Weibel instability and filamentary structures of a relativistic electron beam in plasma

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Overview

• Weibel instability: introductory remarks
  • Role of temperature and collisions.
• 2D and 3D-PIC Simulation results.
• Theoretical model for collisional Weibel instability.
• Conclusions.

Beam filamentation in fast ignition

\[ I > I_A \]

\[ I_A \Rightarrow \text{Alfvén current} = 17\gamma \, kA \]

Such currents can only propagate as charge neutralized currents.
Beam filamentation: Weibel and two-stream instabilities in FI

• The Weibel instability dominates in relativistic conditions.


• In 3D geometry, coupling of both instabilities causes filamentation.

Relevance of temperature and collisions on filamentation

- Role of beam temperature:

  Threshold temperature

  \[
  \frac{T_{\perp}}{mc^2} > \frac{n_b}{n_0}
  \]

  Weibel Instability disappears!


- Role of collisions in return plasma current

  Quite complicated and unclear...

  C. Deutsch et al. PRE (2005)
Simulation Geometry (2D):

The 2D Transverse Geometry

Simulation dimensions \((X \times Y) = (20 \times 20) \lambda_s\)

\[ \lambda_s = c/\omega_p \quad \text{(plasma skin depth)} \]
Simulation Geometry (3D):

Simulation dimensions  \((X \times Y \times Z) = (20 \times 20 \times 20)\lambda_s\)

\[\lambda_s = \frac{c}{\omega_p} \text{ (plasma skin depth)}\]
VLPL PIC Simulation parameters
[2D (3D) Simulations]

Beam and plasma density and simulation parameters

\[ n_b = 0.1, \quad n_p = 0.9, \quad v_b = 0.9, \quad v_p = 0.1 \]

ions are fixed

Simulation parameters

- Mesh of \(160 \times 160\) (\(160 \times 80 \times 20\)) cells
- 64 numerical particles per cell.
- Resolution: \(\delta x = \delta y = 0.125 \lambda_s\)
- Electron beam temperature, \(T_b = 70\) keV
- Collisions frequency \(\nu_{ei}/\omega_{pe} = 0.15\)
Different simulation case

(a) **Cold** electron beam in a *collisionless* background plasma
(b) **Cold** electron beam in a *collisional* background plasma
(c) **Warm** electron beam in a *collisionless* background plasma
(d) **Warm** electron beam in a *collisional* background plasma
2D PIC results
(a) Cold beam in a cold collisionless plasma

\[ B_x \]
\[ E_y \]
\[ n_b \]

\[ T = 15.0 \]
\[ T = 20.0 \]
\[ T = 30.0 \]
\[ T = 90.0 \]
2D PIC results
(b) Cold e-beam in a cold collisional plasma
2D PIC results
(c) Warm e-beam in a cold plasma

2D PIC results
(d) Warm e-beam in a cold collisional plasma
Filaments stopping and collective merging

Honda, Pukhov, Meyer-ter-vehn, PRL, 2000
3D filamentary structures

Can temp. suppress filamentation in 3D geometry?


Instability is not suppressed!
Mag. field structures: 3D simulation [case (a)]
Field energies development: 3D simulations
Theoretical model:  

*Karmakar et al. PRL 101, 255001, (2008)*

The model assumes an infinite beam propagating in z-direction.

The dominant electric and magnetic fields of the beam-plasma system

\[
B_\perp = -e_z \times \nabla_\perp A_z, \quad E_z = -(1/c)(\partial A_z/\partial t), \quad E_\perp = -\left(v_{pz}/c\right)\nabla_\perp A_z,
\]

\[
\nabla^2_\perp A_z = -\frac{4\pi}{c} (J_{bz} + J_{pz})
\]

\[A_z \Rightarrow \text{z component of vector potential}\]

\[J_{bz} \text{ and } J_{pz} \Rightarrow \text{Current densities of the beam and background plasma}\]
Theoretical model (contd.):

Eq. of motions for beam and plasma electrons are

\[
\frac{\partial v_{pz}}{\partial t} + v \cdot v_{pz} = \frac{e}{mc} \frac{\partial A_z}{\partial t};
\]

\[
\frac{d \left( \sum_j v_{j\perp} \right)}{dt} = - \frac{e(v_{jz} - v_{pz})}{mc} \nabla_{\perp} A_z
\]

\( \nu \Rightarrow \text{collision frequency} \)

Conservation of energy equation:

\[
\sum_j \gamma_j mc^2 - \sum_j \frac{m}{2} v_{pz}^2 \epsilon_j \int d^2 x L_z \frac{B_{\perp}}{8\pi} + \int d^2 x L_z \frac{n_0 m v_{pz}^2}{2} = 0
\]
Dispersion relations

For collisionless plasma and long wavelength perturbation $|k_\perp| \ll \omega_p^2/c^2$

$$\omega^2 = \bar{c}_s^2 k_\perp^2$$

$$\bar{c}_s^2 = c_s^2 - c^2 \beta_0^2 n_b/n_0$$

For cold beam $c_s^2 < c^2 \beta_0^2 n_b/n_0 \Rightarrow$ Unstable and leads to the Weibel instability

Dispersion relation for finite plasma resistivity:

$$\omega = c_s^2 k_\perp^2 - \frac{\omega_b^2 \beta_0^2 k_\perp^2}{k_\perp^2 + \frac{k_{pe}^2}{\epsilon + iv/\omega}}$$
Dispersion relations

For smaller collision frequency:

\[ \omega \approx \pm c_s k_\perp - i \nu \frac{c^2 \beta_0^2 n_b}{2 \bar{c}_s^2 n_0} \]

\[ \omega \approx i \nu \frac{c^2 \beta_0^2 n_b}{2 \bar{c}_s^2 n_0} \quad \Rightarrow \text{negative energy mode} \]

Negative energy waves can be destabilized by the resistivity of the plasma

H. Pecseli, Plasma Physics. 17,497 (1975)
Conclusions

2D and 3D PIC simulations show that Weibel instability might be a matter of concern in fast ignition scenario.

Collisions seem to play a deleterious role in the energy transportation to the core. The role is attributed to the generation of negative energy waves in the beam-plasma system.

In 3D geometry, the two-stream instability generated turbulence might provide effective collisionality to the beam-plasma system, which can drive the Weibel instability.