the electric field helps remove the spikes in electric field, but introduces artificial cooling in the electron temperature
- Larger, 2D simulations are underway to determine the better method of modelling the boundary between the two models