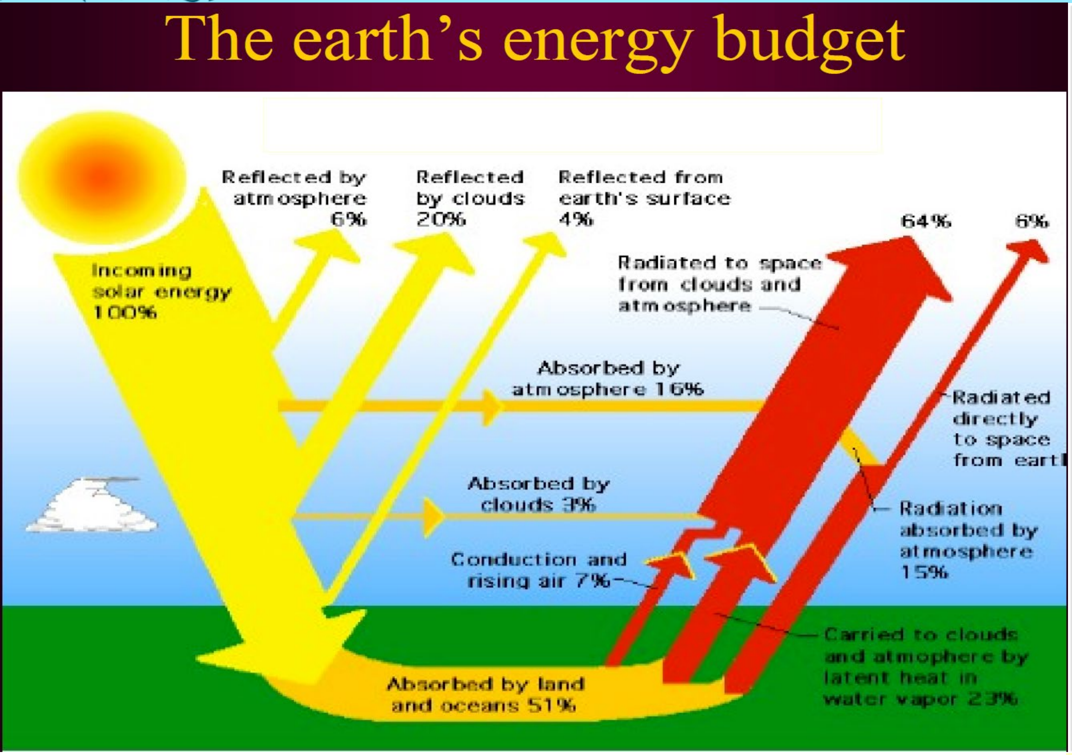
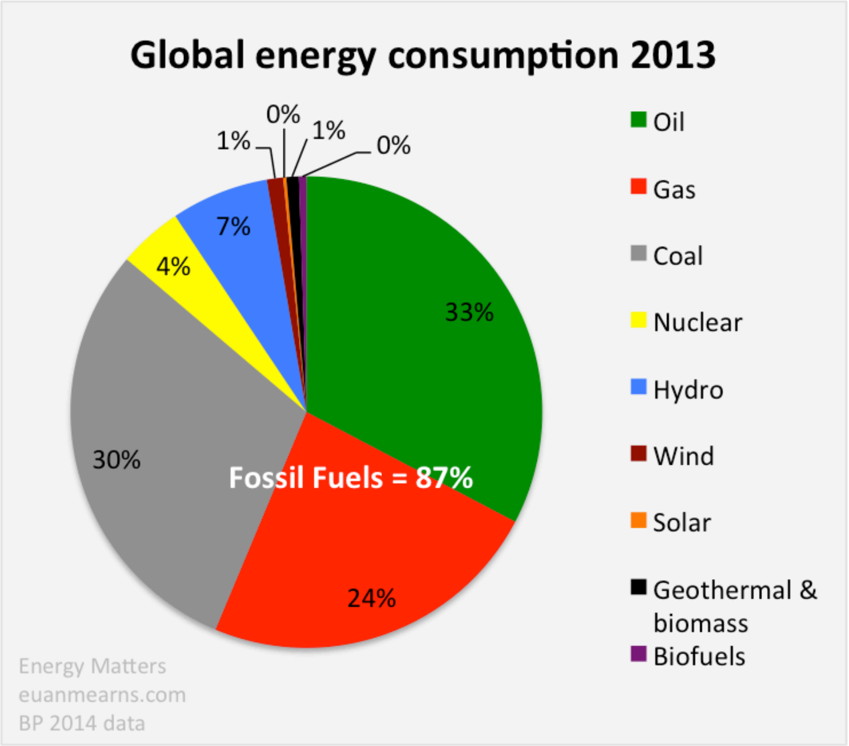
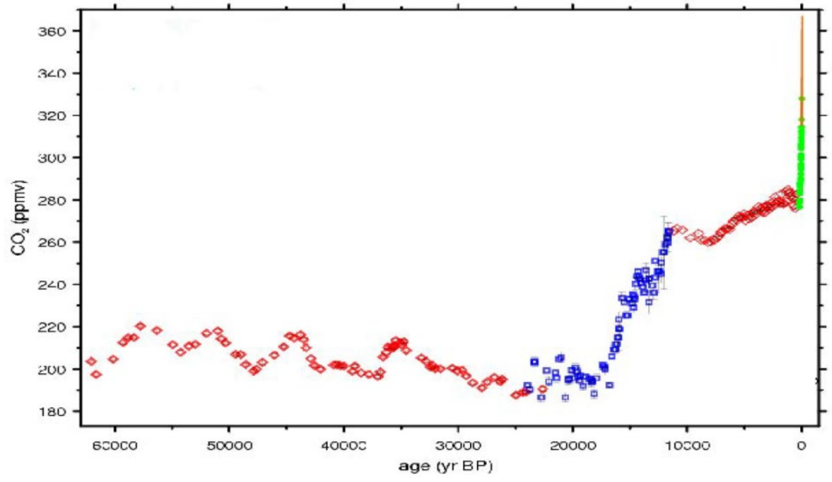
Nuclear Fusion for Energy Production

- ☐ **Why is it so attractive ?**
  - It could guarantee practically unlimited source of energy
  - Ecologic hazards significantly decreased compared to nuclear fission
  - No weapon materials proliferation issues
- ☐ **Why mankind needs a novel energy source**
  - Resources of fossil fuels and fission materials are limited
  - Ecological issues of burning fossil fuels and fission
  - Limitation of renewable energy resources
- ☐ **Global energy balance**
  - Sun delivers to earth 2×10^{24} J/year, global consumption 5.7×10^{20} J = 13699 Mtoe (toe = ton of oilequivalent)
  - Low concentration is the drawback of solar energy

Global graphs



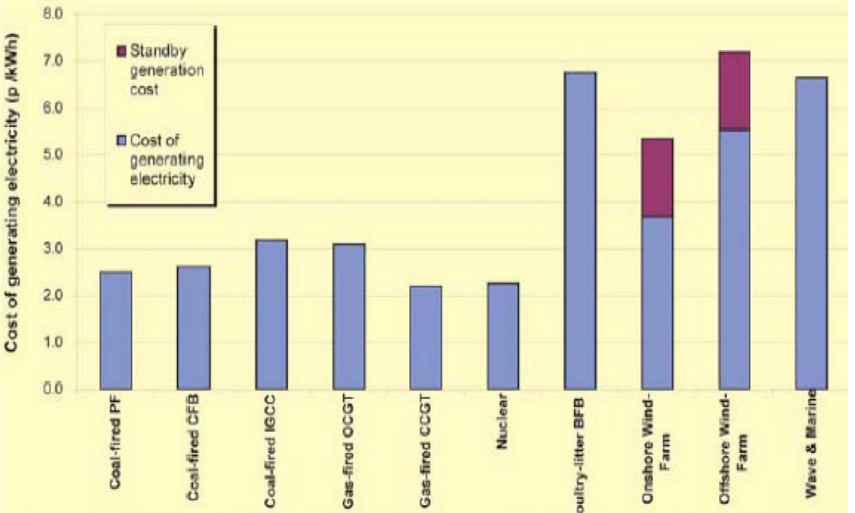




CO$_2$ from 2000 back 60 kyears

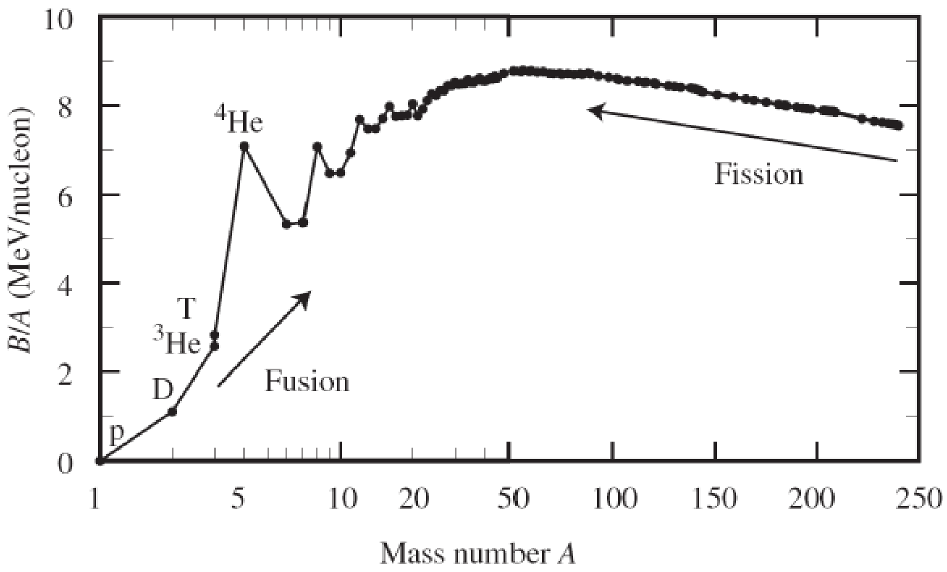
Cost generating electricity

*no cost of CO$_2$ emissions included



Nuclear energy

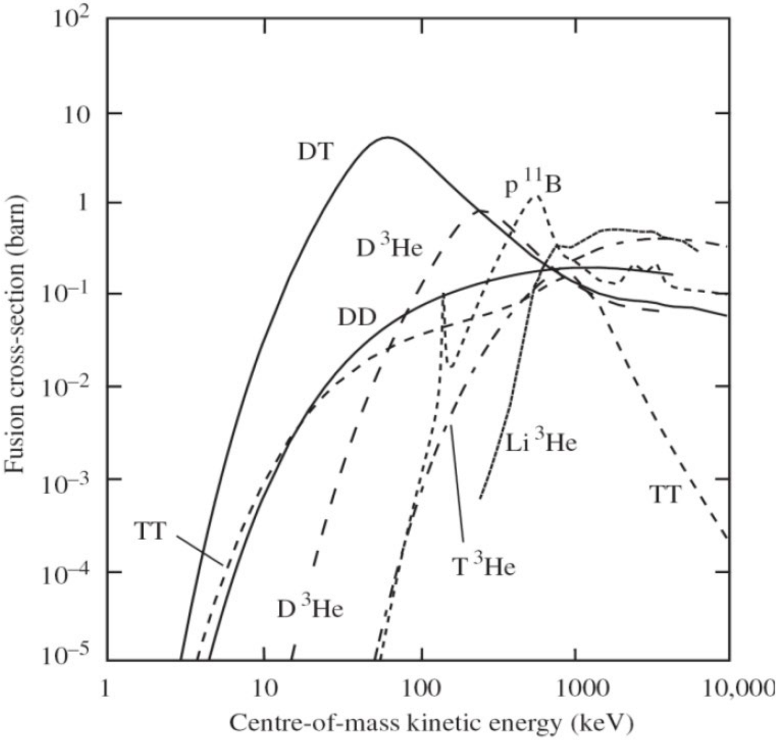
Differences in binding energy per nucleon are exploited for energy production



- The most stable nucleus is ^{56}Fe (Z=26, A = 56)
- Energy gained either by fission of heavy nuclei
- or by fusion of light nuclei
- α particle (^{4}He) – high B/A

- Coulomb repulsion prevents nuclei from fusing – height ~1 MeV
- Fortunately, quantum tunneling enables fusion at lower energy
- Fusion cross-section is ~$10^6\times$ less than for elastic collisions $\Rightarrow$ beam-target interaction cannot produce energy gain

Fusion reactions



- Reaction $D + T \rightarrow n + {}^4He + 17.6\ MeV$
  highest cross-section at low energies
  high energy 340 GJ/g of fuel
  1 g DT $\cong$ 4.5 g ^{235}U $\cong$ 10 t coal
- Tritium is missing in nature, but it can be produced from abundant Li
  $n + {}^6Li \rightarrow {}^4He\ (2.1\ MeV) + T\ (2.7\ MeV)$
  $n + {}^7Li \rightarrow {}^4He + n + T - 2.47\ MeV$
- Ideal ignition temperature (fusion energy = radiation losses) $T_{id} = 4.3$ keV

DT drawback energetic n *(how is energy distributed between n and ^4He ?)*

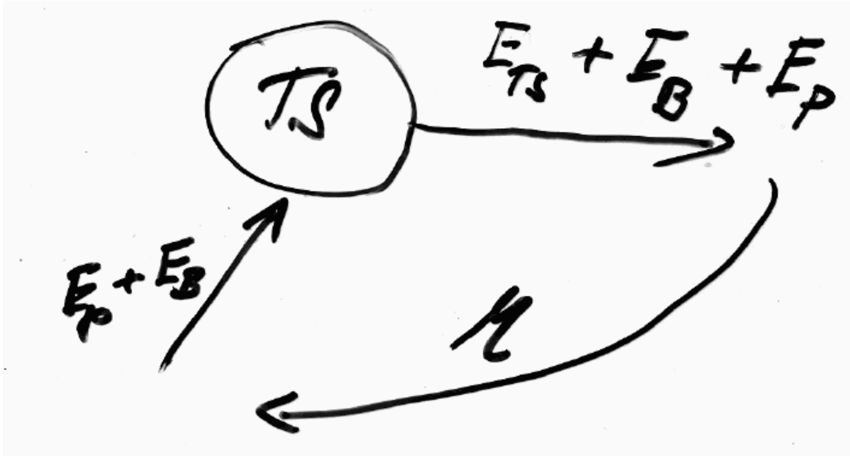
DD reaction – only slow neutrons, higher threshold, lower yield
2 channels $D + D \rightarrow p + T + 4\ MeV$; $D + D \rightarrow n + {}^3He + 3.27\ MeV$

Neutronless fusion – charged products (no radioactivity), T_{id}~100 keV
$p + {}^{11}B \rightarrow 3\times {}^4He + 8.7\ MeV$; $p + {}^6Li \rightarrow {}^4He + {}^3He + 4\ MeV$

Alternative - muon catalysis of fusion

- Muon – 207x heavier than electron, otherwise very similar properties, but half-life $\tau^{\mu}_{1/2} \cong 2.2\,\mu s$
- Lowering of the potential barrier in muonic molecule DTμ, fusion takes place also at room temperature
- Needed source intensity 5×10^{14} μ/s - this can be achieved in future
- Problems – **number of fusions per 1 μ**,
  after fusion 0.8% μ stay attached to α particle and they are lost for fusion, this problem has not been solved despite many years' effort
- Energy needed for generation of 1 μ is at present 6 GeV (though $m_{\mu}c^2$ = 105 MeV), for the above losses of μ, it would be needed to decrease it to 1.5 GeV, and it also has not been reached
- It seems there is **no chance** for energy production

Fusion energy balance



$$Q = \frac{E_F}{E_B + E_p}$$

E_B – bremsstrahlung losses = $\alpha_B n^2 T^{1/2} \tau$

E_p – plasma energy = $2(3/2\, n\, k_B\, T)$

$\eta\,(E_{FS}+E_B+E_p) \geq E_p + E_B \Rightarrow$

$Q \geq 1/\eta - 1 = 1/(1/3) - 1 = 2$

Choice η = 1/3 – Lawson

Fusion energy gain $E_{TS} = ¼\, n^2 \langle\sigma v\rangle\, \varepsilon_S\, \tau$

$$Q = \frac{n\tau\left(\frac{1}{4}\langle\sigma v\rangle_T\, \varepsilon_S\right)}{3k_B T + \alpha_B T^{1/2} n\tau} = f(n\tau, T)$$

$T \cong 10$ keV (1.16×10^8 K)

$n\,\tau \geq 10^{14}$ cm^{-3}s Lawson criterion (1)

2 basic options – $n \sim 10^{14}$ cm^{-3}, $\tau \sim 1$ s – magnetic confinement

– $n \sim 10^{23}$ cm^{-3}, $\tau \sim 10^{-9}$ s - inertial confinement

(less frequent middle option – $n \sim 10^{18}$ cm^{-3}, $\tau \sim 10^{-4}$ s – pinch – dense magnetized plasma) – historically the first one studied

(1)*Lawson criterion derived from energy production arguments (original derivation)*

Fusion power balance

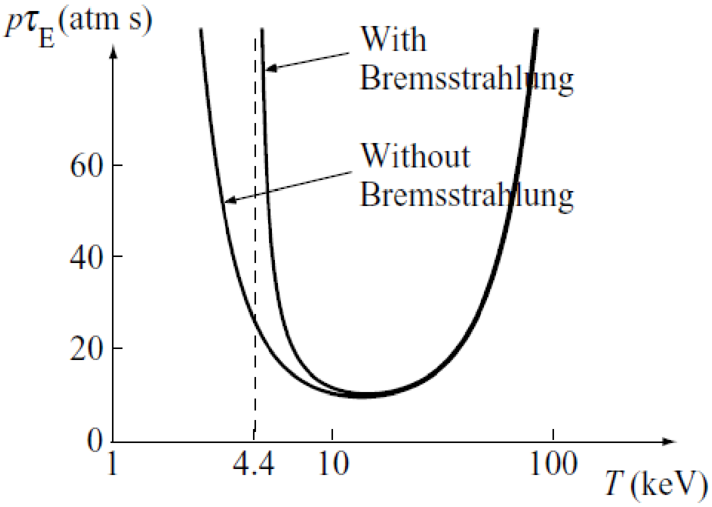
- Fusion products – neutron escapes from fuel but α-particles are basically stopped in the fuel and heat it, let η_α is the part of α energy heating the fuel, then the heating power is

$$S_\alpha = \frac{1}{4}\eta_\alpha E_\alpha n^2 \langle \sigma \mathrm{v} \rangle = \frac{1}{16}\eta_\alpha E_\alpha \frac{P^2}{(k_B T)^2}\langle \sigma \mathrm{v} \rangle$$

- Power loss by radiation (bremstrahlung emission) and due to finite energy confinement time τ_E are

$$S_B = C_B Z_{eff} n^2 T^{1/2} \cong C_B n^2 T^{1/2} = C_B \frac{P^2}{k_b^2 T^{3/2}} \qquad S_C = \frac{3}{2}\frac{P}{\tau_E}$$

- At threshold $S_\alpha = S_B + S_C$. For $\eta_\alpha = 1$ plotted threshold versus T
- Minimum $P\tau_\mathrm{E} \cong 8.3$ bar.s at T = 15 keV corresponds to $n\tau_E = 1.7\times10^{14}$ cm^{-3}s (1)
- At 5 keV threshold $P\tau_\mathrm{E} \cong 36$ bar.s



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(1) *Lawson criterion derived from power balance arguments*

Confinement and burn

- Hot fuel has to be confined for sufficient time τ to allow significant burn fraction Ψ

  Let n_f is cumulative number of fusion reactions in unit volume,

  n_D, n_T is deuterium and tritium density, then

  $$\frac{\mathrm{d}n_f}{\mathrm{d}t} = n_D n_T \langle \sigma \mathrm{v} \rangle$$ for $t=0$ $n_D = n_T = n_0/2$ and $n_f = 0$

  $n_f(t) = \Psi(t) \times n_0/2$ *and* $$\frac{n_0}{2}\frac{d\Psi}{dt} = \frac{n_0{}^2}{4}\langle \sigma \mathrm{v} \rangle (1-\Psi)^2$$

  for constant reaction rate ($T_i \approx 20$ keV $= 2.32 \times 10^8$ K)

  $$\Psi(\tau) = \left(1 + \frac{2}{n_0 \langle \sigma \mathrm{v} \rangle \tau}\right)^{-1}$$ to reach $\Psi = 1/3 \Rightarrow n_0 \tau \geq \langle \sigma \mathrm{v} \rangle^{-1} \approx 10^{15}$ cm^{-3}s

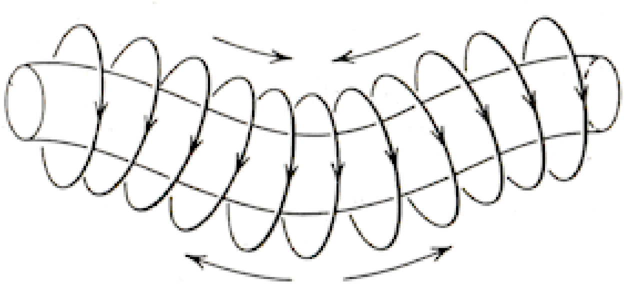
- Confinement
  - Gravitational (stars) – p-p cycle (Sun); CNO cycle (↑T); CC reactions (WD)
  - Magnetic (tokamaks, stellarators, $n_0 \approx 10^{14}$ cm^{-3}, $n_0 \tau_E \geq 10^{14}$ cm^{-3}s, $\tau > \tau_E$)
  - Inertial (direct drive; indirect drive)

Magnetic confinement

Many schemes of magnetic confinement do exist

- Closed systems
  - **Stellarators**
  - **Tokamaks**
  - Multipoles
  - Devices with relativistic electron beam (ASTRON)
- Magnetic mirrors
  - Magnetic cusp
  - Baseball-seam coil
- Pinches
  - z-pinch
  - θ-pinch

Problems – stability – typical **kink**

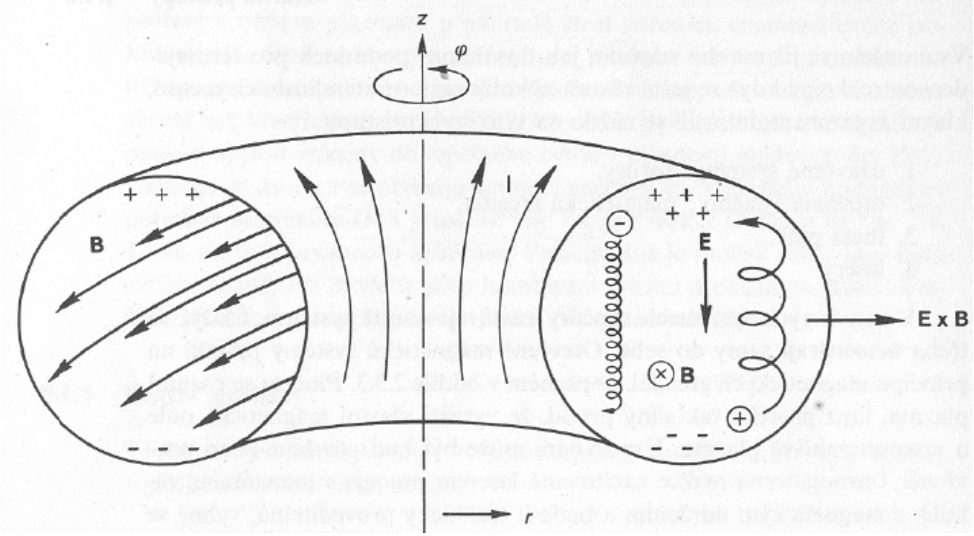


(*a*) Kink instability

Closed systems

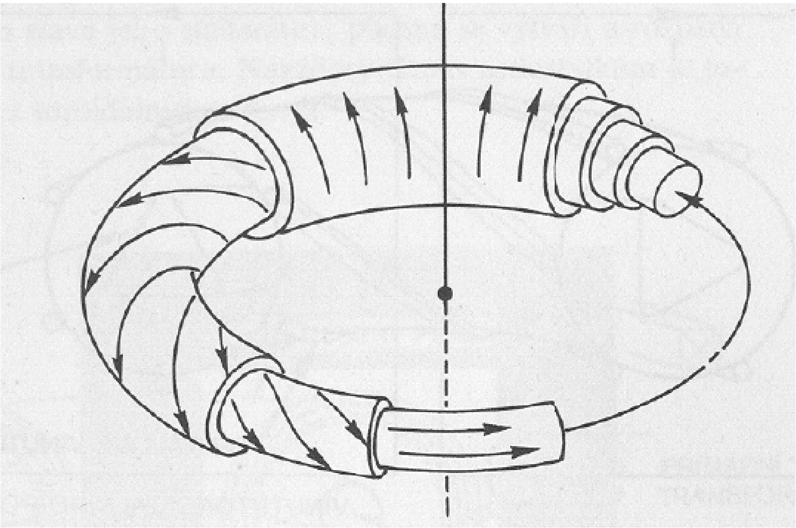
Simple torus is unstable

- Curvature and grad*B* drifts cause electrons and ions drifting to opposite sides
- Space charges ⇒ *E* field
- *E*×*B* drift moves plasma out



Instability mitigation

- Sheared magnetic field
- Magnetic field with minimum inside
- Dynamic stabilization

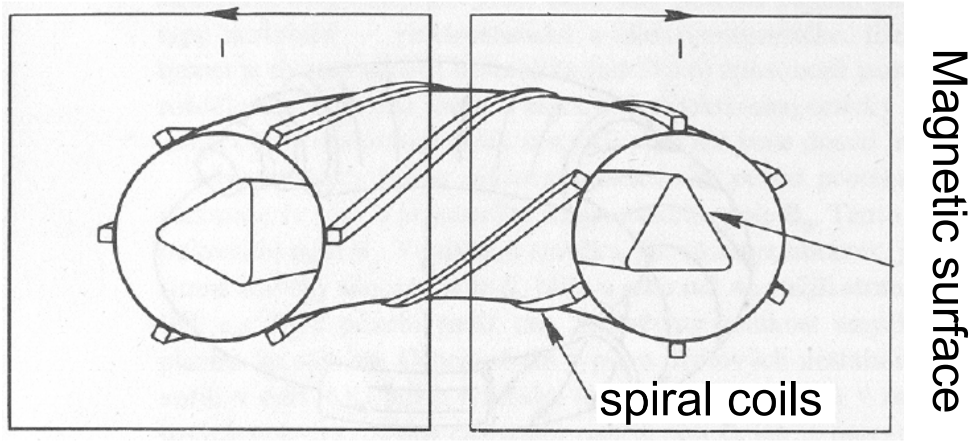


Sheared magnetic field in torus

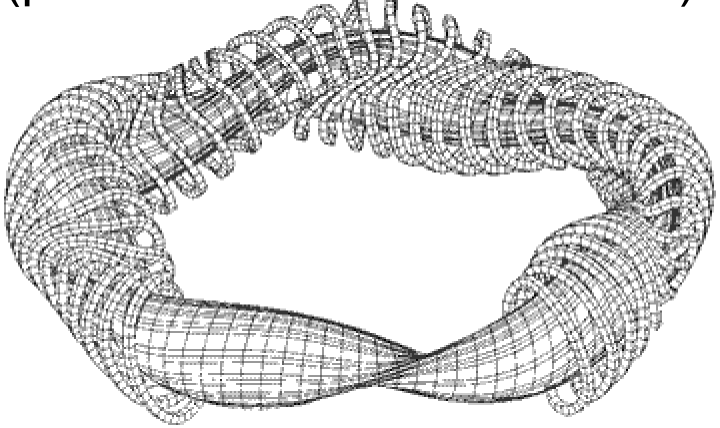
Stellarator

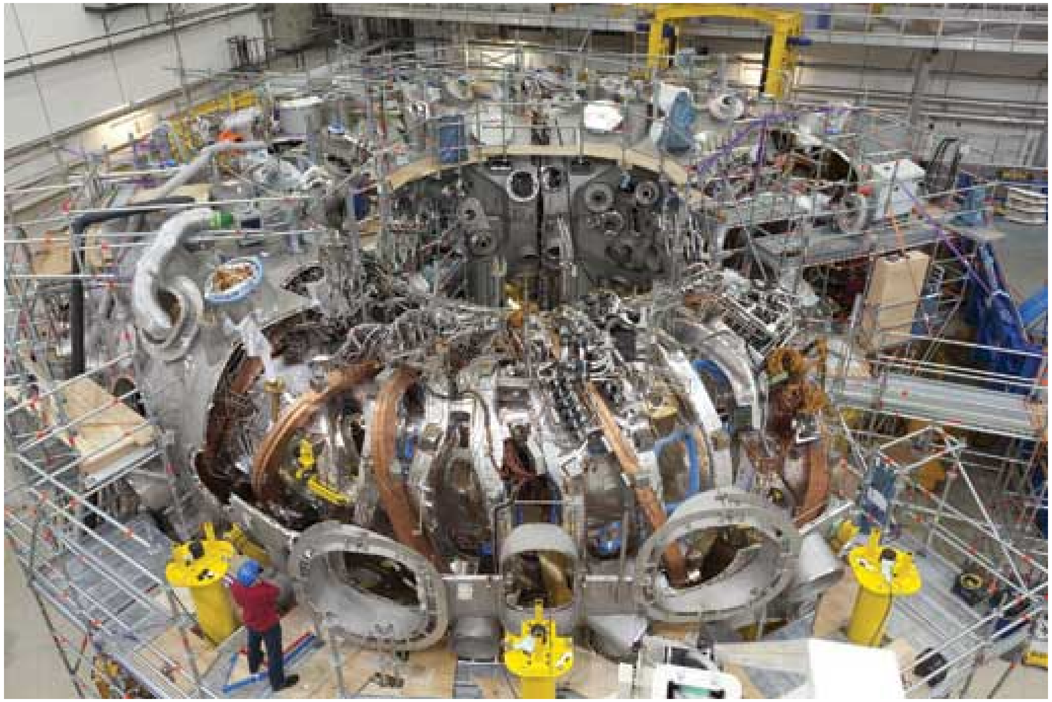
Toroidal equilibrium stationary system – external heating

Magnetic field formed only by external coils, field lines stay at nearly constant minor radius. Field lines form magnetic surfaces, do not leave magnetic surfaces. In 2015, physical experiments started on new supraconductive stellarator Wendelstein-7X in Germany (plasma 30 m^3 x JET 100 m^3)

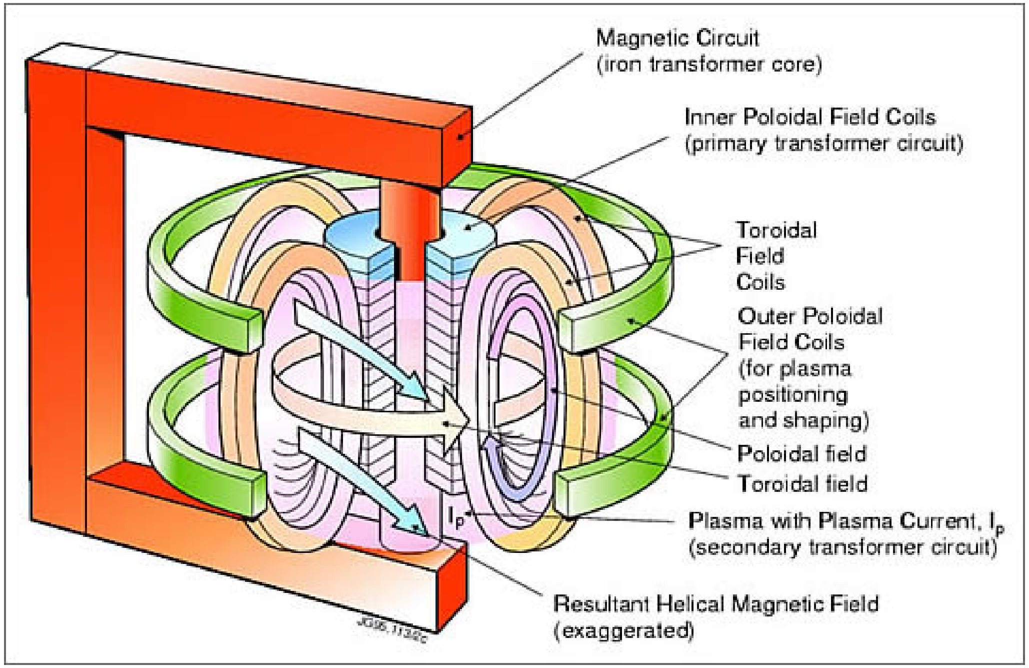


coils of toroidal field





Tokamak



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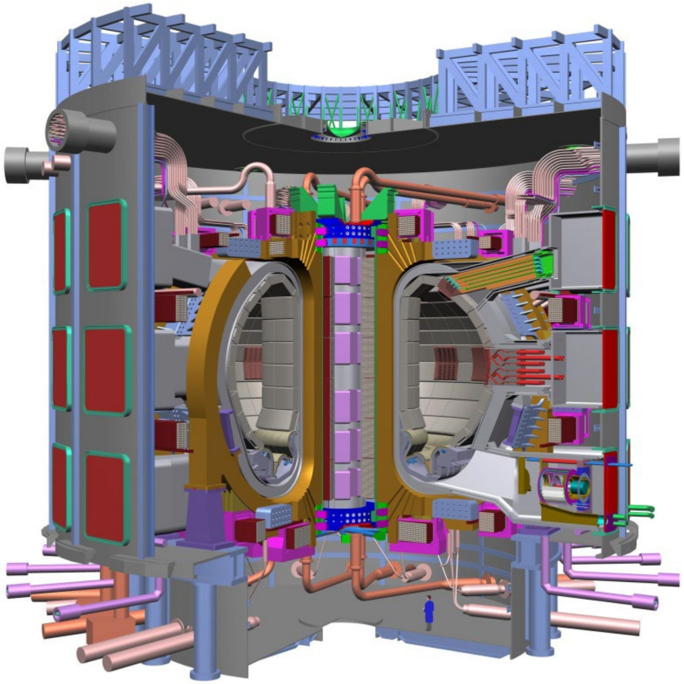


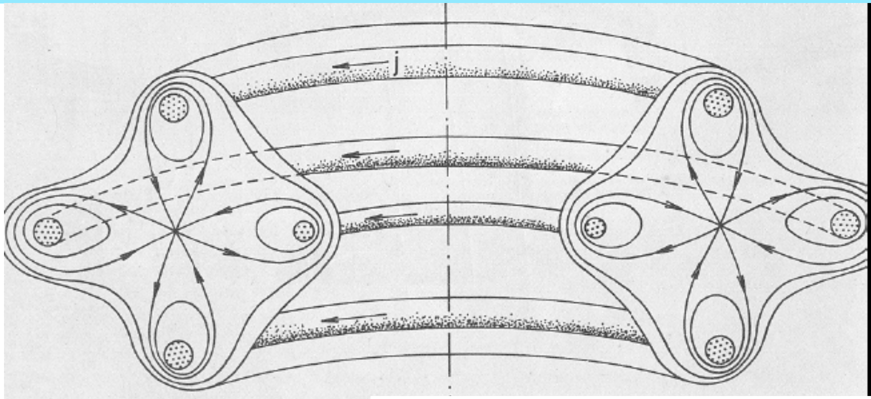
Tokamak (from Russian – toroidal chamber with magnetic coils) – basically a transformer where toroidal plasma acts as the secondary circuit, plasma current creates poloidal field. Besides toroidal field created by external coils , third vertical (poloidal) magnetic field is also needed (external coils)

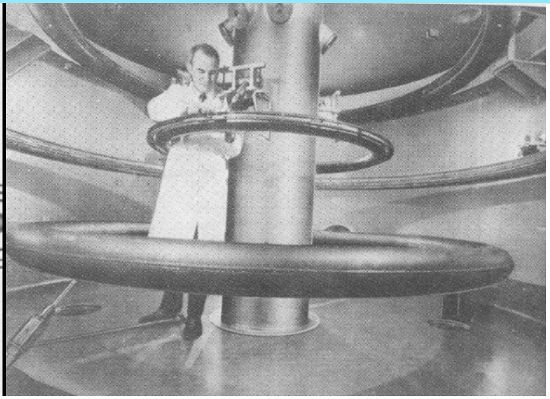
Works in pulsed regime, primary Ohmic heating cannot reach fusion temperature, secondary heating – neutral particle beams or RF antennas

5 big tokamaks in 1980’s– JET (UK), now ITER under construction (2025?)

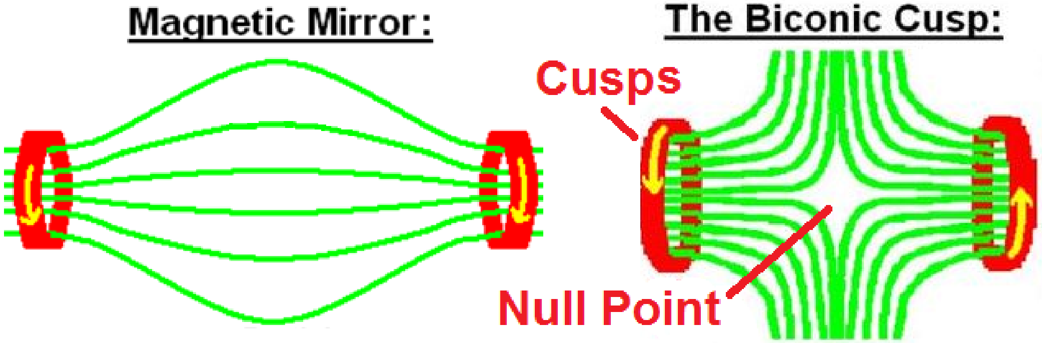
Mutipoles, magnetic mirrors

Multipoles – with parallel conductors in toroidal shape form minimum-B configuration that is MHD stable

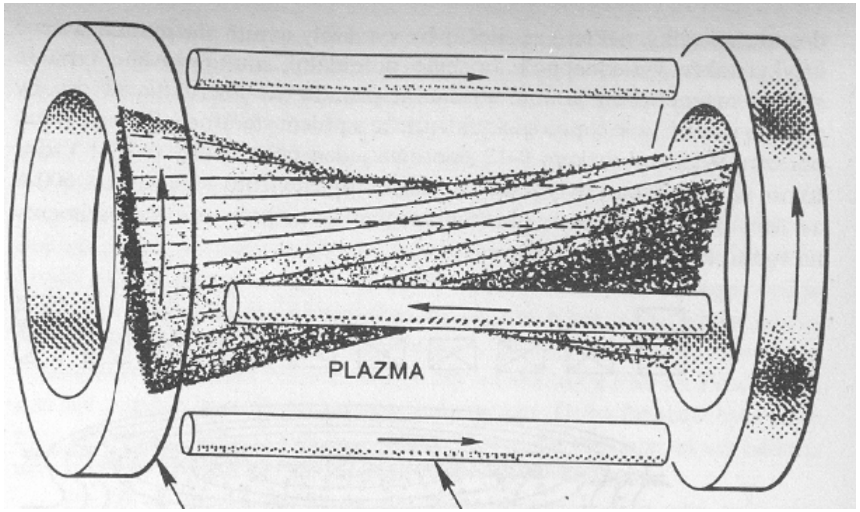


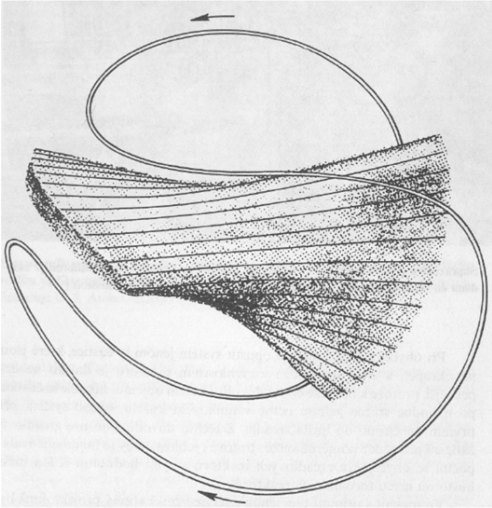


Ordinary magnetic mirror is also unstable, but magnetic cusp is stable.



Stable configuration is achieved by adding Ioffe rods. The topologically same configuration is achieved in the baseball-seam coil

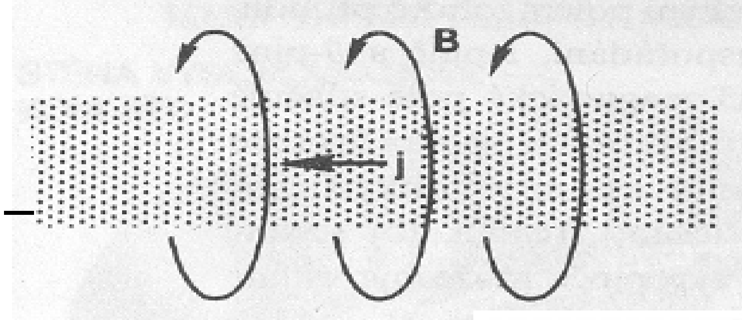


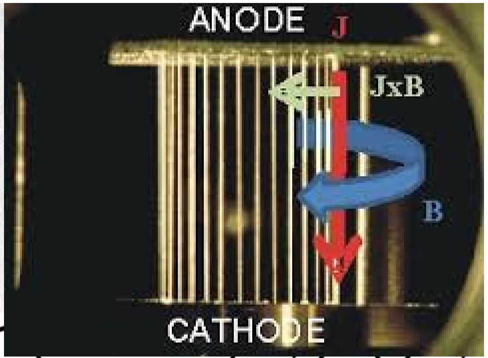


Pinch – z-pinch and θ-pinch

Z-pinch – magnetic field created by high-current discharge can compress it – pinch effect

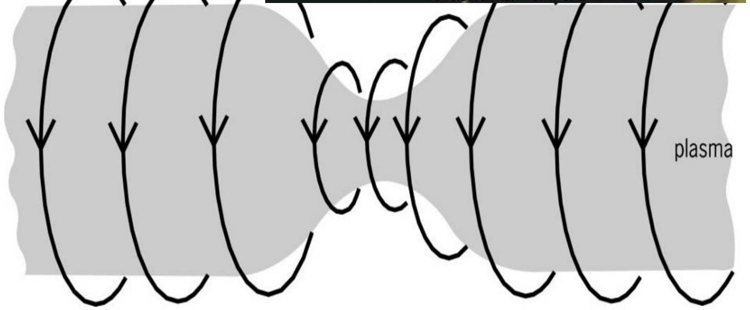
classical z-pinch unstable equilibrium





z-pinch; wire-array z-pinch

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(p+\frac{B^2}{2\mu_0}\right)=\frac{1}{\mu_0}\left(\vec{B}\nabla\right)\vec{B}=-\frac{B^2}{\mu_0 r}$$



sausage instability

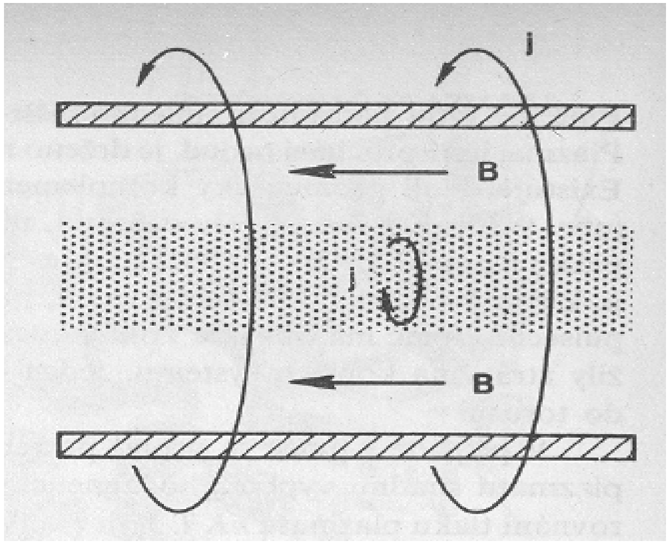
$$B=\frac{I}{2\pi r\,\varepsilon_0\,c^2}\Rightarrow$$ $I^2 = 2\times10^7\ N\ k_B T$, *electron number per length* $N=\pi R^2 n$

***the* Bennett relation**

Z-machine in Sandia National Laboratory, USA

θ-pinch – current in θ direction in the outer shell induces opposite θ current in plasma column

surprisingly stable, may be also used in toroidal geometry

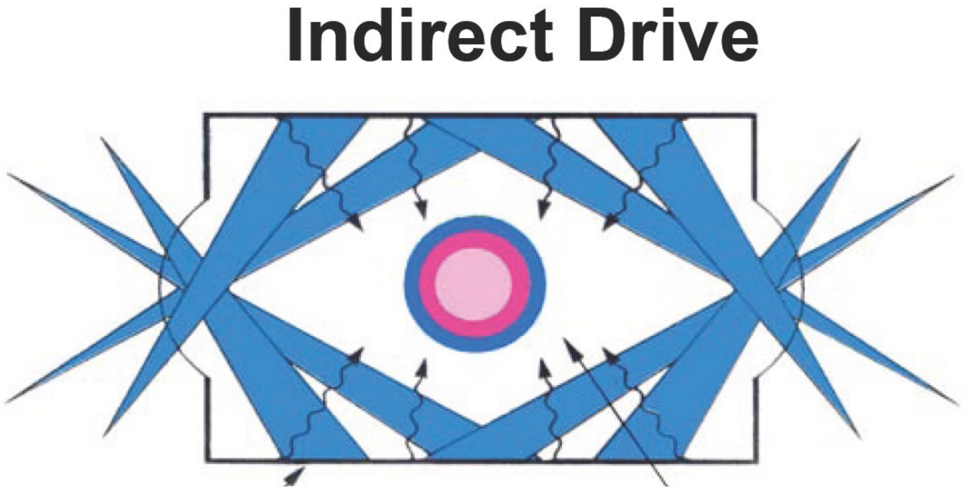


θ-pinch

Inertial fusion – compression necessity

- Inertial confinement – hardly any confinement, due to inertia disassembling of hot fuel takes final time
- Spherical hot fuel assembly assumed (radius R), then $\tau \approx R/3c_s$ (ion sound velocity ~ $T^{1/2}$) and $n_0 \cong \rho/2.5m_p \Rightarrow$ $\Psi = \dfrac{\rho R}{\rho R + H_B}$, $H_B \cong 6.3$ g/cm^2, $\Psi = 1/3 \Rightarrow \rho R = 3$ g/cm^2
- Fuel pressure P [bar] $\approx 8\times10^8 \rho T_i$ [keV]
- In ICF (inertial confinement fusion) conditions: $\rho R \approx 3$ g/cm^2, $T \approx 10$ keV $\Rightarrow PR \sim 3\times10^{10}$ bar×cm $\Rightarrow$ $E \sim PV \sim 3\times10^9 R^2$ [J]
- If $E \sim 300$ kJ can be delivered to the fuel, then $R \sim 100$ μm, $P \sim 3$ Tbar, $\rho \sim 300$ g/cm^3 (solid DT density $\rho_{DT} = 0.25$ g/cm^3, $m_{DT} \sim 1.25$ mg)
- *How to achieve such tremendous pressures and densities?* **Carefully tuned spherical implosions !!**

Indirect and direct drive

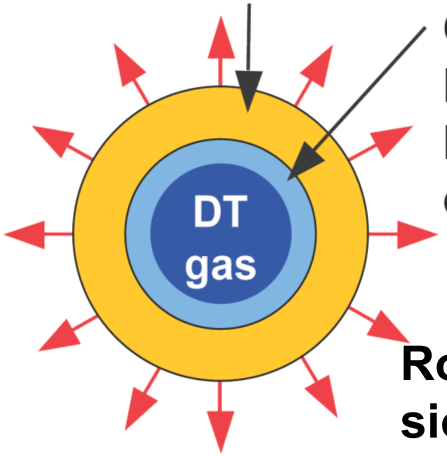


Lasers, heavy ion beams or Z-pinches produce in a miniature cavity called hohlraum X-rays that ablate capsule

Lasers or heavy ion beams directly irradiate and ablate the capsule

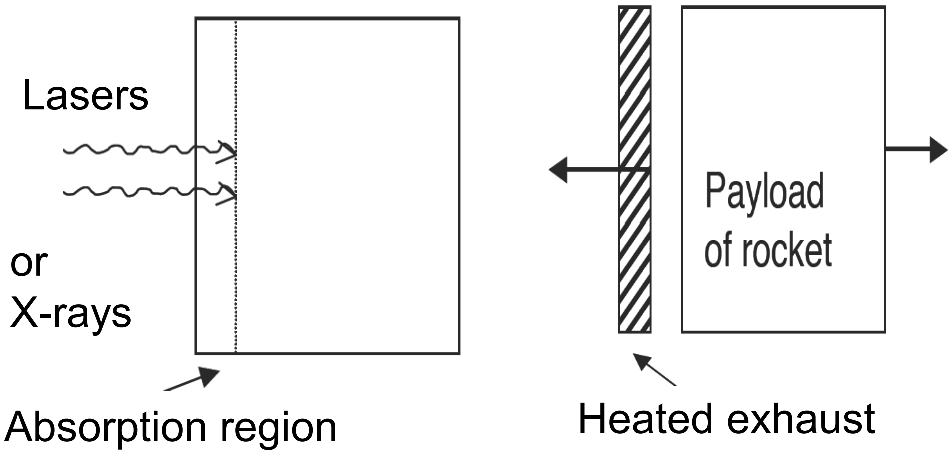
Ablation and compression

Low-Z ablator for efficient absorption



Cryogenic Fuel for Efficient compression

Rocket-like implosion of nearly Fermi-degenerate fuel



- Acceleration comes from particle momentum
- Irradiance is balanced by the outflow of heated material
- For ID $I_{X\text{-}ray} \approx \sigma_{SB} T_r^4 \sim nTc_s \Rightarrow P_{abl} \sim \sigma_{SB} T_r^4/c_s \sim T_r^{3.5}$
- Typically $T_r \approx 300$ eV $\Rightarrow I_{X\text{-}ray} \approx 8\times10^{14}$ W/cm$^2 \Rightarrow P_{abl} \approx 100$ Mbar
- Similar laser intensities used, short λ to avoid fast electron preheat
- Stagnation (max. compression) $P_{sg}V_{sg} \approx P_{abl}V_0$ and $P_{sg} \sim 10^4\ \mathrm{P}_{\mathrm{abl}}$ $\Rightarrow V_{sg} \sim 10^{-4}\ V_0 \Rightarrow R_0/R_{sg} \sim 20$ *(like compressing football to a pea)*

Energy flow and gain

- Driver energy E_D is coupled with efficiency η_c to capsule
- Capsule energy $\eta_c E_D$ is converted with hydro (rocket) efficiency η_H into energy of imploding fuel
- Typically

Typically	Direct Drive (DD)	Indirect Drive (ID)
η_c	0.8	0.2
η_H	0.1	0.2

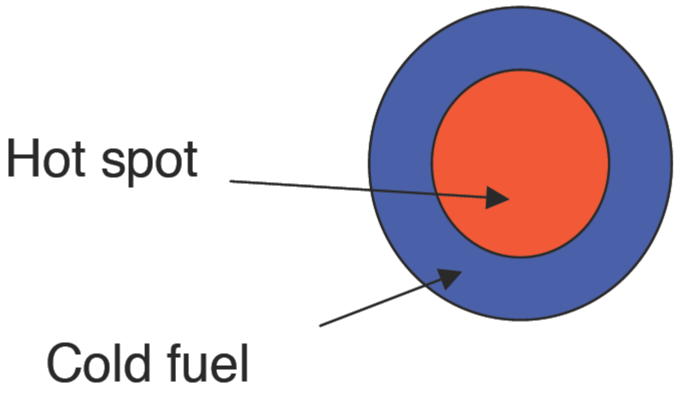
- So overall efficiency η_T is ~0.08 for DD and ~0.04 for ID
- Theoretical fusion energy gain seems high

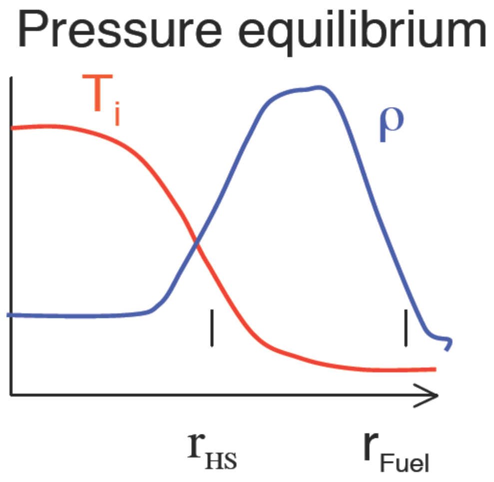
$$G \simeq \frac{17.6\ \mathrm{MeV}}{4\times {}^{3}\!/_{2}\, k_B T}\cdot\Psi \sim \frac{17.6\ \mathrm{MeV}}{6\times 5\,\mathrm{keV}}\Psi = 580\,\Psi \sim 190$$

- But overall target gain is $G_T \sim \eta_T G \sim 16$ (DD) and ~8 (ID). Considering the efficiency of heat conversion into electricity and electricity into driver energy, this is far too low.
- So volume heated DT cannot work for energy production.

Spark ignition

- Solution – heat a small part of the fuel to high *T* and fusion α particles heat the surrounding cold dense fuel and fusion burn wave propagates



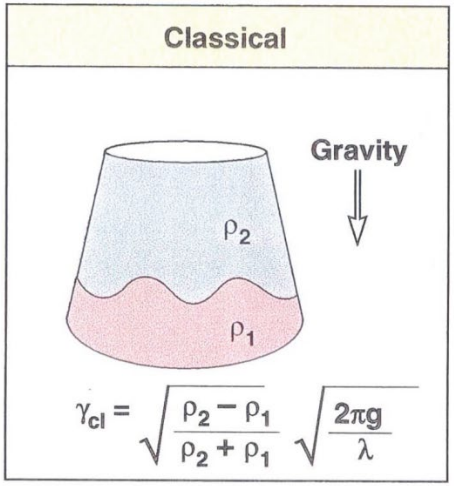


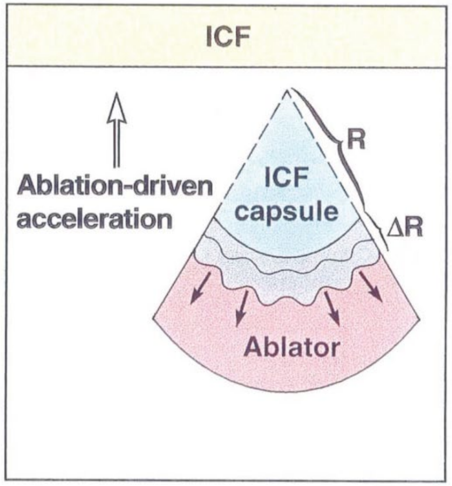
Isobaric fuel assembly

- This works fine in 1D spherical numerical simulations, but life is not 1D
- **Mixing must be avoided** of cold fuel into hot spot material
- Implosion symmetry is important issue, small non-unifomities are magnified when shell is imploded to very small radius
- Hydroinstabilities during implosion are major concern

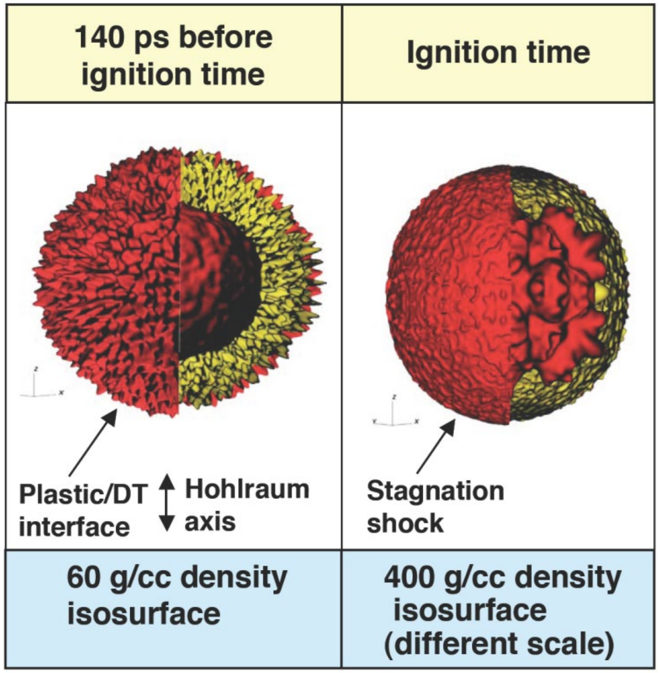
Rayleigh-Taylor instabilities (RTI)

- RTI is major concern
- RTI may appear where $\nabla\rho . \nabla P < 0$
- Classically interface between upper heavier fluid and lighter one below





- When approaching ablation surface from outside - density ↑ pressure ↓
- Deceleration phase - unstable region
- At stagnation – perturbations lead to mix of cold fuel with hot spot that can quench the burn
- 3D calculations are used to assess capsule performance in the presence of perturbations

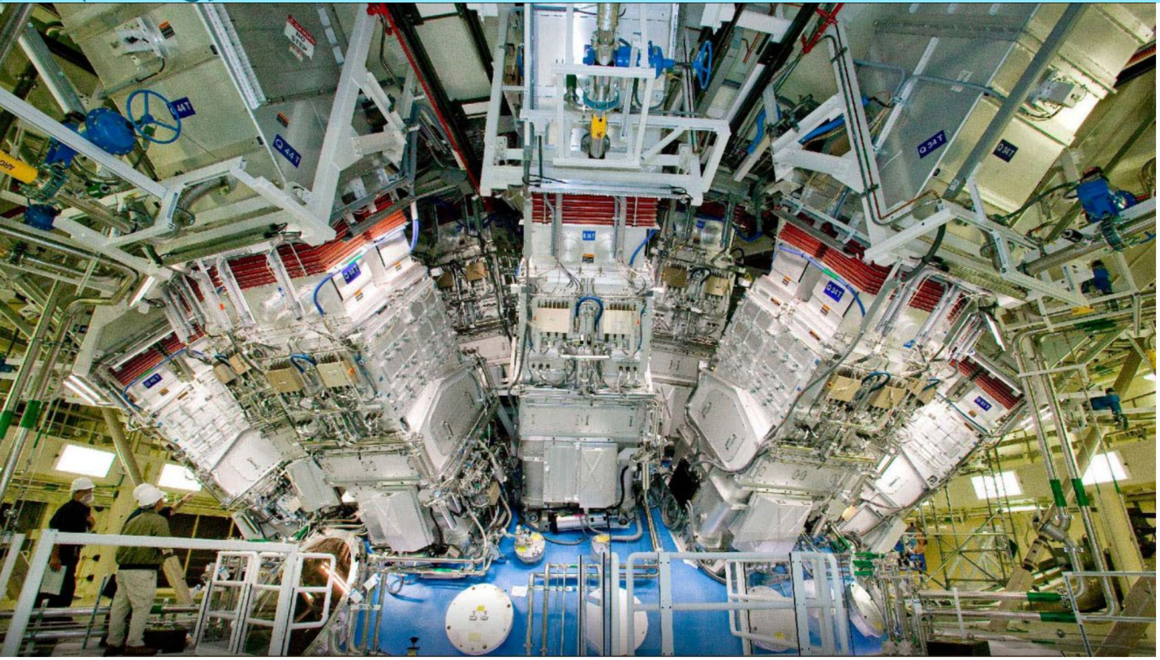


National Ignition Facility

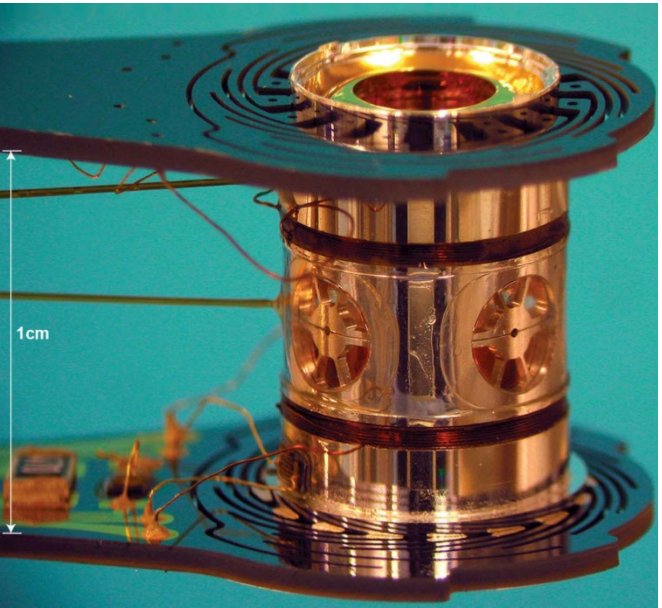
Similar lab. LMJ near Bordeaux starts operation now

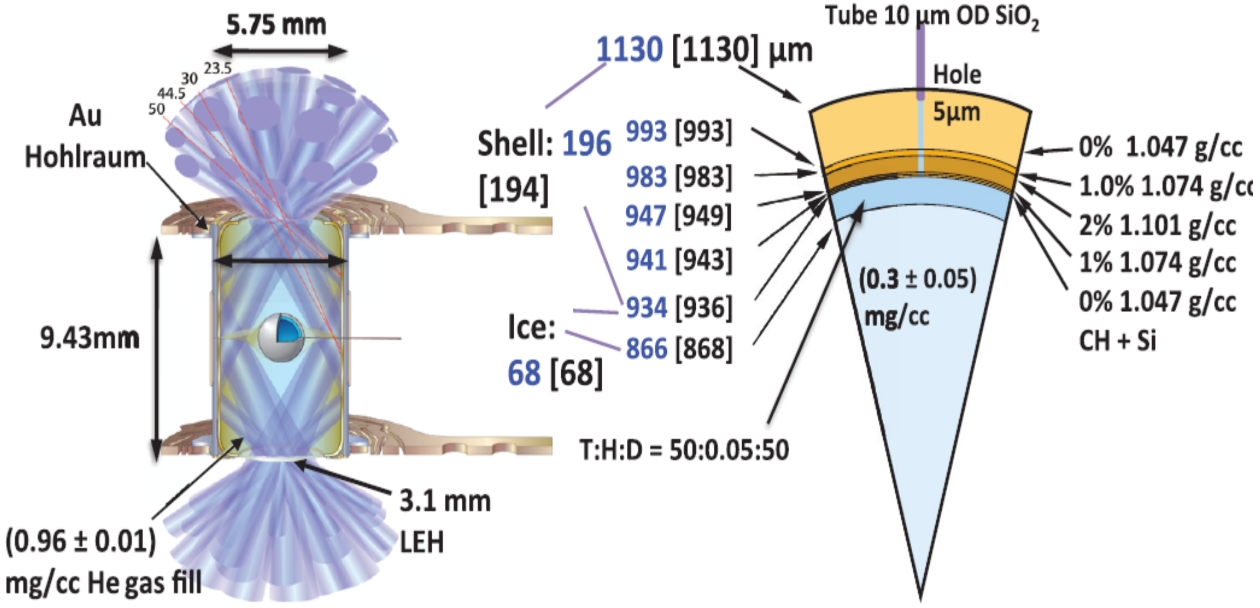
- 1 building, 5 hectares (2 soccer fields)
- Height 10-stores house
- 10 years construction
- 30 years operation
- > 4 G$ - financed for maintaining nuclear weapons stockpile
- Indirect drive - primary as similar to H bomb
- 192 beams of Nd-laser in 48 quads converted to 3ω – 1.8 MJ in 20 ns shaped pulses
- 1 shot/8 hours, $\eta < 1$ %
- Full energy 2009
- **Operates perfectly**

NIF interaction chamber and targets

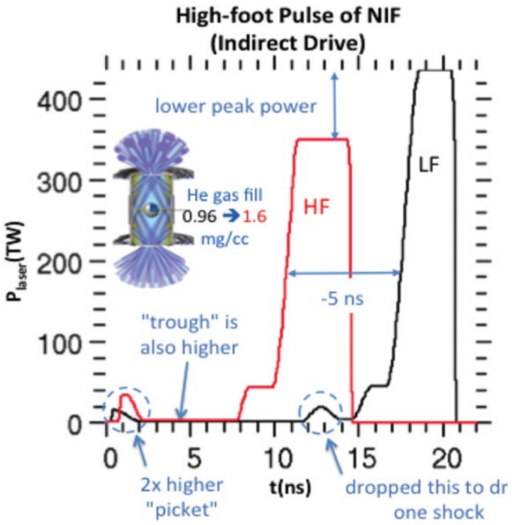


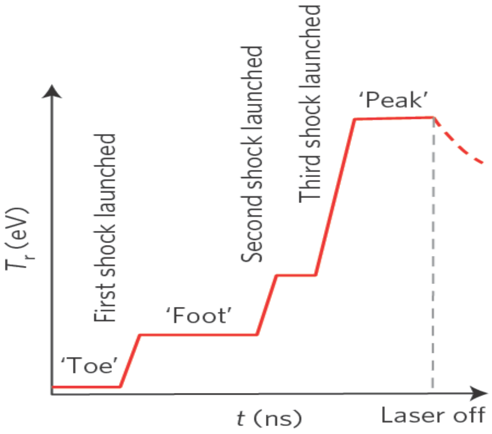
Target chamber ∅ 10 m
view from equator
(diagnostics and target)
Laser beams from upper
and lower side
Cryogenic target at 17.3 K
Hohlraum, lasers in 4 cones
Capsule with plastic layered
ablator doped by Si





Experiments

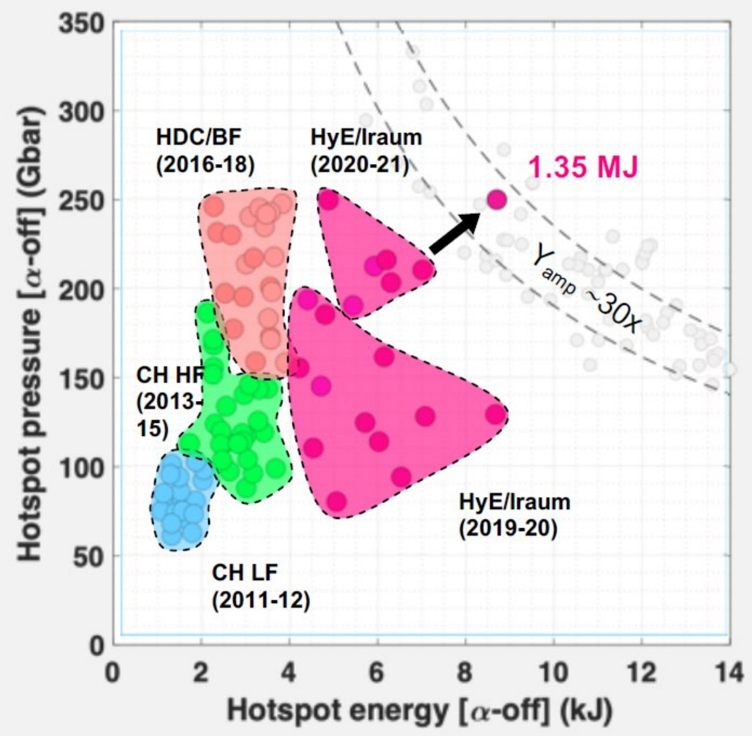




- Hohlraum and capsule must be perfectly matched with laser pulse, capsule precise shape
- The original scheme developed over 10 years was based on 4 shocks with low energy picket ⇒ **low foot** radiation $T_r \Rightarrow$ small 1st shock ⇒ to keep fuel at low adiabat ($P/P_{\text{Fermi}} \sim 1.45$)
- Outer cones laser λ is tuned to modify cross-beam energy transfer (CBET) and reach macroscopically symmetric capsule irradiation (time dependence still uncertain, modelling capability insufficient)
- In the point design – peak T_r = 300 eV, $v_{impl} \cong 370$ km/s, P= 375 Gbar, gain ~ 10 (5×10^{18} neutrons)
- Instabilities and fuel mix underestimated in simulations ⇒ max. fusion yield ~10^{15} neutrons
- Unstable growth of baroclinic vorticity ($\nabla\rho\times\nabla P/\rho^2$) seeded by the tent
- Partial cure – **high foot** + 3-shocks - stability ↑, gain ↑, predictability ↑ (price paid– adiabat ↑⇒ lower compression)

History of improvements

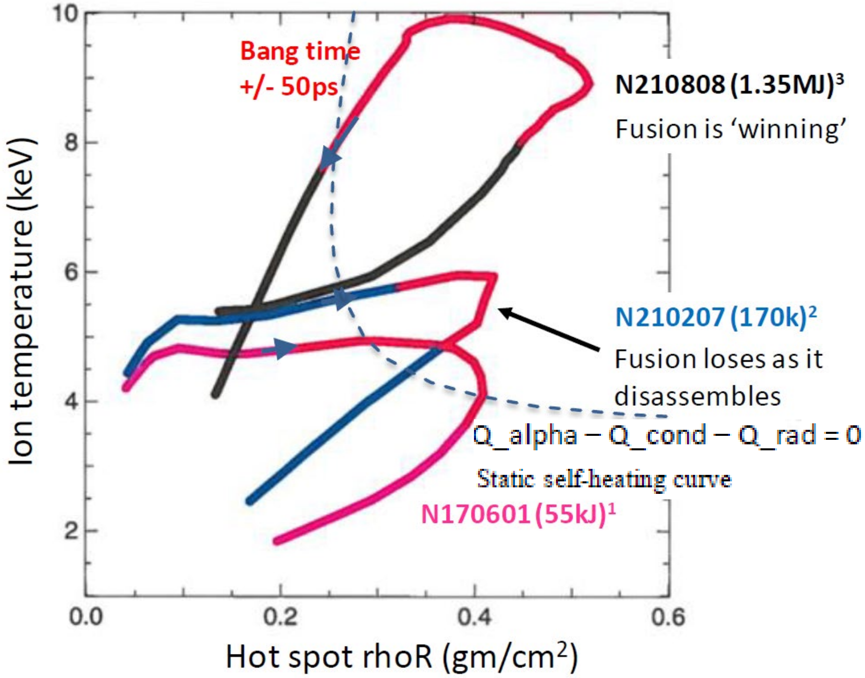
- **High foot** – foot T_r ~ 90 eV (1.5×T_r for low foot) to increase ablation velocity and density scale length ⇒ ablative RT instability is suppressed, but higher adiabat ($P/P_{Fermi} \cong 2.5$) reduces convergence ratio
- 1.9 MJ of 3ω radiation led to released fusion energy 26 kJ > **2x** fuel energy, doubling fusion yield due to **α-particle self-heating** (2013-2014)

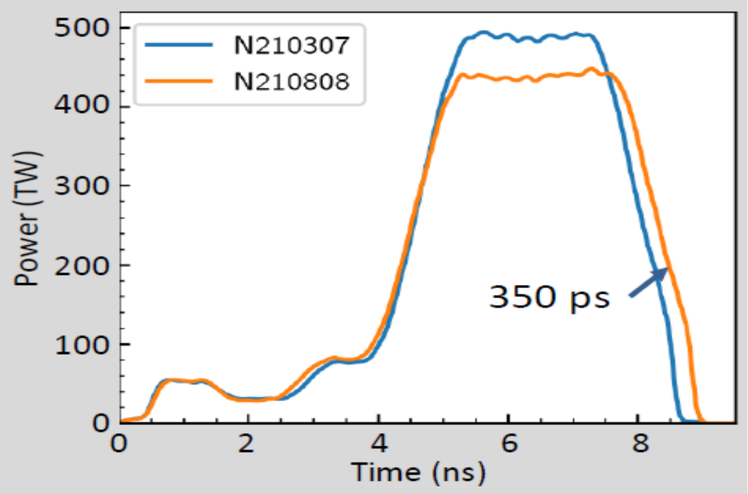


- **Diamond (HDC/BF) capsule** (+ lower gas fill of depleted uranium hohlraum) increased released fusion energy to 54 kJ in 2018
- **HybridE/Iraum** bigger capsule radius (910 → 1100 μm) in slightly bigger hohlraum (∅ 6.2 → 6.4 mm) led to higher hot-spot energy
- **HybridE** with radius 1050 μm 2020-21 used **frequency detuning** between inner and outer laser cone, Feb 21 yield ~170 kJ, burning plasma regime when up to bang time α **energy > work by pressure**

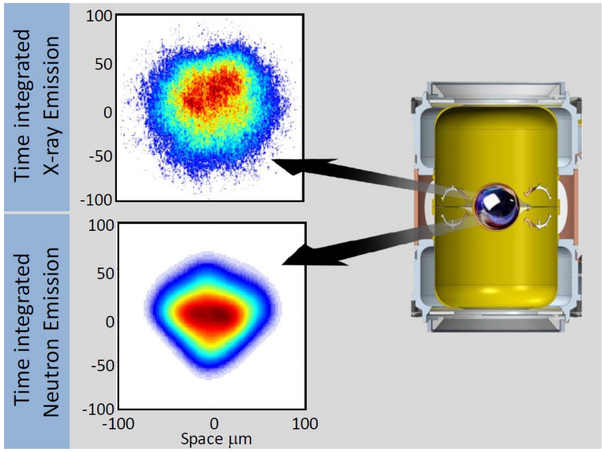
Ignition shot parameters

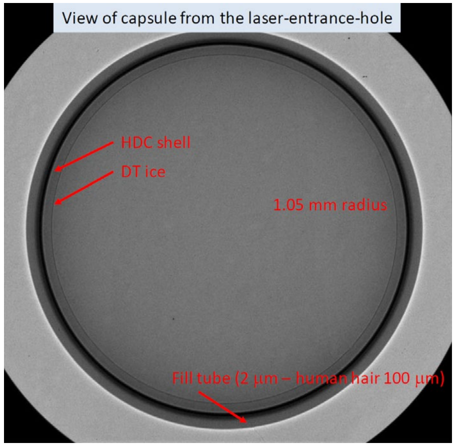
- Inertial fusion **ignition and burn** on Aug 8, 2021 – **fusion energy 1.3 MJ** still less than laser energy 1.9 MJ (breakeven not achieved)
- HDC (high-density carbon) shell capsule (3.48 g/cc) – shorter laser pulse
- Gold-plated depleted Uranium hohlraum
- Lower density He-gas fill (0.3 mg/cc)
- Wavelength separation (1-2 Å) between inner and outer laser cones
- Bigger capsule - 1050 μm inner radius, thickness 78 μm, inner 5 μm undoped, then 20 μm doped by W
- Thicker DT layer (65 μm)
- Narrower DT fill tube ∅ 2 μm
- The best quality capsule (pits and voids reduced 100× from 2018)
- Smaller laser entrance holes ∅ 3.1 mm (laser beams had to be repointed)
- Reduce coasting time (bang time – laser end) – lower max laser power - 440 TW

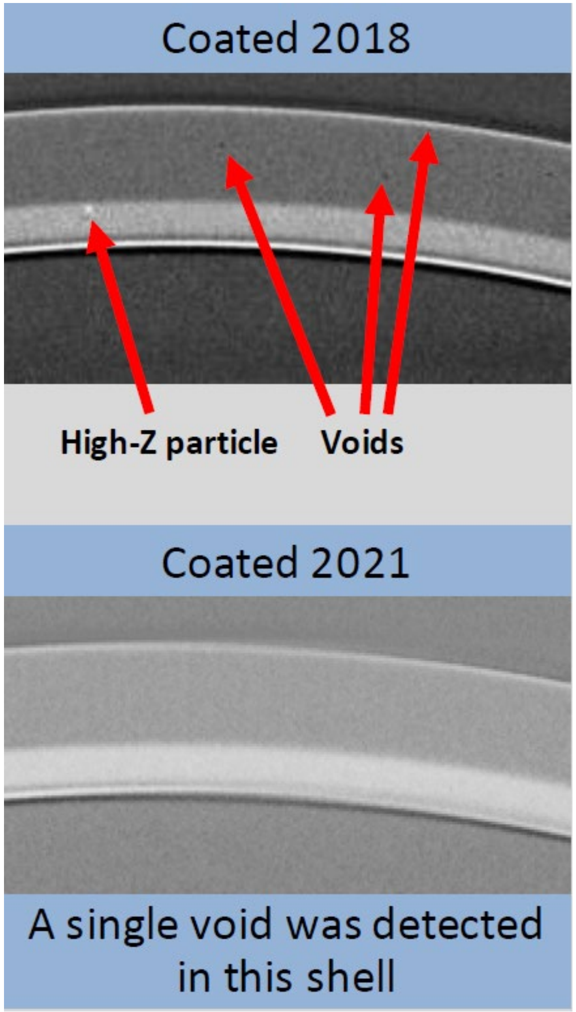




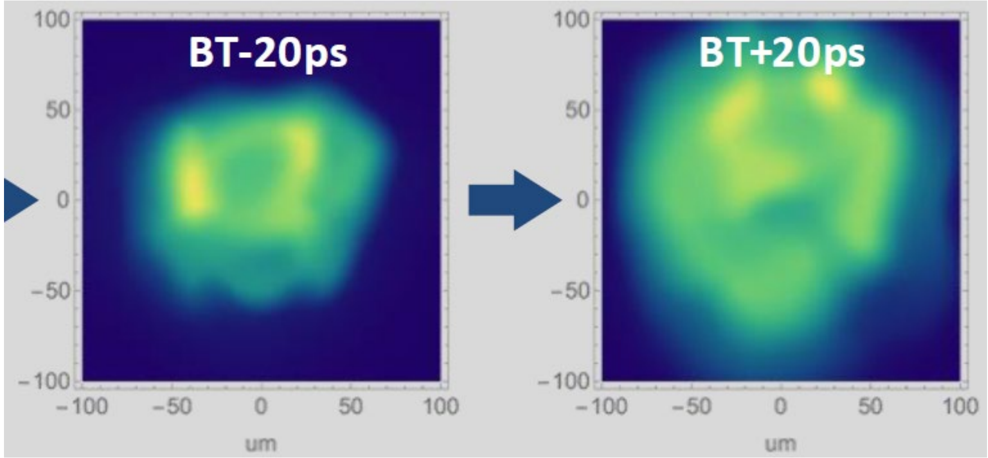
Ignition shot







250 diagnostics fielded



Measured X-ray emission in ignition shot 20 ps before after bang time (maximum compression)

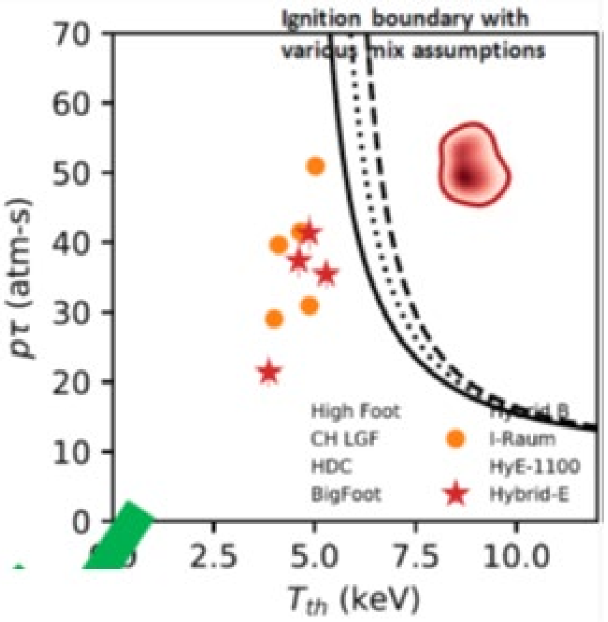
Ignition and breakeven shots

- Hot spot temperature evolution

$$c_{DT}\frac{dT}{dt} = f_\alpha Q_\alpha - f_B Q_B - Q_e - \frac{1}{m} p \frac{dV}{dt}$$

α-heating Q_α ~ $T^{3.6}$, work *pdV* heats hot spot before bang time and cools it after

- **Ignition - α-heating > losses**, 70–80% of α heats hot spot (∅100 μm), 20-30 % heats surrounding DT
- Max $\rho_c \approx$ 100 g/cc, $p_c \approx$ 450 Gbar, burn time ~90 ps
- α energy >250 kJ, radiation loss ≈60 kJ, work ≈20 kJ
- Capsule absorbed energy ≈ 230 kJ, capsule gain ≈ 6
- **Fusion energy 1.35 MJ**, power 15 PW, mix ~10%
- Burnt 2% of DT fuel (NIF energy input – 320 MJ/shot)
- 3 repeat experiments reach 25 – 50% fusion yield (but all > early 21, all > capsule absorbed energy)

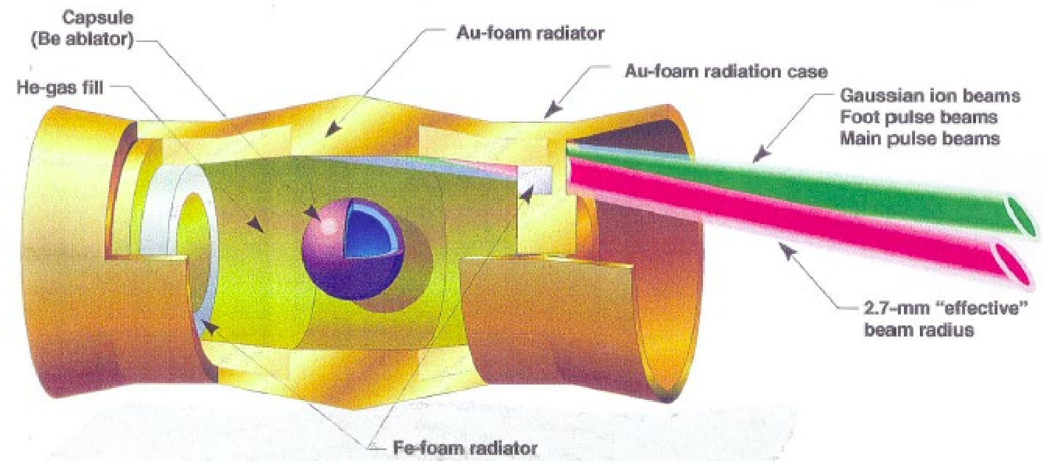


Breakeven (positive energy balance) – Dec 5, 2022

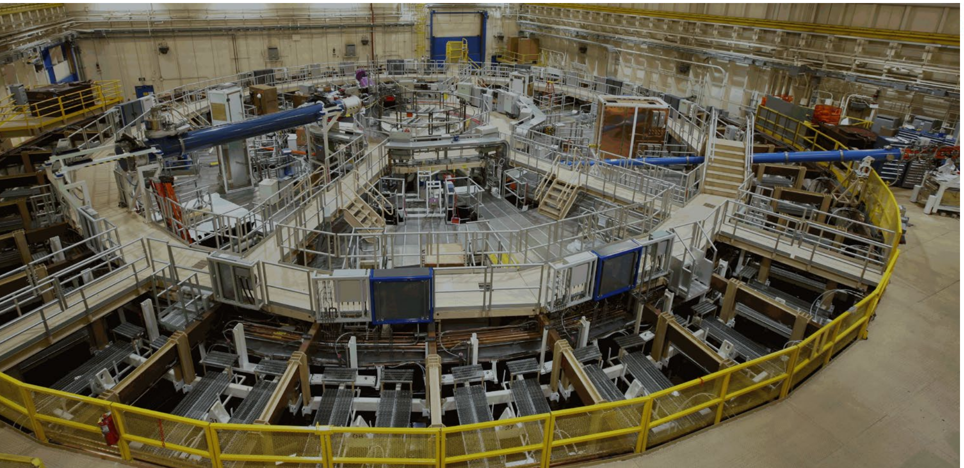
- Laser energy increased to **2.05 MJ**, released fusion energy **3.15 MJ** (gain ~1.5)
- 8% thicker capsule corresponding to laser energy increase, better protection against hydro instability growth
- Difference in laser wavelength increased form 0.25 to 0.275 nm

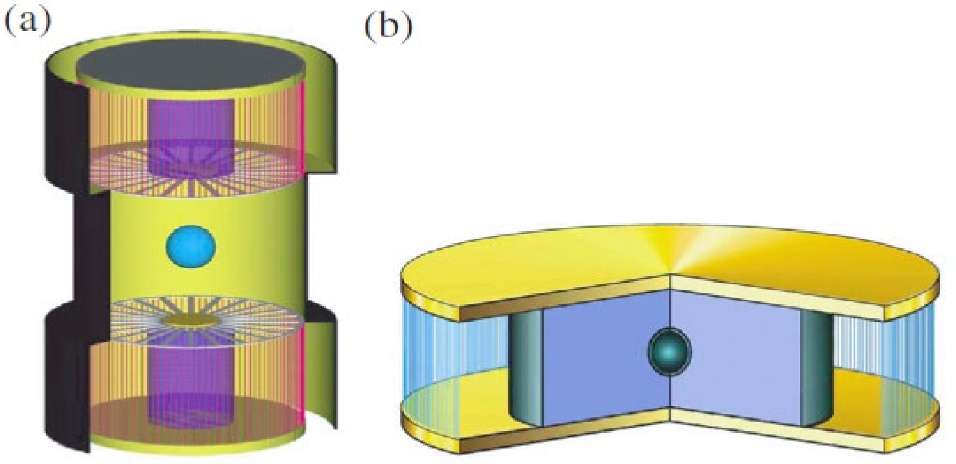
Alternative drivers

- **Nuclear explosion** – in Halite/Centurion program in 1980’s X-rays from underground nuclear test shined into a hohlraum and ignited inertial fusion in a capsule
- **Heavy ion beam** - driver with efficiency >40% and 10 Hz feasible, direct and direct-indirect schemes also possible, no big installation



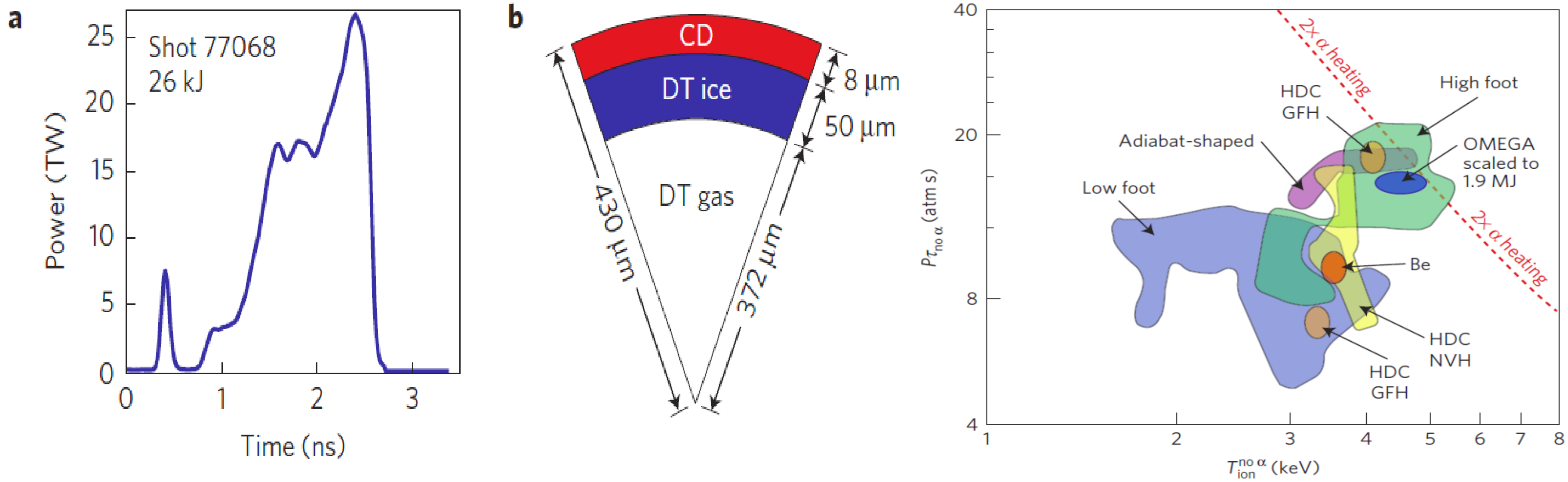
- **Z-pinch (wire array)** – Z-machine Sandia 2 MJ X-ray energy, 15% plug-to-X-rays efficiency





Direct drive

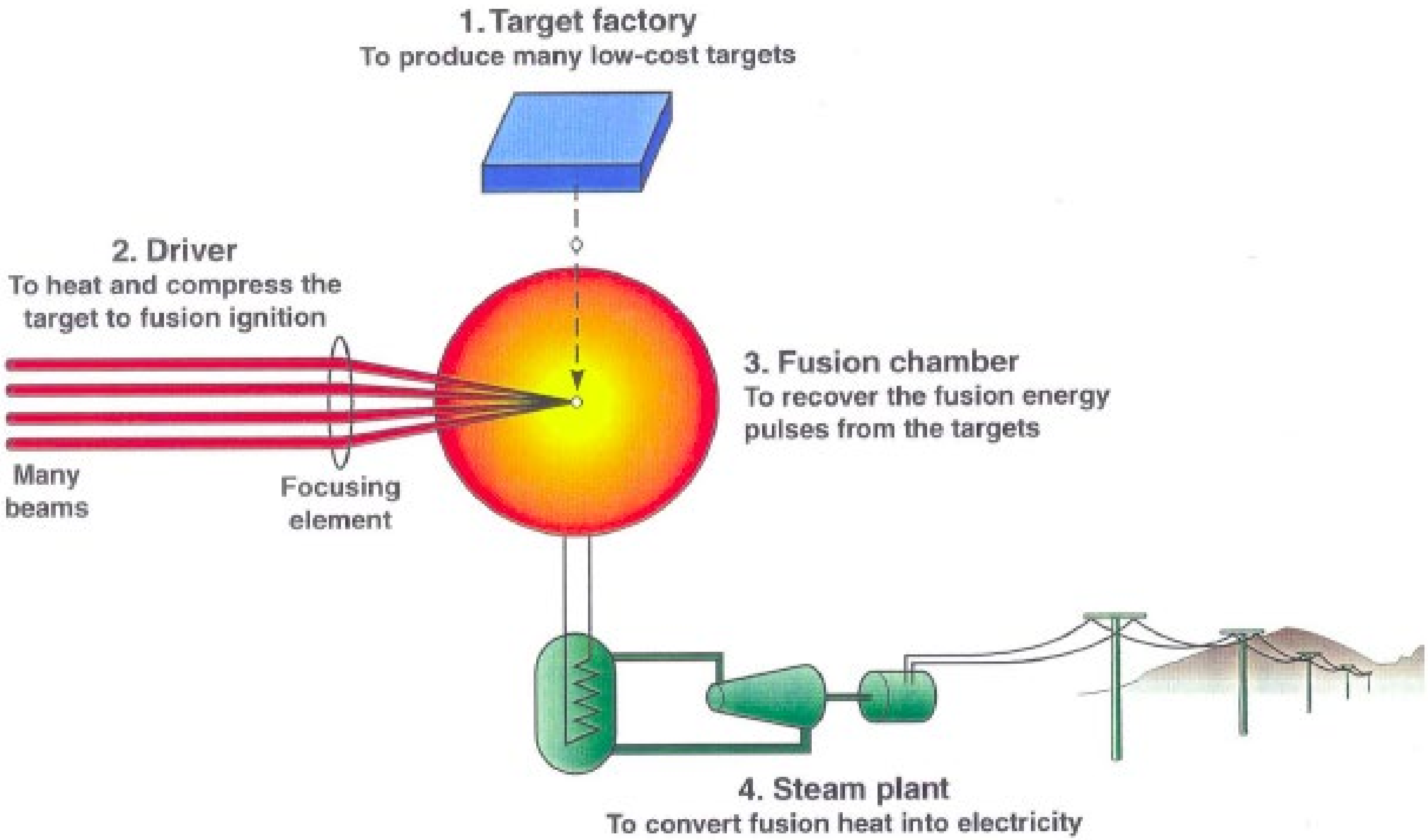
- Main experiments at Omega laser in LLE Univ. Rochester, USA
- 60 laser beams symmetrically irradiate cryogenic capsule
- Total laser energy 30 kJ, laser smoothing techniques adopted
- Intensity slightly < 10^{15} W/cm^2 to control laser-plasma instabilities
- Implosion performance scales hydrodynamically to ~2x α heating at NIF energy (higher η_C , but lower implosion quality)
- NIF – polar direct drive (symmetric impossible) – preliminary experiments started, enhanced losses due to CBET



Advanced fusion schemes

- Advanced schemes use external means to increase temperature of compressed (either by DD or ID) fuel
  - **Fast ignition (FI)** – energetic particle beam (electrons or ions)
  - **Shock ignition (SI)** – spherically convergent shocks
  - **Magnetized ICF** or magneto-inertial fusion (**MIF**) – magnetic fields
- The basic idea of FI and SI is to use long (ns) laser pulse for compression to reach sufficient ρR with low temperature and then to use short (ps) pulse to heat and ignite the fuel
- Though idea of decoupling compression and heating was proposed earlier, interest started with emergence of high-power ultrashort-pulse lasers (chirped pulse amplification)

Inertial fusion energy



1. Target factory
To produce many low-cost targets

2. Driver
To heat and compress the target to fusion ignition

Many beams

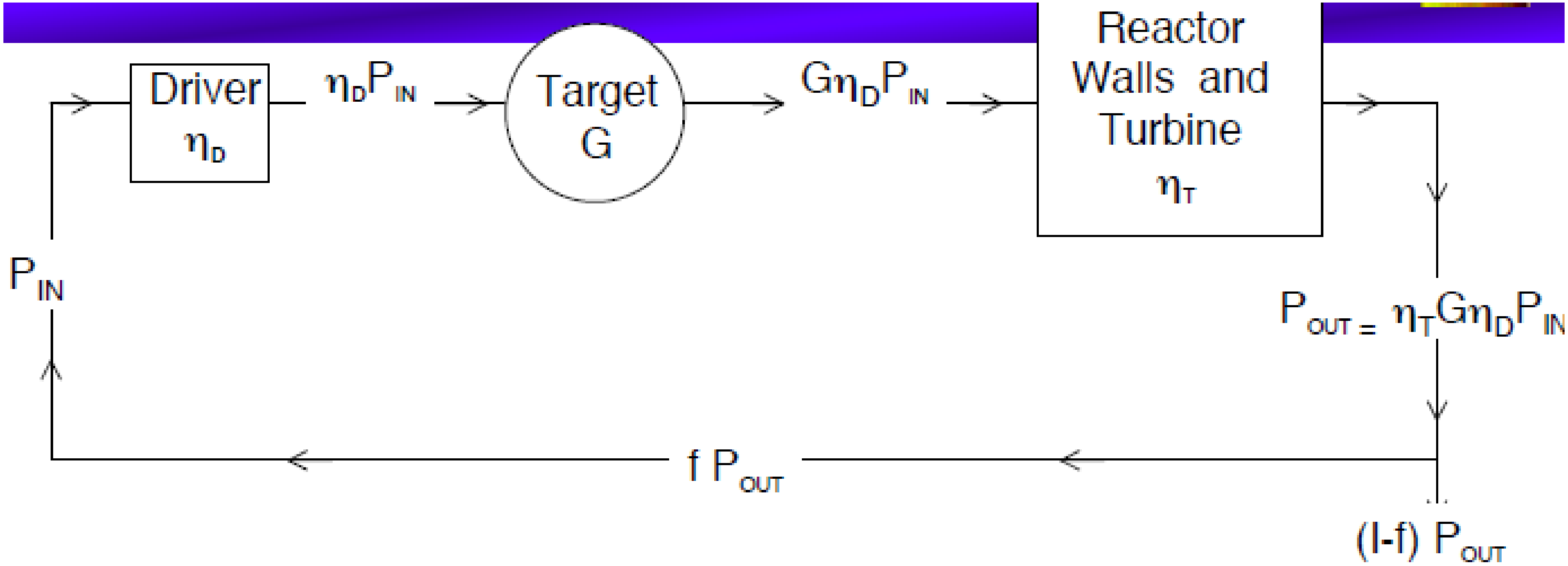
Focusing element

3. Fusion chamber
To recover the fusion energy pulses from the targets

4. Steam plant
To convert fusion heat into electricity

A power-plant driver would fire about five targets per second to produce as much electricity as today's 1000-megawatt power plant

IFE power balance



$$P_{IN} = f\,P_{out} = f\,\eta_T\,G\,\eta_D\,P_{IN} \Rightarrow f\,\eta_T\,G\,\eta_D = 1$$

The cycling power should not be too high, let $f = 0.25$

With $\eta_T = 0.4 \Rightarrow \boldsymbol{\eta_D\,G \geq 10}$

Representative numbers for (1-f) P_{out} = 1000 MW block and 10% driver efficiency (η_D) – $f = 0.23$, $P_{IN} = 300$ MW, driver 6 MJ, 5 Hz (30 MW), G = 100, output 3 GW (600 MJ), $\eta_T = 0.43$, $P_{out} = 1.3$ GW

Reactor target chamber

- 600 MJ is the energy released in explosion of 1/7 ton TNT
  - However, damage to chamber is caused by momentum p and $p = m\,v = (2\ \mathrm{E}\ \mathrm{m}\ )^{1/2}$ and for $m_{DT} = 5.4$ mg (burn fraction 1/3) is the momentum equivalent of explosion of $m_{\mathrm{TNT}} = 29\ \ g$
  - Protecting the first wall against radiation introduces more mass
- **Reactor chamber requirements**
  - Regenerate chamber conditions for target injection, driver beam propagation, and ignition at sufficiently high rates
  - Protect chamber structures for several to many years or allow easy replacement of inexpensive modular components
  - Extract fusion energy in high-temperature coolant, regenerate tritium
  - Reduce radioactive waste generation, inventory, and possible release fractions low enough to meet no-public-evacuation standards
- Chamber cost accounts for 7 – 15 % of power station cost

IFE power plant concepts

- Many concepts propose and analyzed, here just 1 example
- **HYLIFE–II** – heavy ion driver, uses oscillating liquid jets of FLIBE (a F, Li and Be molten salt) to protect fusion chamber from neutrons and also to produce Tritium